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FOR THE YEAR MDCCCLXXXV.

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Adjudication of the Medals of the Royal Society for the year 1885, by the President and Council.

The Copley Medal to Professor August Kekulé, For. Mem. R.S., for his researches in Organic Chemistry.

A Royal Medal to Professor David Edward Hughes, F.R.S., for his Electric and Magnetic Researches, and his invention of the Microphone and the Induction Balance.

A Royal Medal to Professor Edwin Ray Lankester, F.R.S., for his discoveries concerning the Embryology and Morphology of the Mollusca, and his services to Embryology and Animal Morphology in general.

The Davy Medal to Professor Jean Servais Stas, For. Mem. R.S., for his researches on the Atomic Weights.

The Bakerian Lecture, "On the Corona of the Sun," was delivered by Dr. Huggins, F.R.S.

The Croonian Lecture was not delivered.

III. *On the connexion between Electric Current and the Electric and Magnetic Inductions in the surrounding field.**

By J. H. POYNTING, *M.A., late Fellow of Trinity College, Cambridge, Professor of Physics, Mason College, Birmingham.*

Communicated by Lord RAYLEIGH, *M.A., D.C.L., F.R.S.*

Received January 31,—Read February 12, 1885.

IN a paper published in the Philosophical Transactions for 1884 (Part II., pp. 343–361), I have deduced from MAXWELL'S equations for the electromagnetic field the mode in which the energy moves in the field. The result there obtained is that the energy moves at any point perpendicularly to the plane containing the directions of the electric and magnetic intensities, and in the direction in which a right-handed screw would move if turned round from the positive direction of the electric intensity to the positive direction of the magnetic intensity. The quantity crossing the plane per unit area per second is equal to the product of the two intensities multiplied by the sine of the included angle divided by 4π.†

Hence it follows that the energy moves along the intersections of the two sets of level surfaces, electric and magnetic, where they both exist, their intersections giving, as it were, the lines of flow. In the particular case of a steady current in a wire

* [Added July 15.—Since the reading of the paper I have found a remarkable passage in FARADAY'S 'Experimental Researches,' vol. 1, p. 529, § 1659, which I give below. The words I have put in *italics* might be regarded as the starting point of the views which I have attempted to develop in this paper. "§ 1659. According to the beautiful theory of AMPÈRE, the transverse force of a current may be represented by its attraction for a similar current and its repulsion of a contrary current. May not then the equivalent transverse force of static electricity be represented by that lateral tension or repulsion which the lines of inductive action appear to possess (1304)? Then, again, *when current or discharge occurs between two bodies, previously under inductrical relations to each other, the lines of inductive force will weaken and fade away, and, as their lateral repulsive tension diminishes, will contract and ultimately disappear in the line of discharge.* May not this be an effect identical with the attractions of similar currents, *i.e.*, may not the passage of static electricity into current electricity, and that of the lateral tension of the lines of inductive force into the lateral attraction of lines of similar discharge, have the same relation and dependence, and run parallel to each other?"]

† I here adopt the simpler term "Electric Intensity," denoted by E.I., instead of "Electromotive Intensity," for the force which would act on a small body charged with unit of positive electrification. The magnetic intensity, *i.e.*, the force which would act on a unit north-seeking Pole, will be denoted by M.I.

where the electrical level surfaces cut the wire perpendicularly to the axis, it appears that the energy dissipated in the wire as heat comes in from the surrounding medium, entering perpendicularly to the surface.

In that paper I made no assumption as to the transfer of the electric and magnetic inductions—the electric and magnetic conditions—through the medium, merely considering the movement of energy. I now propose to develop a hypothesis as to the transfer of the inductive condition in the medium, and its movement inwards upon current-bearing wires.

The value of the electric induction at any point in an isotropic medium is equal to $K \times E.I./4\pi$, and the direction of the induction coincides with that of the intensity. MAXWELL terms this electric induction "displacement," but I think that "induction" is preferable, as it implies no hypothesis beyond that of some alteration in the medium, which can be described by a vector. The value of the magnetic induction is equal to $\mu \times M.I.$, and its direction coincides with that of the magnetic intensity.

If we symbolise the electric and magnetic conditions of the field by induction tubes running in the directions of the intensities, the tubes being supposed drawn in each case so that the total induction over a cross section is unity, then we have reason to suppose that the electric tubes are continuous except where there are electric charges, while the magnetic tubes are probably in all cases continuous and re-entrant.

In the neighbourhood of a wire containing a current, the electric tubes may in general be taken as parallel to the wire while the magnetic tubes encircle it. The hypothesis I propose is that the tubes move in upon the wire, their places being supplied by fresh tubes sent out from the seat of the so-called electromotive force. The change in the point of view involved in this hypothesis consists chiefly in this, that induction is regarded as being propagated sideways rather than along the tubes or lines of induction. This seems natural if we are correct in supposing that the energy is so propagated, and if we therefore cease to look upon current as merely something travelling along the conductor carrying it, and in its passage affecting the surrounding medium. As we have no means of examining the medium, to observe what goes on there, but have to be content with studying what takes place in conductors bounded by the medium, the hypothesis is at present incapable of verification. Its use, then, can only be justified if it accounts for known facts better than any other hypothesis.

The basis of MAXWELL'S *Electromagnetic Theory.*

MAXWELL'S Electromagnetic Theory rests on three general principles.

I. The first principle consists in the assumption that energy has position, *i.e.*, that it occupies space. The electric and magnetic energies of an electromagnetic system reside therefore somewhere in the field. It is an inevitable conclusion that they are present wherever the electric and magnetic intensities can be shown to exist. For

instance, suppose a small electrified body placed in a field where there is electric intensity; then the body will be acted on by force and will receive energy which appears as the energy of motion, the electric energy at the same time decreasing. If energy has position that which is now in the body must have come into it through the surrounding space, or it was present in that space before the body took it up. The alternative that it appeared in the body without passing through the space immediately surrounding the body need not be discussed. Hence the existence of electric intensity implies the existence of electric energy in the place where the electric intensity is capable of manifestation. Similarly magnetic energy accompanies magnetic intensity. The inductive condition of the medium imagined by FARADAY is due then to its modification when containing energy. MAXWELL has shown that all the energy is accounted for on the supposition that the electric energy per unit volume at any point is $K(E.I.)^2/8\pi$, and that the magnetic energy is $\mu(M.I.)^2/8\pi$. He has given in his 'Elementary Treatise on Electricity,' p. 47, another way of describing the distribution of energy which will be more useful for my purpose. If the field be mapped out by unit induction tubes—either electric or magnetic—*i.e.*, tubes drawn so that the total induction over every cross section of a tube is unity, and if these tubes be divided into cells of length such that the difference of potential or the line integral of the intensity between the two ends of each cell is unity, then each cell contains, if electric, half a unit of energy, if magnetic $\frac{1}{8\pi}$ of a unit, the divisor 4π being introduced by the difference in definition of the two inductions. MAXWELL terms these unit cells.

II. The second principle is in part experimental, viz.:—that the line integral of the electric intensity round any closed curve is equal to the rate of decrease of the total magnetic induction through the curve. This is verified by experiment when the curve is drawn through conducting material. MAXWELL supposes it to be true in all cases, that is, he supposes that electric induction can be produced in insulators by means of magnetic changes, without the presence of charges on conductors, and is therefore led to identify the growth and decrease of electric induction with current.

III. The third principle is also in part experimental, viz.:—that the line integral of the magnetic intensity round any closed curve is equal to $4\pi\times$ current through the curve. This is verified by experiment when the current is in a wire, and MAXWELL supposes it to be also true in the case where there is change of electric induction in an insulator. The supposition is justified by Prof. ROWLAND'S well-known experiment.

From these three principles MAXWELL deduces his general equations of the Electromagnetic Field. I have stated them in full as I propose to modify the second and third principles, and I wish to make quite clear the nature of the proposed changes.

Modification of the Second Principle.

I propose to replace the second principle by the following:—*Whenever electromotive force is produced by change in the magnetic field, or by motion of matter through the field, the E.M.F. per unit length or the electric intensity is equal to the number of tubes of magnetic induction cutting or cut by the unit length per second, the E.M.F. tending to produce induction in the direction in which a right-handed screw would move if turned round from the direction of motion relatively to the tubes towards the direction of the magnetic induction.**

In order that the results obtained from this should agree with those obtained from MAXWELL'S statement of the principle, it is necessary that change in the total quantity of magnetic induction passing through a closed curve should always be produced by the passage of induction tubes through the curve inwards or outwards. In some instances this is undoubtedly the case, as, for instance, where a part of a circuit moves so as to cut a fixed magnetic field, or where a magnet moves in the neighbourhood of a circuit. Here the E.M.F. is equal to the number of tubes cut by the wire per second, and its seat is that part of the wire cutting the tubes. In other cases, as, for instance, where the wire is between the poles of an electromagnet whose magnetising current is changing, we have no direct experimental evidence of the movement of the induction in or out. But the induction tubes are closed, and to make them thread a circuit we might expect that they would have to cut through the boundary. The alternative seems to be that they should grow or diminish from within, the change in intensity being propagated *along* the tubes. This would be inconsistent with their closed nature, unless the energy were instantaneously propagated along the whole length, and is further negatived by the theory of the transfer of energy, which implies that the energy flows transversely to the direction of the tubes. I shall suppose, then, that alteration in the quantity of magnetic induction through a closed curve is always produced by motion of induction tubes inwards or outwards through the bounding curve.

* Taking the electric intensity as always perpendicular to the plane of motion of the magnetic tubes through a point, and equal to the number cut per second by unit length of the normal to the plane of motion, we can easily show that the component of the intensity in any other direction will be equal to the number of tubes cut by a line of unit length in that direction. For let OA represent a small length drawn perpendicular to the plane of motion, and let OP represent a line drawn in any direction making θ with OA. Draw AP perpendicular to OA, and meeting OP in P. Then the same number of tubes will cut both OA and OP, since AP is parallel to their plane of motion. If the number cutting OA be $E \times OA$, where E is the number cutting unit length, and therefore equal to the resulting intensity, the number cutting unit length of OP will be $E.\frac{OA}{OP} = E \cos\theta$, or the component of the intensity along OP.

Modification of the Third Principle.

The third principle admits of similar analysis, according to which we may regard the magnetic intensity along a closed curve as due to the cutting of the curve by tubes of electric induction. If we regard the line integral of the magnetic intensity round a tube of induction as measuring the magnetomotive force—employing a useful term suggested by Mr. BOSANQUET—we may put the modification in the following form :—

Whenever magnetomotive force is produced by change in the electric field, or by motion of matter through the field, the magnetomotive force per unit length is equal to $4\pi\times$ *the number of tubes of electric induction cutting or cut by unit length per second, the magnetomotive force tending to produce induction in the direction in which a right-handed screw would move if turned round from the direction of the electric induction towards the direction of motion of the unit length relatively to the tubes of induction.*

This is the most general form of the principle, but we shall only require the more special statement which immediately follows from it : that the line integral of the M.I. round any curve is equal to $4\pi\times$ the number of tubes passing in or out through the curve per second.

We have reasons exactly similar to those given in the last case for supposing that any change in the total electric induction through a curve is caused by the passage of induction tubes in or out across the boundary. The alternative that change takes place by propagation from the ends, seems inconsistent with the theory of the transverse flow of energy.

I shall postpone the discussion of the modifications of the general equations of the electromagnetic field following from these changes in the fundamental principles, and proceed to discuss the bearing which they have upon the nature of currents in conductors.

A straight wire carrying a steady current.

Let AB represent a wire in which is a steady current from A to B. The direction of the electric induction in the surrounding field near the wire, if the field be homogeneous, is parallel to AB.

Let E be the value of the electric intensity, or the difference of potential per unit length perpendicular to the level surfaces, and let R be the resistance of the wire per unit length. Then $C=\frac{E}{R}$ where C is the current, and C is uniform throughout the circuit. The magnetic intensity in the immediate neighbourhood of the wire at a distance r from the axis of the wire is $\frac{2C}{r}$.

The hypothesis proposed as to the nature of the current is that C electric induction tubes close in upon the wire per second. The wire is not capable of bearing a

continually-increasing induction, and breaks the tubes up, as it were, their energy appearing finally as heat.*

Fig. 1.

Let us see how this hypothesis accounts for known facts, when aided by the two principles just laid down.

It accounts at once for the constancy of the current at all parts of the wire in the steady state, in so far as it reduces this constancy to a particular case of the law according to which there is the same total induction over all cross sections of a tube. If, for instance, there were more induction entering at A than at B, then more tubes must be entering at A, and so there would be an increase in the number of tubes left in the medium about B, or the field would not be steady.

Further, if we draw any closed curve embracing the wire once, we may apply the third principle to give us the line integral of the magnetic intensity round the curve. For this is a case where change is certainly going on in the electric field, and the magnetomotive force is due to this change. The field being steady, if C tubes enter the wire and are there broken up, C tubes must cross through any encircling curve to supply their place, or the line integral of the magnetic intensity round the curve is equal to $4\pi\times$ number of tubes passing through the boundary per second, *i.e.*, $4\pi C$. If the curve be a circle of radius r, with its centre in the axis and plane perpendicular thereto, the intensity at any point of this circle will be tangential to it, and equal to

$$\frac{4\pi C}{2\pi r}=\frac{2C}{r}.$$

The known constancy of the line integral of the magnetic intensity round the wire, which the hypothesis thus accounts for, almost seems to force the hypothesis upon us,

* May we not say that the tubes are dissolved. The term seems to suggest that the induction is not destroyed, but only loses its continuity. Probably this is the case; for on the electromagnetic theory of radiant energy, when the wire is heated, it sends out the energy it received, again in the electromagnetic form.

if we regard the field as caused by the inward flowing of the energy rather than by something propagated out from the wire.

Assuming that the induction tubes bring in their energy, the quantity is easily found. The number of unit cells per unit length is equal to the difference of potential per unit length, or E. Hence the energy per unit length of each tube is $\frac{E}{2}$, since each cell contains a half unit. If C tubes disappear in the wire per second they yield up $\frac{CE}{2}$ of energy per unit length. Now the total energy dissipated per unit length is CE per second. Or the movement inwards of the electric induction will only account for half of the energy. The other half must be accounted for by the movement inwards of the magnetic induction. This movement of the magnetic induction is suggested by the existence of electric induction, which cannot be ascribed to statical charges.

The electric intensity is E. Hence E tubes of magnetic induction must move in per second, cutting unit length parallel to the axis of the wire, in accordance with the second principle, and it will easily be seen that the inward motion gives the right direction of the electric intensity. The line integral of the magnetic intensity round a tube is $4\pi C$, the tubes being closed rings. Hence there are $4\pi C$ unit cells in the length. Since each of these contains $\frac{1}{8\pi}$ of energy the quantity per tube $=\frac{4\pi C}{8\pi}=\frac{C}{2}$. E tubes entering the wire per second will carry in $\frac{CE}{2}$ of energy, the other half to be accounted for.

We can in a similar manner trace the dissipation of the energy, which we must suppose taking place within the wire. The line integral of the magnetic intensity round a circle, with its centre in the axis of the wire, is constant up to the wire, and equal to $4\pi C$. Within the wire it gradually diminishes as the circle contracts. At a distance r from the centre it is $4\pi C.\frac{r^2}{a^2}$ when a is the radius of the wire. If we assume this intensity to be still due to the passage inwards of the tubes of electric induction only, $\frac{Cr^2}{a^2}$ cross inwards per second at a distance r, the difference between this number and the C tubes entering the outer boundary being destroyed and their energy dissipated. The energy thus dissipated per unit length between the outer boundary and a coaxal cylinder of radius r will be $\frac{EC}{2}\left(1-\frac{r^2}{a^2}\right)$ per second. If $r=0$ the whole of the electric energy is dissipated. It would appear, then, that we may represent the dissipation of the electric energy by the total destruction of the tubes all through their length.

The value of the electric intensity being E throughout the wire the number of tubes of magnetic induction cutting unit length parallel to the axis is the same at all parts, viz., E per second. Hence, the magnetic tubes are not destroyed as the electric tubes

are. But the line integral of the magnetic intensity round the tubes diminishes as they approach the axis, being $4\pi\frac{Cr^2}{a^2}$ round that at distance r. The number of unit cells diminishes, and, therefore, the energy per tube is less, the decrease being due to that dissipated. Thus the energy entering in the E tubes at the outer boundary is $\frac{4\pi CE}{8\pi}$ or $\frac{CE}{2}$. That crossing in E tubes at a distance r is $\frac{4\pi}{8\pi}\frac{Cr^2}{a^2}E=\frac{CE}{2}\frac{r^2}{a^2}$. The difference $\frac{CE}{2}\left(1-\frac{r^2}{a^2}\right)$ being dissipated.

Hence it appears that the energy dissipated per second may be represented as half electric half magnetic, the electric energy being dissipated by the breaking up of the tubes, and their disappearance while the magnetic energy is dissipated by the shortening of the tubes, and their final disappearance by contraction to infinitely small dimensions of the diameters of the rings by which we may represent them. At all points therefore outside and inside the energy crossing any surface may be represented as equally divided between the two kinds.

As we know the value of the induction at any point, or the number of tubes passing through unit area, and as we also know the number of tubes cutting the boundary it is easy, on the assumption that the tubes move on unchanged, to calculate their velocity. Of course this velocity is purely hypothetical, as we cannot examine minutely into the medium and observe what goes on there. Probably, if we could observe with sufficient minuteness we should find unevennesses in the induction. If the velocity of the tubes has any physical meaning it is that these unevennesses are carried forward with that velocity. To illustrate this let us suppose that we have water flowing through a glass tube at a steady rate. We have nothing to show that the water is moving past any point in the tube beyond its disappearance at the entrance and its appearance at the exit, but knowing the cross section of the tube, *i.e.*, the quantity of water in any part of it, and the quantity entering and leaving it is easy to assign a velocity to the water in the tube which shall account for the observed amount entering and leaving. This velocity is to a certain extent hypothetical. But if we examine the tube with a sufficient magnifying power to show particles of dust in the water the existence of the velocity receives a more direct proof. I do not know whether we should have any right to expect a similar proof of the motion of induction even if we had the means of observation.

To find the hypothetical velocity of the electric induction tubes let us calculate the number of tubes passing through a circular band with radii r and $r+dr$ and centre in the axis of the wire, and lying in a plane perpendicular to the axis. The intensity being E the induction is $\frac{KE}{4\pi}$, and therefore the area of cross section of each tube is $\frac{4\pi}{KE}$, since area × induction is unity. The number passing through the circular band is therefore $2\pi r dr\cdot\frac{KE}{4\pi}=\frac{KE r dr}{2}$.

Since C tubes move in through the inner circle per second, $\frac{KEr dr}{2}$ tubes move in in $\frac{KEr dr}{2C}$ of a second, *i.e.*, all the tubes passing through the band will have just moved in in this time. The outermost tubes therefore describe the space dr in time $\frac{KEr dr}{2C}$, or the velocity is $\frac{2C}{KEr}$. Now we know that if R be the resistance per unit length $C=\frac{E}{R}$. Hence we may put the velocity in the form

$$\frac{2}{KR}\frac{1}{r}$$

which is independent of the current.

To take a special case let us calculate the velocity just outside the boundary of a copper wire, the specific resistance of copper being 1642 in electromagnetic measure. Then if a be the radius of the wire

$$R=\frac{1642}{\pi a^2}$$

and $K=\frac{1}{v^2}$ where v is the ratio of the units, which in air may be taken as 3×10^{10}.

Then the velocity

$$=\frac{2v^2\pi a^2}{1642a}$$

$$=\frac{2\times9\times10^{20}\pi a}{1642}$$

$$=345\times10^{16}a$$

At greater distances the velocity will be less, diminishing according to the inverse distance.

The hypothetical velocity of propagation of the magnetic induction may be calculated in a similar manner. The intensity at a distance r from the axis is $\frac{2C}{r}$ and the induction is $\frac{2\mu C}{r}$. The area of each tube is therefore $\frac{r}{2\mu C}$, and the number lying in a ring of rectangular section with depth unity and internal and external radii r and $r+dr$, will be $1\times dr\div\frac{r}{2\mu C}=\frac{2\mu C dr}{r}$.

But E tubes move in per second through the inner face of the ring, so that $\frac{2\mu C dr}{r}$ tubes move in in time $\frac{2\mu C dr}{Er}$, or this is the time taken by the outermost tubes to move across the ring describing a distance dr. The velocity is therefore

$$\frac{Er}{2\mu C}=\frac{R r}{2\mu}$$

which is again independent of the current.

If the current-bearing wire is copper, $R=\frac{1642}{\pi a^2}$, and with $\mu=1$ the velocity becomes

$$\frac{1642r}{2\pi a^2}$$

We cannot assign a velocity to the electric tubes within the wire since the number is diminishing as their energy dissipates. But the magnetic tubes crossing unit length parallel to the axis are still unchanged in number, so that we may assign a velocity to them. This velocity means that with the known value of the magnetic induction this velocity will give the number crossing inwards required to produce electric intensity E.

The velocity will be found equal to $\frac{Ea^2}{2\mu Cr}$ or $\frac{Ra^2}{2\mu r}$.

In the case of a copper wire this becomes

$$\frac{1642}{2\mu\pi r}$$

Discharge of a condenser through a fine wire.

Let us suppose that we have a condenser consisting of two parallel plates A and B and charged with equal and opposite charges. Then we know that there will be electric induction between the two plates, and that according to MAXWELL'S theory the energy of the system is stored there. We may form an idea of the distribution of the energy by drawing the unit induction tubes, each starting from and ending in unit quantity of electricity, and dividing these into unit cells by the level surfaces, drawn at unit difference of potential (fig. 2). If the dimensions of the plates be

Fig. 2.

great compared with their distance apart, then nearly all the cells will be between the two plates, and since each cell contains half a unit of energy, nearly all the energy is there. There will, however, be slight induction, and therefore some small quantity of energy in the surrounding space.

Now let the two plates be connected by a wire. Discharge takes place, and we are fairly justified, from the heat in the wire and the transient magnetic effects, in saying that a current has been in the wire from the positive to the negative plate, or the wire was for the time being in the same relation to the surrounding medium as the wire in the case just considered, the condition of affairs, however, not being steady.

Let us suppose the wire to have a very great resistance, in order that, at least in imagination, we may lengthen out the time of discharge. On the ordinary current theory, combined with MAXWELL'S "displacement" theory, the medium between the plates has returned from the strained condition, denoted by "displacement" from the positive to the negative plate, causing displacement through the plates and along the wire, the displacement being in the same direction all round the circuit. This is generally, I think, supposed to take place by the recovery of the medium between the plates causing displacement in the metal immediately in front of it, the displacement being analogous to the forcing of water along a pipe corresponding to the plates and wire, by the recovery from strain of some substance placed in a chamber corresponding to the space between the plates.

According to the hypothesis here advanced we must suppose the lessening of the induction between the plates—induction being used with the same physical meaning as MAXWELL'S displacement—to take place by the divergence outwards of the induction tubes. We may picture them as taking up the positions of successive lines of induction further and further away from the space between the plates, their ends always remaining on the plates. They finally converge on the wire, and are then broken up and their energy dissipated as heat. At the same time some of the energy becomes magnetic, this occurring as the difference of potential between the plates lowers, so that the tubes contain fewer unit cells.

The magnetic energy will be contained in ring-shaped tubes which will expand from between the plates and then contract upon some other part of the circuit. To illustrate the movement of the electric induction tubes let us suppose them to be represented by elastic strings stretched between the two plates. Then the motion of the tubes outwards would be roughly represented by pulling the elastic strings outwards and doubling them back close against the wire, their ends being still attached to the plates. It is evident that if any ring surround the wire each of the strings must break through it in order to reach the wire. Hence the total number of strings cutting any ring surrounding the wire is the same wherever the ring be placed. Similarly the total number of tubes of electric induction cutting any curve encircling the wire is the same, and therefore the line integral of the magnetic intensity round the curve integrated throughout the time of discharge is the same, or the total magnetic effect is the same at all parts of the circuit. It is not necessary to suppose that a tube enters the wire at the same moment throughout its whole length; indeed, the experiments of WHEATSTONE on the so-called velocity of electricity prove clearly that

this is not the case, for in those experiments the tubes reached air breaks near the two ends of the wire before they reached a break in the middle.

We cannot by this general reasoning show that the energy entering any length of the wire will be proportional to the resistance of that length—the result obtained by RIESS. Indeed, this cannot always be the case. For instance, imagine a condenser discharged by two wires connected to the two plates of another condenser of greater capacity, whose plates are again connected by a fine wire of enormous resistance, through which the discharge can only take place slowly. Then the energy dissipated in the wires will not to a first approximation depend on their resistances but on the ratios of the capacities, that in the wire of high resistance bearing to that in the other wires the ratio of the less capacity to the greater. Probably RIESS's results only hold when the discharge takes place in such a way that it may be looked upon at any one moment as approximately in the steady state.

We have shown that the magnetic measure of the total current is the same all along the wire. Probably also the chemical measure is the same—meaning by the chemical measure whatever interchanging or turning round of molecules may occur when induction takes place in a conductor. For even if a tube does not enter the wire at the same time throughout its length, an end part, say, entering first, the point of attachment of the tube to the conductor being transferred from the plate to somewhere along the wire, this transference of the point of attachment from molecule to molecule implies the same amount of chemical change within the wire as if the tube entered all at the same moment. It will not, however, take place equally throughout the cross section as it does in the steady state.

Probably we only have the simultaneous disappearance of all parts of a tube when the wire follows a line of electric induction, and has its resistance per unit length proportional to the intensity which would exist there if the wire were removed.

The hypothesis here advanced is in accordance with MAXWELL'S doctrine of closed currents. For the induction dissipated at one part of the circuit has come there from another part where relatively to the circuit it ran in the opposite direction. The total result is equivalent to the addition of so many closed induction tubes to the circuit, the induction running the same way relatively to the circuit throughout.

If the two plates of the condenser are not connected by a wire but are discharged gradually by the imperfect insulation of the dielectric, then we must suppose that the tubes of induction in this case are dissipated *in situ*, the induction simply decaying at a rate depending on its amount and upon the conductivity of the dielectric. We may still represent this process by a closed current by regarding the loss of induction $\left(\text{MAXWELL'S } -\frac{df}{dt}\right)$ and the quantity of induction dissipated (MAXWELL'S p), as two different quantities. We have then $p+\frac{df}{dt}=0$ or we have two equal and opposite currents. But this seems artificial. It is more natural to look upon the process

merely as a decay of electric induction without movement inwards of fresh induction tubes, and therefore without the formation of magnetic induction.

I have discussed the case of discharge of a condenser at some length, as we can here realise more easily what goes on at the source of energy. The results obtained suggest that a similar action occurs at the source of energy or seat of the electromotive force in other cases where we do not know the distribution of induction, and are obliged to guess at the action.

A circuit containing a voltaic cell.

We may pass on from the discharge of a condenser to the consideration of the current in a circuit containing a voltaic cell. The chemical theory of the cell will be here adopted—in fact, the hypothesis I am endeavouring to set forth has no meaning on the voltaic metal-contact theory.

Let us suppose the cell to consist of zinc and copper plates, a vessel of dilute sulphuric acid, and copper wires attached to each plate which on junction complete the circuit. For simplicity I shall disregard the effect of the air and suppose that it is a neutral gas causing no induction.

We shall begin by supposing the circuit open. Then we know that on immersion there will be temporary currents in the wires, the quantities of these currents depending on the electrostatic capacity of the system composed of the wires. The currents last till the wires have received charges such that they are, say at difference of potential V. If the terminals are connected to a condenser the temporary currents may be easily detected by a galvanometer in the circuit. They are in no way to be distinguished in kind from the permanent current which will be established when the circuit is complete, except that they are of short duration and in general very small. There is no reason then to suppose that the action in the cell is different from that which takes place when the current is permanent, and I think we may safely assume that FARADAY'S law of electrolysis holds according to which the quantity of electricity flowing along either wire is proportional to the quantity of chemical action—or, in the form appropriate here, the number of tubes of induction produced is proportional to the quantity of chemical action.

Let Q be the total quantity of electricity upon the positive terminal; then $\frac{QV}{2}$ is the total energy thrown out into the dielectric.

Let z be the quantity of zinc consumed per unit of electricity, then Qz is the total quantity consumed in the charging of the terminals. Let E be the energy set free by each quantity z of zinc consumed, after all actions in the cell have been provided for. E then is the E.M.F. which the cell will have on the closure of the circuit, as long as the chemical actions remain the same, for z corresponds to the passage of a unit of electricity or the production of one tube, and we know that the energy set free by C units is CE.

Now while the charges are gathering and while the potential difference of the terminals is gradually increasing, the energy required to add equal increments of charge will also increase, and the charging will cease when the amount of energy given up by a given amount of chemical action in the cell is equal to the amount required to add the corresponding charge to the terminals. For to suppose the action to go beyond this is to suppose that the energy thrown out into the space between the terminals is greater than that yielded by the battery.

Let dQ be the last quantity of charge added to the terminals. This requires energy $V dQ$.

The corresponding quantity of zinc consumed is $z dQ$, giving up energy $E dQ$.

The condition of equilibrium is that

$$V dQ = E dQ$$

or

$$V = E$$

which agrees with the result of experiment that the difference of potential of the terminals in open circuit is equal to the E.M.F. of the cell immediately after closure.

It may be noticed that the total quantity of energy extracted from the battery is

$$QE = QV$$

while the electric energy left in the medium is

$$\frac{QV}{2}$$

or half the energy has been converted into heat in the wires.

We will now consider the distribution of level surfaces in the field while the circuit is still open. There will be $V-1$ surfaces between the terminals dividing each tube into V cells. None of these will cut the homogeneous parts of the circuit, since the whole of each of these must be at one and the same potential.

They can only cut the circuit by passing through the regions where there is contact of dissimilar bodies. We will neglect the contact of the zinc and copper, as the difference of potential there is insignificant compared with that at the two surfaces, zinc-acid and copper-acid.

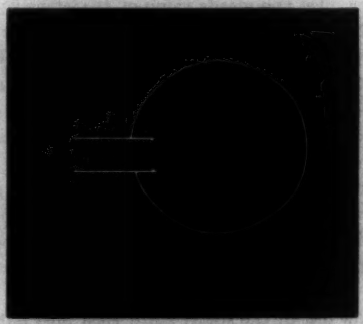
Now we know that the energy of the cell is put out at the zinc-acid contact, but the amount is greater than that obtained from a consideration of the E.M.F. of the cell, for some energy is absorbed again, probably, at the copper-acid contact in the evolution of hydrogen. There is probably, then, induction between the acid and the zinc, and between the acid and the copper, these resembling the spaces between the plates of two condensers, the acid being at a higher potential than either. But if a given amount of induction disappears from the zinc-acid contact and appears at the terminals, more energy is lost at the former than appears at the latter. Hence all the cells have not been transferred from one to the other, or the difference of potential

zinc-acid is greater than V. Then more than V—1 level surfaces pass between the zinc and the acid, the excess over V—1 going round and passing between the copper and the acid, somewhat as in fig. 3, where A, B, are the metal plates. The surfaces

Fig. 3.

are roughly sketched and numbered, on the supposition that the zinc terminal is at 0, the copper at 5, and the acid at 8. They have probably the same shape as those which would be produced by condensers at A and B with the wires attached, respectively, to one terminal of each, the other terminals being connected together and the charges adjusted, so that the difference of potential of the two terminals at A was 3, while that at B was 8.

Fig. 4.



Let us now suppose the circuit closed. Then the level surface will cut the circuit at various points, somewhat as in fig. 4.

The energy being dissipated in the wire, the cell will continually send out fresh energy, the induction tubes, which proceed from the acid to the zinc, diverging outwards in the same way as described in the discharge of a condenser. They bend round, and finally go into the circuit, the energy they carry being used for the necessary molecular changes, and finally appearing as heat in the circuit—except at the copper-acid contact where there is a crowding in of level surfaces, and therefore a convergence of more energy, which is required to set the hydrogen free.

At the same time magnetic ring-shaped tubes will be continually sent out from the zinc-acid contact, expanding for a time and then contracting again on various parts of the circuit and also giving up their energy.

There is, therefore, a convergence of tubes of electric induction on the circuit, running in the same direction throughout, viz., from copper to zinc outside the cell, and from zinc to copper inside, except between the zinc and acid, where there is a divergence of tubes in which the induction runs in the opposite way. But a divergence of negative tubes causes magnetic intensity in the same direction as, and may therefore be considered as equivalent to, a convergence of positive tubes. The current may therefore be said to go round the circuit in the same way throughout.

The tendency to a steady state in which the current or the number of induction tubes broken up per second is the same at all parts of the circuit, admits of simple explanation. We know, as the result of experiment given by OHM's law, that $C=\frac{E}{R}$ where R is the resistance per unit length and E the electric intensity. Until we can explain the molecular working of the current, *i.e.*, the mode in which the induction tubes are broken up, we must accept OHM's law as a simple fact. Let us suppose that we have not yet arrived at the steady state, so that in some part of the circuit the electric intensity is less than in the steady state, while in another part it is equal to it or greater. Let the steady value of the intensity be E, the actual value in the former part E′, and in the latter E″. By OHM's law the number of tubes absorbed by the wire per second is given by $C'=E'/R$, and $C''=E''/R$, in the two parts respectively, so that $C'<C''$ since $E'<E''$ or less tubes are being destroyed in the first than in the second part. But all the tubes are sent out from the source of the energy, and are only destroyed in the circuit, being otherwise continuous and with their two ends in the circuit. Hence, if more tubes are destroyed at one part than another, the parts of the tubes not yet destroyed will gather in the medium surrounding the part where fewer are destroyed, increasing the induction there, and so raising the intensity in the wire and therefore the number of tubes destroyed. The field can evidently only be steady when the number of tubes destroyed in all parts of the circuit is the same.

But it does not follow that in the steady state each tube enters the wire along its whole length at the same moment. This would imply that the axis of the wire is a line of electric induction perpendicular everywhere to the level surfaces. If we draw

the level surfaces due to the seats of induction at the contacts of acid and metal, they will probably be somewhat as drawn in fig. 4. If now the wire is not so arranged as to follow with properly adjusted resistances a line of induction for these surfaces, but pursues an irregular course, then the level surfaces will be much distorted, and the distribution of the induction will be greatly altered.

We may ascribe this alteration to a distribution of electricity along the wire, the quantity in any small area on the surface of the wire being equal to the difference between the number of tubes which have entered and the number which have left that area since the beginning of the system. We have a familiar example of this in the charging of deep-sea cables. Another example is afforded by a condenser with terminals connected to two points in the circuit. The plates of the condenser are then virtually parts of the circuit.

The effect of a junction of two wires, say of the same diameter, but of different specific resistances, upon the level surface will resemble that of a charge upon the separating surface. This can be seen in a general way from the fact that the level surfaces must cut the wire with the higher specific resistance at intervals shorter than those at which it cuts the other wire.

If there be an insulated conducting body, say a metal sphere, near the circuit, we know that in the steady state there is no electric intensity, and therefore no current within it; consequently there is no movement of energy and no movement of induction through it. We can see how this condition is arrived at. As the first tubes of electric and magnetic induction come up to the sphere they will enter it, and the parts of the electric induction tubes thus entering will be broken up, causing a transient current in the sphere. The parts of the tubes left in the medium will end on the sphere giving a negative charge on the end nearer the regions of higher potential, and a positive charge on the end nearer the regions of lower potential. This will go on until such charges have accumulated that the sphere becomes itself a level surface. When this point is reached no more energy can enter the sphere, and the parts of the magnetic tubes within it cease to move.

The charges formed on the wire or on neighbouring conductors are to be distinguished from ordinary statical charges in this: that their existence depends on the existence of the current, and therefore on the motion of magnetic induction. If the current is stopped by a break in the circuit, so that the motion of the magnetic induction ceases, the electric induction ceases and the charges are all lost. We should expect, therefore, to find that these charges can be described in terms of the magnetic motions which have occurred and are occurring in the system.

Current produced by motion of a conductor in a magnetic field.

We may explain by general reasoning the production of a current by motion of a part of a circuit so as to cut the tubes of magnetic induction. We will consider the

simple case of a slider AB, fig. 5, running on two parallel rails, AC BD, with a fixed cross piece CD, the tubes of magnetic induction running from above downwards through the paper. Let AB move so as to enlarge the circuit. We know from experiment that this tends to cause a current in the direction AC DB.

Fig. 5.

As AB moves through the field its motion tends to cause electric intensity in the direction BA. At the same time its kinetic energy is being continually converted into electric and magnetic energy which travels to the rest of the circuit there to be dissipated, that is, there must be a divergence of energy from AB. Instead then of a convergence of positive tubes running from B to A, we shall have what is magnetically equivalent—a divergence of negative tubes or tubes running from A to B, their motion outwards being accompanied by tubes of magnetic induction running round in the same way as if there were an ordinary current from B to A. These magnetic tubes must be supposed to move outwards in order to account for the direction of the electric intensity.*

When these electric and magnetic tubes converge upon the rest of the circuit they will evidently form a current running in the direction AC DB. We have here taken, just as in the case of the condenser and the voltaic cell, the lessening of negative induction by its motion outwards, as equivalent to the increase of positive induction by its motion inwards, and we have considered both of them to indicate the application of electric intensity in the same direction in the conductor.

If instead of considering AB as a whole we break it up into elements, each element will be a source of diverging negative tubes, and the remainder of AB will to that element be a part of the rest of the circuit. Hence some of the energy sent out from the element will converge on and be dissipated in AB, or AB will be heated just as the rest of the circuit.

The general equations of the electromagnetic field.

We can easily obtain equations corresponding to and closely resembling those of MAXWELL by means of the principles upon which this paper is founded.

The assumption that if we take any closed curve the number of tubes of magnetic

* [Added July 15.—The above must not be regarded as an attempt to explain the production of electric induction by the motion of a conductor in a magnetic field, but merely as an attempt to show how the induction arising in the moving part of a circuit finds its way into the rest of the circuit.]

induction passing through it is equal to the excess of the number which have moved in over the number which have moved out through the boundary since the beginning of the formation of the field, suggests a historical mode of describing the state of the field at any moment.

Let a, b, c be the components of magnetic induction at any point O. Consider a small area $dy\ dz$ close to the point, then the number of tubes passing through the area $dy\ dz$ will be $a dy\ dz$. This will be equal to the difference between those which have come in and those which have gone out.

Fig. 6.

Let $L dx$, $M dy$, $N dz$ denote the numbers of tubes which have cut the lengths dx, dy, dz since the beginning of the system, those being positive which have tended to produce electric intensity in the positive direction along the axes, and those being negative and therefore subtracted which have tended to produce intensity in the opposite direction. Let us consider the number which have come into the area $OBCD = dy\ dz$ (fig. 6). The number which have come in across OB is $-M dy$ ($-$because the movement of tubes passing through $dy\ dz$ in the positive direction must be outwards to produce E.I. along OB). The number which has passed out across CD is $-\left(M + \frac{dM}{dz} dz\right) dy$. The difference is $\frac{dM}{dz} dy\ dz$. The number which has come in across OC is $+N dz$ ($+$ because the movement of tubes passing through $dy\ dz$ in the positive direction must be inwards to produce E.I. along OC). The number which has passed out across BD is $\left(N + \frac{dN}{dy} dy\right) dz$. The difference is $-\frac{dN}{dy} dy\ dz$.

The number still passing through $dy\ dz$ is therefore $\left(\frac{dM}{dz} - \frac{dN}{dy}\right) dy\ dz$.

Equating this to the actual induction through the area, viz.,

$$a dy\ dz$$

and performing the same process for the corresponding areas $dz\ dx$, $dx\ dy$, we obtain

$$\left.\begin{aligned} a &= \frac{dM}{dz} - \frac{dN}{dy} \\ b &= \frac{dN}{dx} - \frac{dL}{dz} \\ c &= \frac{dL}{dy} - \frac{dM}{dx} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (1)$$

Comparing these with MAXWELL'S equations (vol. ii., p. 216) we see that

$$\frac{dM}{dz}-\frac{dN}{dy}=\frac{dH}{dy}-\frac{dG}{dz}$$

with two similar equations, F, G, H being the components of the vector potential. We should obtain MAXWELL'S equations if we defined F, G, H to be the number of tubes which would cut the axes per unit length if the system were to be allowed to return to its original unmagnetic condition, the tubes now moving in the opposite direction. According to MAXWELL, the vector whose components are F, G, and H "represents the time integral of the electromotive force which a particle placed at the point (x, y, z) would experience if the primary current were suddenly stopped" (vol. ii., 2nd Ed., p. 215). If the electric intensity is produced by the motion of magnetic induction, then our definition of F, G, H will by the second fundamental principle agree with MAXWELL'S statement.

If u, v, w be the components of current—including, of course, under currents, growth of induction—we have from the third principle MAXWELL'S equations E (vol. ii., p. 233), which on multiplying by μ become when μ is constant

$$\left.\begin{aligned} 4\pi\mu u&=\frac{dc}{dy}-\frac{db}{dz}\\ 4\pi\mu v&=\frac{da}{dz}-\frac{dc}{dx}\\ 4\pi\mu w&=\frac{db}{dx}-\frac{da}{dy}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (2)$$

Combining these with equations (1) (as in MAXWELL, vol. ii., pp. 236–7), and writing $-\nabla^2$ for $\frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}$ we obtain

$$\left.\begin{aligned} 4\pi\mu u&=-\nabla^2 \mathrm{L}-\frac{d}{dx}\left(\frac{d\mathrm{L}}{dx}+\frac{d\mathrm{M}}{dy}+\frac{d\mathrm{N}}{dz}\right)\\ 4\pi\mu v&=-\nabla^2 \mathrm{M}-\frac{d}{dy}\left(\frac{d\mathrm{L}}{dx}+\frac{d\mathrm{M}}{dy}+\frac{d\mathrm{N}}{dz}\right)\\ 4\pi\mu w&=-\nabla^2 \mathrm{N}-\frac{d}{dz}\left(\frac{d\mathrm{L}}{dx}+\frac{d\mathrm{M}}{dy}+\frac{d\mathrm{N}}{dz}\right)\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (3)$$

These equations only differ in sign from MAXWELL'S, and are therefore to be solved in the same way.

It is easy to see by substitution that if we assume

$$\left.\begin{aligned} L' &= -\mu\iiint\frac{u}{r}dxdydz \\ M' &= -\mu\iiint\frac{v}{r}dxdydz \\ N' &= -\mu\iiint\frac{w}{r}dxdydz \\ H &= \frac{1}{4\pi}\iiint\left(\frac{dL}{dx}+\frac{dM}{dy}+\frac{dN}{dz}\right)\frac{1}{r}dxdydz \end{aligned}\right\} \quad \ldots\ldots\ldots (4)$$

then the following will be solutions

$$\left.\begin{aligned} L &= L' - \frac{dH}{dx} \\ M &= M' - \frac{dH}{dy} \\ N &= N' - \frac{dH}{dz} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (5)$$

It is evident that we may add to the right-hand side of equations (5) $\frac{d\phi}{dx}, \frac{d\phi}{dy}, \frac{d\phi}{dz}$ respectively, where ϕ is any function of x, y, z, since these will disappear from (3) and also from (1).

The electric intensity, in so far as it depends upon magnetic motions, will consist of two terms, one depending upon the motion of the material at the point (its components being found as in MAXWELL, vol. ii., p. 227, note), the other upon the motion of magnetic induction about the point. We may add a third term, arising from any electrical distribution with a potential ψ.

If there is no material motion we shall have

$$\left.\begin{aligned} P &= \frac{dL}{} - \frac{d\psi}{dx} \\ Q &= \frac{dM}{dt} - \frac{d\psi}{dy} \\ R &= \frac{dN}{dt} - \frac{d\psi}{dz} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (6)$$

Substituting from (4) and (5) we get

$$\begin{aligned} P &= -\mu\iiint\frac{du}{dt}\frac{1}{r}dxdydz - \frac{1}{4\pi}\frac{d}{dx}\iiint\left(\frac{d}{dx}\frac{dL}{dt}+\frac{d}{dy}\frac{dM}{dt}+\frac{d}{dz}\frac{dN}{dt}\right)\frac{1}{r}dxdydz - \frac{d\psi}{dx} \\ &= -\mu\iiint\frac{du}{dt}\frac{1}{r}dxdydz - \frac{1}{4\pi}\frac{d}{dx}\iiint\left(\frac{dP}{dx}+\frac{dQ}{dy}+\frac{dR}{dz}\right)\frac{1}{r}dxdydz \\ &\qquad - \frac{1}{4\pi}\frac{d}{dx}\iiint\left(\frac{d^2\psi}{dx^2}+\frac{d^2\psi}{dy^2}+\frac{d^2\psi}{dz^2}\right)\frac{1}{r}dxdydz - \frac{d\psi}{dx}, \end{aligned}$$

substituting for $\frac{dL}{dt}$, &c., from (6).

The last two terms cancel each other, and we get

$$P=-\mu\iiint\frac{du}{dt}\frac{1}{r}dxdydz-\frac{1}{4\pi}\frac{d}{dx}\iiint\left(\frac{dP}{dx}+\frac{dQ}{dy}+\frac{dR}{dz}\right)\frac{1}{r}dxdydz \quad . \quad . \quad . \quad . \quad (7)$$

or if we put

$$\frac{dP}{dx}+\frac{dQ}{dy}+\frac{dR}{dz}=4\pi\rho$$

and

$$V=\iiint\frac{\rho}{r}dxdydz,$$

then

$$P=-\mu\iiint\frac{du}{dt}\frac{1}{r}dxdydz-\frac{dV}{dx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

with similar equations for Q and R.

If the system is steady $\frac{du}{dt}$, $\frac{dv}{dt}$, $\frac{dw}{dt}$ are all zero, and then

$$P=-\frac{dV}{dx}, \quad Q=-\frac{dV}{dy}, \quad R=-\frac{dV}{dz}.$$

The quantity ρ, of which V is the potential, will be zero within non-conducting homogeneous parts of the field, for there

$$f=\frac{KP}{4\pi}, \quad g=\frac{KQ}{4\pi}, \quad h=\frac{KR}{4\pi},$$

and

$$\frac{dP}{dx}+\frac{dQ}{dy}+\frac{dR}{dz}=\frac{4\pi}{K}\left(\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}\right)=0$$

since no charges can reside within a homogeneous non-conducting medium. Or, stating it in another way, all the induction tubes brought into any part of such a medium remain there without dissipation, a charge in a non-homogeneous medium being due to unequal amounts of dissipation of induction in different parts of the medium.

But ρ will have value at surfaces separating dissimilar substances either in the insulating or conducting parts of the medium. For in the former the induction is continuous, while the intensity is discontinuous, and in the latter the current or rate of destruction of induction may be continuous, but the relation between intensity and current changes discontinuously with the conductivity. At surfaces separating insulators from conductors ρ may have value, as, for instance, at the surfaces of the plates of a condenser with its terminals connected with two points in a circuit, or at the surface of an insulated conductor near the circuit. It is also to be noted that ρ will have values at the seat of electromotive force.

The values of the components of magnetic induction a, b, c are not in any way dependent on ρ. For taking the first of equations (1) and substituting from (5) we have

$$a=\frac{dM}{dz}-\frac{dN}{dy}=\frac{dM'}{dz}-\frac{dN'}{dy}-\frac{d^2H}{dydz}+\frac{d^2H}{dzdy}=\frac{dM'}{dz}-\frac{dN'}{dy} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where $\mathbf{M}'$ and $\mathbf{N}'$ depend on the currents in the system and not on the charges.

Comparing our equations with MAXWELL'S we see that the important point of difference is that we can no longer put the quantity corresponding to his J equal to zero, J being given by $\frac{dF}{dx}+\frac{dG}{dy}+\frac{dH}{dz}$.

This does not affect the determination of velocity of propagation of disturbance in a homogeneous non-conducting medium.

For in such a medium we shall have

$$u=\frac{df}{dt}=\frac{K}{4\pi}\frac{dP}{dt}$$

with corresponding values for v and w.

Substituting in (3) the first equation becomes

$$K\mu\frac{dP}{dt}=-\nabla^2L-\frac{d}{dx}\left(\frac{dL}{dx}+\frac{dM}{dy}+\frac{dN}{dz}\right)$$

differentiating with respect to t

$$K\mu\frac{d^2P}{dt^2}=-\nabla^2\frac{dL}{dt}-\frac{d}{dx}\left(\frac{d}{dx}\frac{dL}{dt}+\frac{d}{dy}\frac{dM}{dt}+\frac{d}{dz}\frac{dN}{dt}\right)$$

and putting $\frac{dL}{dt}=P+\frac{d\psi}{dx}$

$$K\mu\frac{d^2P}{dt^2}=-\nabla^2P-\frac{d}{dx}\nabla^2\psi-\frac{d}{dx}\left(\frac{dP}{dx}+\frac{dQ}{dy}+\frac{dR}{dz}\right)+\frac{d}{dx}\nabla^2\psi=-\nabla^2P \quad . \quad . \quad . \quad (10)$$

since

$$\frac{dP}{dx}+\frac{dQ}{dy}+\frac{dR}{dz}=\frac{4\pi}{K}\left(\frac{df}{dx}+\frac{dg}{dy}+\frac{dh}{dz}\right)=0$$

within a homogeneous non-conductor.

This gives the velocity of propagation of electric induction equal to $1/\sqrt{K\mu}$.

We can also obtain the corresponding equation for the magnetic induction.

Substituting in (3) for u, v, and w in terms of P, Q, and R, as above, differentiating the second with respect to z, and the third with respect to y, and subtracting

$$K\mu \frac{d}{dt}\left(\frac{dQ}{dx}-\frac{dR}{dy}\right)=-\nabla^2\left(\frac{dM}{dz}-\frac{dN}{dy}\right)$$

then from (6)

$$K\mu \frac{d}{dt}\left(\frac{d}{dt}\frac{dM}{dz}-\frac{d^2\psi}{dxdy}-\frac{d}{dt}\frac{dN}{dy}+\frac{d^2\psi}{dydz}\right)=-\nabla^2\left(\frac{dM}{dz}-\frac{dN}{dy}\right)$$

or

$$K\mu \frac{d^2}{dt^2}\left(\frac{dM}{dz}-\frac{dN}{dy}\right)=-\nabla^2\left(\frac{dM}{dz}-\frac{dN}{dy}\right)$$

or from (1)

$$K\mu \frac{d^2\alpha}{dt^2}=-\nabla^2\alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

whence the velocity of propagation of magnetic induction is also equal to $1/\sqrt{K\mu}$.

It would seem that in some cases, such as that of the field surrounding a straight wire with a steady current, the electric intensity may be regarded as entirely due to the motion of magnetic induction, and its components will therefore be $\frac{dL}{dt}, \frac{dM}{dt}, \frac{dN}{dt}$.

But in other cases it would seem that the electric induction cannot be wholly due to the motion of magnetic induction, and we must therefore introduce the terms involving ψ. If, for instance, the electric and magnetic intensities were inclined at an angle θ, we should have to suppose the electric intensity E to be produced by the motion of the component of magnetic induction I perpendicular to E, viz., $\mu I \sin\theta$, the other component $\mu I \cos\theta$ being at rest. To produce intensity E, E tubes must cut unit length in the direction of E per second; and since the value of the magnetic induction is $\mu I \sin\theta$, this requires a velocity v, given by $v.\mu I \sin\theta = E$ or $v = E/\mu I \sin\theta$. Now we can easily imagine a case where E and I coincide, as, for instance, a condenser with its planes parallel to the axis of a wire carrying a current, and its terminals connected with two points in the wire. Here $I \sin\theta = 0$, and v is infinite. Or we have to suppose the electric intensity to be produced by the movement of tubes of induction of no intensity with infinite velocity, a statement without physical meaning.

But it is, perhaps, worth noting that if we suppose that the electric intensity is produced by the motion of magnetic induction, and that the magnetic intensity is produced by the motion of the electric induction, each carrying their energy with them, the right quantity of energy crosses the unit area.

For E magnetic tubes, with $I \sin\theta$ unit cells per unit length, will carry across unit area in the plane of E and I a quantity $\frac{EI \sin\theta}{8\pi}$, or half the energy which actually crosses the plane. If $I \sin\theta$ is due to the motion of electric tubes, then $I \sin\theta/4\pi$ tubes must cut unit length in the direction of $I \sin\theta$ per second. The number of unit cells per unit length is E, and therefore the motion of the tubes will carry a quantity of energy $\frac{EI \sin\theta}{8\pi}$, or the other half actually crossing.

The equations which have been obtained in the foregoing manner by the aid of the

hypothesis of movement of magnetic induction, may also be obtained without any special hypothesis as to the motion of the induction tubes, merely assuming that growth of induction through a curve is accompanied by electric intensity round the curve. Instead of connecting L, M, N with the number of tubes which have cut the axes, we start with the following definitions :—

Let L, M, N denote the time integrals of the components of the electric intensity parallel to the axes since the origin of the system, so that

$$L=\int P dt \qquad M=\int Q dt \qquad N=\int R dt,$$

then

$$P=\frac{dL}{dt} \qquad Q=\frac{dM}{dt} \qquad R=\frac{dN}{dt}.$$

If a, b, c be components of magnetic induction, since the growth of induction through a curve is equal to the line integral of the electric intensity round a curve in the negative direction, we have

$$\frac{da}{dt}=\frac{d}{dz}-\frac{dR}{dy}=\frac{d}{dt}\left(\frac{dM}{dz}-\frac{dN}{dy}\right)$$

with corresponding equations for $\frac{db}{dt}$ and $\frac{dc}{dt}$.

Integrating with respect to t from the origin of the system, when all the quantities were zero

$$\left.\begin{aligned} a&=\frac{dM}{dz}-\frac{dN}{dy}\\ b&=\frac{dN}{dx}-\frac{dM}{dz}\\ c&=\frac{dL}{dy}-\frac{dM}{dx}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (1')$$

equations the same in form as equations (1).

As before we obtain equations (3), (4), and (5), while instead of (6) we have the simple equations $P=\frac{dL}{dt}$ and the two others.

Substituting for $\frac{dL}{dt}$ we obtain an equation of the same form as (7), which may also be put into the form (8). Equations (9) and (10) will also follow.

Just as we have obtained equations by considering the growth of the magnetic induction to its present state so we may obtain corresponding equations by considering the growth of the electric induction.

Let $\frac{Adx}{4\pi}$, $\frac{Bdy}{4\pi}$, $\frac{Cdz}{4\pi}$ be the algebraic sum of the number of electric induction tubes

which have cut dx, dy, dz drawn from a point in such a way as to create magnetic intensities in the positive direction along dx, dy, dz.

The excess of the number of tubes which have passed in over those which have passed out through the boundary of any area will be equal to the time integral of the total current through the area.

The components of the total current are

$$u=p+\frac{df}{dt} \qquad v=q+\frac{dg}{dt} \qquad w=r+\frac{dh}{dt}$$

p, q, and r being the components of the conduction current or the number of tubes dissipated per second, and f, g, h the components of the induction actually existing.

As in the last case, if we put $f'=\int u dt$, &c., we at once obtain the equations

$$\left.\begin{aligned} 4\pi f' &= \frac{dC}{dy}-\frac{dB}{dz} \\ 4\pi g' &= \frac{dA}{dz}-\frac{dC}{dx} \\ 4\pi h' &= \frac{dB}{dx}-\frac{dA}{dy} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (12)$$

Corresponding to the current equations (2) we have three equations obtained from the condition that the rate of increase of magnetic induction through an area is equal to the integral of the electric intensity round it in the negative direction. These are

$$\left.\begin{aligned} \frac{da}{dt} &= \frac{dQ}{dz}-\frac{dR}{dy} \\ \frac{db}{dt} &= \frac{dR}{dx}-\frac{dP}{dz} \\ \frac{dc}{dt} &= \frac{dP}{dy}-\frac{dQ}{dx} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (13)$$

If C_1 is the specific conductivity we may by OHM's law put the current equations after integrating in the form

$$f'=C_1\int P dt+\frac{KP}{4\pi}$$

$$g'=C_1\int Q dt+\frac{KQ}{4\pi}$$

$$h'=C_1\int R dt+\frac{KR}{4\pi}$$

whence in media where K is constant

$$\frac{dg'}{dz}-\frac{dh'}{dy}=C_1\int\left(\frac{dQ}{dz}-\frac{dR}{dy}\right)dt+\frac{K}{4\pi}\left(\frac{dQ}{dz}-\frac{dR}{dy}\right)$$

$$= C_1\int\frac{da}{dt}dt+\frac{K}{4\pi}\frac{da}{dt} \text{ from (13)}$$

$$= C_1 a+\frac{K}{4\pi}\frac{da}{dt}$$

with two similar equations.

Finding the values of the left hand from (12) we obtain

$$\left.\begin{aligned} 4\pi C_1 a+K\frac{da}{dt} &= -\nabla^2 A-\frac{d}{dx}\left(\frac{dA}{dx}+\frac{dB}{dy}+\frac{dC}{dz}\right)\\ 4\pi C_1 b+K\frac{db}{dt} &= -\nabla^2 B-\frac{d}{dy}\left(\frac{dA}{dx}+\frac{dB}{dy}+\frac{dC}{dz}\right)\\ 4\pi C_1 c+K\frac{dc}{dt} &= -\nabla^2 C-\frac{d}{dz}\left(\frac{dA}{dx}+\frac{dB}{dy}+\frac{dC}{dz}\right)\end{aligned}\right\} \quad \ldots\ldots (14)$$

If we assume

$$A' = -\frac{1}{4\pi}\iiint\left(4\pi C_1 a+K\frac{da}{dt}\right)\frac{1}{r}dxdydz$$

with corresponding values for B′ and C′ and

$$M = \frac{1}{4\pi}\iiint\left(\frac{dA}{dx}+\frac{dB}{dy}+\frac{dC}{dz}\right)\frac{1}{r}dxdydz$$

then

$$\left.\begin{aligned} A &= A'-\frac{dM}{dx}\\ B &= B'-\frac{dM}{dy}\\ C &= C'-\frac{dN}{dy}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (15)$$

are solutions of (13).

We may obtain by substitution from (15) in (12) values for f', g', h' corresponding to the values of the magnetic induction in (9), viz.:

$$4\pi f' = \frac{dC'}{dy}-\frac{dB'}{dz}$$

and two others; where A′, B′, C′ are given in terms of the magnetic induction as above.

It is only in special cases, such as that of a straight wire with a steady current, that the magnetic intensity will be equal to 4π times the number of electric induction tubes passing through unit length per second. In all cases the line integral of the magnetic intensity round a closed curve is equal to 4π times the number of electric tubes passing through the boundary, but the electric tubes may be more crowded in

some parts than in others, while the magnetic intensity is not altered in a corresponding manner. For instance, the magnetic tubes will be continued through an insulated conductor in the field, while in the steady state no electric tubes pass through it. But each element adds to the line integral the quantity which, after Mr. Bosanquet, I have called the magnetomotive force, this being equal to 4π times the number of electric tubes passing through the element. But it only adds it on integrating round the whole of the closed curve.

The intensity at any point will therefore be the resultant of the intensities produced by the magnetomotive forces in the various elements. Perhaps the simplest mode of finding it is as follows.

The components of the magnetomotive force produced in a cube dx, dy, dz parallel to the three edges will be

$$\frac{dA}{dt}dx, \frac{dB}{dt}dy, \frac{dC}{dt}dz.$$

for $\frac{1}{4\pi}\frac{dA}{dt}, \frac{1}{4\pi}\frac{dB}{dt}, \frac{1}{4\pi}\frac{dC}{dt}$, are by definition the rates at which electric tubes are cutting unit lengths parallel to the axes.

But these magnetomotive forces would be produced by currents round the cube in planes perpendicular to the axes respectively, and equal to

$$\frac{1}{4\pi}\frac{dA}{dt}dx, \frac{1}{4\pi}\frac{dB}{dt}dy, \frac{1}{4\pi}\frac{dC}{dt}dz.$$

for the line integral of the intensity round a curve threading a current is $4\pi\times$ current. But the magnetic intensity at any point due to a current is equal to that of a magnetic shell of strength (*i.e.*, intensity $\times$ thickness), equal numerically to the current bounding the shell.

If we suppose the thickness of the shell equal to that of the cube, the effect is the same as if the cube were magnetised with intensity having components

$$\frac{1}{4\pi}\frac{dA}{dt}, \frac{1}{4\pi}\frac{dB}{dt}, \frac{1}{4\pi}\frac{dC}{dt}.$$

The potential of such a distribution of magnetisation is (Maxwell, vol. ii., p. 29, equation (23)).

$$V=\frac{1}{4\pi}\iiint\left(\frac{dA}{dt}\frac{dp}{dx}+\frac{dB}{dt}\frac{dp}{dy}+\frac{dC}{dt}\frac{dp}{dz}\right)dxdydz$$

where $p=\frac{1}{r}$, and the magnetic intensity is given by

$$\alpha=-\frac{dV}{dx}, \quad \beta=-\frac{dV}{dy}, \quad \gamma=-\frac{dV}{dz}.$$

It may be noticed that in a steady field $\frac{dA'}{dt}$, $\frac{dB'}{dt}$, $\frac{dC'}{dt}$ are all zero, so that

$$V=-\frac{1}{4\pi}\iiint\left(\frac{d}{dx}\frac{dM}{dt}\frac{dp}{dx}+\frac{d}{dy}\frac{dM}{dt}\frac{dp}{dy}+\frac{d}{dz}\frac{dM}{dt}\frac{dp}{dz}\right)dxdydz$$

where

$$M=\frac{1}{4\pi}\iiint\left(\frac{dA}{dx}+\frac{dB}{dy}+\frac{dC}{dz}\right)\frac{1}{r}dxdydz$$

We may obtain equations of the same form as those given in (14) without any hypothesis as to the movement of electric induction tubes, merely assuming that the total current through a curve is equal to $4\pi\times$ line integral of magnetic intensity round the curve.

We start with the following definitions. Let A, B, C be the time integrals of the components of magnetic intensity since the origin of the system.

Then

$$A=\int\alpha dt, \qquad B=\int\beta dt, \qquad C=\int\gamma dt$$

and

$$\alpha=\frac{dA}{dt}, \qquad \beta=\frac{dB}{dt}, \qquad \gamma=\frac{dC}{dt}$$

We have the equations

$$4\pi u=\frac{d\gamma}{dy}-\frac{d\beta}{dz}$$

and two others.

Integrating with respect to t we have

also

$$\left.\begin{aligned} 4\pi\int udt=4\pi f'&=\frac{dC}{dy}-\frac{dB}{dz}\\ 4\pi g'&=\frac{dA}{dz}-\frac{dC}{dx}\\ 4\pi h'&=\frac{dB}{dx}-\frac{dA}{dy}\end{aligned}\right\}$$

which are of the same form as (12).

Hence exactly as before we obtain equations (14) and their solutions (15).

The equations for the magnetic intensity are now

$$\alpha=\frac{dA}{dt}, \qquad \beta=\frac{dB}{dt}, \qquad \gamma=\frac{dC}{dt}$$

If we differentiate (14) with respect to t, and substitute from these equations for magnetic intensity, we obtain

$$4\pi\mu C_1\frac{d\alpha}{dt}+K\mu\frac{d^2\alpha}{dt^2}=-\nabla^2\alpha-\frac{d}{dx}\left(\frac{d\alpha}{dx}+\frac{d\beta}{dy}+\frac{d\gamma}{dz}\right)$$

with corresponding equations for β and γ.

Differentiating the second of these with respect to z, and the third with respect to y, and subtracting, we obtain

$$4\pi\mu C_1\frac{du}{dt}+K\mu\frac{d^2u}{dt^2}=-\nabla^2u$$

with corresponding equations for v and w.

These correspond to MAXWELL's equations (7), p. 395.

In conclusion it may be remarked that the equations found in this paper give the same expression for the rate of Transfer of Energy as that in my previous paper derived from MAXWELL's equations involving F, G, and H.

IV. *On some Applications of Dynamical Principles to Physical Phenomena.*

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§ 1. The tendency to apply dynamical principles and methods to explain physical phenomena has steadily increased ever since the discovery of the principle of the Conservation of Energy. This discovery called attention to the ready conversion of the energy of visible motion into such apparently dissimilar things as heat and electric currents, and led almost irresistibly to the conclusion that these too are forms of kinetic energy, though the moving bodies must be infinitesimally small in comparison with the bodies which form the moving pieces of any of the structures or machines with which we are acquainted. As soon as this conception of heat and electricity was reached mathematicians began to apply to them the dynamical method of the Conservation of Energy, and many physical phenomena were shown to be related to each other, and others predicted by the use of this principle; thus, to take an example, the induction of electric currents by a moving magnet was shown by von Helmholtz to be a necessary consequence of the fact that an electric current produces a magnetic field. Of late years things have been carried still further; thus Sir William Thomson in many of his later papers, and especially in his address to the British Association at Montreal on "Steps towards a Kinetic Theory of Matter," has devoted a good deal of attention to the description of machines capable of producing effects analogous to some physical phenomenon, such, for example, as the rotation of the plane of polarisation of light by quartz and other crystals. For these reasons the view (which we owe to the principle of the Conservation of Energy) that every physical phenomenon admits of a dynamical explanation is one that will hardly be questioned at the present time. We may look on the matter (including, if necessary, the ether) which plays a part in any physical phenomenon as forming a material system and study the dynamics of this system by means of any of the methods which we apply to the ordinary systems in the Dynamics of Rigid Bodies. As we do not know much about the structure of the systems we can only hope to obtain useful results by using methods which do not require an exact knowledge of the mechanism of the system. The method of the Conservation of Energy is such a method, but there are others which hardly require a greater knowledge of the structure of the system and yet are capable of giving us more definite information than that principle when used in the

ordinary way. LAGRANGE'S equations and HAMILTON'S method of Varying Action are methods of this kind, and it is the object of this paper to apply these methods to study the transformations of some of the forms of energy, and to show how useful they are for coordinating results of very different kinds as well as for suggesting new phenomena. A good many of the results which we shall get have been or can be got by the use of the ordinary principle of Thermodynamics, and it is obvious that this principle must have close relations with any method based on considerations about energy.

LAGRANGE'S equations were used with great success by MAXWELL in his 'Treatise on Electricity and Magnetism,' vol. ii., chaps. 6, 7, 8, to find the equations of the electromagnetic field.

In order to confine this paper to a reasonable size I shall limit myself to the consideration of the relations existing between various phenomena in elasticity, heat, electricity and magnetism, but even with this limitation it will only be possible to consider a few of the more prominent out of the many phenomena to which the method can be applied.

When we apply LAGRANGE'S or HAMILTON'S methods to discuss the motion of any material system we have first to choose coordinates which can fix the configuration of the system, and then to find an expression for the kinetic energy in terms of these coordinates and their differential coefficients with respect to the time. Before applying these methods therefore to any physical phenomenon we must have coordinates which are sufficient to fix the configuration of the system which takes part in the phenomenon we are considering. The notation which we shall use will be as follows :—

To fix the geometrical configuration of the system, *i.e.*, to fix the position in space of any bodies of finite size which may be in the system, we shall use coordinates denoted by the letters $x_1, x_2 \ldots, x_n$, and when we want to denote a geometrical coordinate generally without reference to any one in particular we shall use the letter x.

To fix the configuration of the strains in the system we shall in any particular case use the ordinary strain components a, b, c, e, f, g (THOMSON and TAIT'S 'Natural Philosophy,' vol. ii., § 669), but as it will be convenient to have a letter typifying the coordinates generally we shall use the letter w for this purpose.

To fix the electrical configuration of the system we shall use coordinates denoted by the letters $y_1, y_2 \ldots$, and y for the typical coordinate, where y in a dielectric is what MAXWELL calls an electric displacement and in a conductor the time integral of a current.

To fix the configuration of the magnetic field by coordinates in such a way that LAGRANGE'S equations can be used I have found it necessary, for reasons which will be given later, to introduce two coordinates to fix the magnitude of the intensity of magnetisation at a point : one of these is what we may call a kinosthenic* coordinate,

* I am indebted to Professor J. P. POSTGATE, Fellow of Trinity College, Cambridge, for this word.

i.e., one which only enters into the expression for the kinetic energy through its differential coefficient with respect to the time, so that the kinetic energy is not an explicit function of this coordinate, but only of its rate of change; the other coordinate is of a geometrical kind. This way of fixing the intensity of magnetisation is the mathematical analogue of AMPÈRE's theory of magnetism, which supposes that electrical currents flow round the molecules of bodies, and that a body is magnetised when its molecules are so arranged that the normals to the planes of the currents which flow round them are not distributed uniformly in all directions. Thus the kinosthenic coordinates would be the ones fixing the molecular currents, and the geometrical ones those fixing the arrangement in space of the normals to the planes of the currents. We shall call the kinosthenic coordinate ζ, and the geometrical one η, and suppose they are chosen so that the intensity of magnetisation is $\eta\xi$ where ξ is the generalised component of momentum of the type ζ.

We must now consider how to represent the temperature of a body so as to bring it within the power of dynamical methods. In the ordinary kinetic theory of gases the temperature is taken to be the mean kinetic energy of translation of the molecules of the gas, and there are reasons for believing that a similar thing may be true for bodies in the solid and liquid as well as in the gaseous state. We shall therefore suppose that the temperature is represented by that part of the kinetic energy in unit of volume which involves the squares and products of the velocities of a system of coordinates denoted by the symbols $u_1, u_2 \ldots, u_m$. A moment's consideration will show that the u's must be kinostenic coordinates, *i.e.*, that the kinetic energy is a function of the differential coefficients of these quantities with respect to the time and not of the quantities themselves; and since the kinetic energy cannot be altered if we reverse the motion of the system whose kinetic energy is supposed to be measured by the temperature, the expression for the kinetic energy cannot contain any terms which involve the product of a "u" velocity with one of the type x, y, z, or w. We may therefore take $u_1, u_2 \ldots, u_m$ to be principal coordinates, and suppose that that part of the kinetic energy per unit of volume which depends upon the squares of their velocities, and which we shall call Θ, is given by the equation

$$\Theta = \tfrac{1}{2}\{[u_1u_1]\dot{u}_1^2 + [u_2u_2]\dot{u}_2^2 + \ldots\} \quad \ldots\ldots\ldots \quad (1)$$

or if $v_1, v_2 \ldots$ be the momenta corresponding to $u_1, u_2 \ldots$

$$\Theta = \tfrac{1}{2}\left\{\frac{v_1^2}{[u_1u_1]} + \frac{v_2^2}{[u_2u_2]} + \ldots\right\} \quad \ldots\ldots\ldots \quad (2)$$

and this form is more convenient for some purposes than the preceding one. Since the temperature can be fixed by one coordinate when it is uniform, we must suppose,

at any rate when things are in a steady state, that there are $(m-1)$ linear relations between the m quantities $\dot{u}_1, \dot{u}_2 \ldots$. We might easily alter the equations of motion so as to allow for this relation, or we might eliminate $(m-1)$ of the quantities $\dot{u}_1, \dot{u}_2 \ldots$, but it is more convenient to use the expressions for Θ given in equations (1) and (2): the reason for this is that when any property of a body depends upon the temperature it does not depend upon the value of one of the u's more than another, but only on the value of Θ: the u's never occur except as parts of the expression Θ.

We have at present only considered kinetic energy and have not said anything about potential energy, but we shall see that if we give a sufficiently wide meaning to the term material system we can explain all the effects produced by potential energy by considerations about kinetic energy alone. There is an advantage gained by doing this, because kinetic energy appears to be a much more fundamental conception than potential energy. When we can explain any phenomenon as a property of the motion of bodies, we have got what may be called a physical explanation of the phenomenon, and any further explanation must be rather metaphysical than physical; it is not so, however, with regard to potential energy, the use of this quantity cannot in any ordinary sense of the word be said to explain any physical phenomenon, it does little more than embody the results of experiments in a form well adapted to mathematical investigation. An investigation similar to that given in THOMSON and TAIT's 'Treatise on Natural Philosophy,' vol. i., p. 320, 2nd edition, shows that the effects produced by the potential energy of a system (A) can be explained by changes in the kinetic energy of another system (B) connected with (A). We must suppose that there are portions of matter connected with the system (A) and capable of motion which are not fixed by any of the coordinates which we have already spoken about, viz., the geometric, electric, magnetic, elastic, and temperature coordinates, and that the coordinates fixing the position of these portions of matter enter the expression for the kinetic energy only through their different coefficients. An analogous case is that of a sphere surrounded by water; in order to fix the configuration of the water it would be necessary to use an infinite number of coordinates, but these would enter the expression for the kinetic energy of the sphere and water only through their differential coefficients. For the sake of brevity we have called coordinates of this kind kinosthenic coordinates.

The coordinates fixing the configuration of the portions of matter mentioned above are kinosthenic coordinates. We shall denote them by the letters $\chi_1, \chi_2 \ldots, \chi_n$. We shall suppose for the sake of simplicity that there are no terms containing the product of a differential coefficient of one of the χ's with a differential coefficient of one of the geometric, elastic, electric, magnetic, or temperature coordinates.

Since $\chi_1, \chi_2 \ldots$ do not enter into the expression for the kinetic energy T of the systems (A) and (B), we have by LAGRANGE's equations

$$\left.\begin{aligned} \frac{d\mathrm{T}}{d\dot{\chi}_1} &= c_1 \\ \frac{d\mathrm{T}}{d\dot{\chi}_2} &= c_2 \\ &\cdots \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $c_1, c_2 \ldots, c_n$ are constants.

We can find the values of $\dot{\chi}_1, \dot{\chi}_2 \ldots$ from these equations in terms of $c_1, c_2 \ldots$, and substitute them in the expression for T, which will then be of the form

$$\mathfrak{T}+\mathrm{K}$$

where $\mathfrak{T}$ is a homogeneous quadratic function of the differential coefficients of the x, y, z and w coordinates; in fact, the kinetic energy of the system (A) and K is a quadratic function of the c's involving it may be the coordinates x, y, z and w, but not their differential coefficients; K is evidently equal in magnitude to the kinetic energy of the system (B). Equations (19) on page 323 of THOMSON and TAIT'S 'Natural Philosophy,' vol. i., 2nd ed., show that if we consider only the kinetic energy, LAGRANGE'S equation takes the form

$$\frac{d}{dt}\frac{d\mathfrak{T}}{d\dot{x}} - \frac{d\mathfrak{T}}{dx} + \frac{d\mathrm{K}}{dx} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

but if the system A had possessed potential energy equal to V the equation (considering A alone) would have been

$$\frac{d}{dt}\frac{d\mathfrak{T}}{d\dot{x}} - \frac{d\mathfrak{T}}{dx} + \frac{d\mathrm{V}}{dx} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Thus the effect of the system (B) on (A) is the same as if (A) possessed potential energy equal to K, which is, as we saw, the kinetic energy possessed by the system B, which is fixed by the kinosthenic coordinates. Thus we may look on the potential energy of any system (A) as being the kinetic energy of a kinosthenic system (B) connected with A; and so we may regard all energy as kinetic. If we do this it will simplify some of the dynamical principles very much. We may take as our fundamental principle HAMILTON'S Principle of Varying Action, as we can very readily deduce from this principle the ordinary dynamical methods, such as LAGRANGE'S and HAMILTON'S equations. But if all the energy is kinetic, then by the principle of the conservation of energy the magnitude of the kinetic energy remains constant, and the principle of least action takes the very simple form that, with a given quantity of energy, any material system will by its unguided motion pass from one configuration to another in the least possible time, where, of course, in the phrase material system we include the kinosthenic systems whose motion produce the same effects as the

potential energy of the original system, and two configurations are not supposed to coincide unless the configuration of these systems coincide also.

We shall, however, in the following investigations sometimes make use of the potential energy in the usual way, as the analysis is the same, and the use of the ordinary method saves a good deal of explanation.

§ 2. Let us now go on to consider the various terms in the expression for the kinetic energy of a material system, taking into account the geometric, electric, magnetic, and thermal conditions of the system. We are dealing with five sets of coordinates—the sets we have denoted by the letters x, y, z, u, w.

The kinetic energy T will be of the form

$$\begin{aligned} \mathrm{T}=\tfrac{1}{2}\{&[x_1x_1]\dot{x_1}^2+2[x_1x_2]\dot{x_1}\dot{x_2}+\ldots \\ &+[y_1y_1]\dot{y_1}^2+2[y_1y_2]\dot{y_1}\dot{y_2} \\ &+2[xy]\dot{x}\dot{y}+\ldots \\ &+\ldots \qquad \} \end{aligned}$$

where the quantities denoted by $[x_1x_1]$, $[y_1y_1]$, $[x_1y_1]$, may be functions of the coordinates x, y, z, u, w.

The various terms in T can be divided into fifteen types; there are five sets, one corresponding to each of the coordinates x, y, z, u, w, where the terms are of the same character as

$$[x_1x_1]\dot{x_1}^2+2[x_1x_2]\dot{x_1}\dot{x_2}$$

where each term involves the squares of the velocities of the coordinates of one kind, or the product of two velocities of the same kind: it is evident that each of these five types can exist in actual dynamical systems.

There are ten sets of the type

$$[xy]\dot{x}\dot{y}$$

involving the product of the differential coefficients of two coordinates of different kinds: we can see, however, that some of these can not exist in any actual material system. Thus, for example, since the u coordinates only enter through the temperature, there can be no terms involving the product of the differential coefficient of u and the differential coefficient of any of the other coordinates: this consideration reduces the ten sets to six. To determine whether terms of the type of any particular set exist or not we must determine what the consequences would be if terms of this type did exist; if these are contrary to experience we conclude that terms of this type do not exist. We can determine these consequences in the following way. Suppose we have a term in the kinetic energy equal to

$$(\lambda\mu)\dot{\lambda}\dot{\mu}$$

where λ and μ may be any of the five kinds of coordinates which we are considering. We have by LAGRANGE's equations

$$\frac{d}{dt}\frac{dT}{d\dot{\lambda}} - \frac{dT}{d\lambda} = \text{external force of the type } \lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If instead of T we consider only the term $(\lambda\mu)\dot{\lambda}\dot{\mu}$ we see that it requires the existence of a force of the type λ equal to

$$-\left[(\lambda\mu)\ddot{\mu} + \frac{d}{d\mu}(\lambda\mu)\dot{\mu}^2 + \Sigma\frac{d}{d\nu}(\lambda\mu)\dot{\mu}\dot{\nu}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and a force of the type μ equal to

$$-\left[(\lambda\mu)\ddot{\lambda} + \frac{d}{d\lambda}(\lambda\mu)\dot{\lambda}^2 + \Sigma\frac{d}{d\nu}(\lambda\mu)\dot{\lambda}\dot{\nu}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

when ν is a coordinate of any other type.

Thus as it is clearer to take a definite case, suppose that λ is the geometrical coordinate x, and μ then electrical coordinate y; then if the term $(xy)\,\dot{x}\dot{y}$ occurred in the expression for the kinetic energy, the mechanical force produced by a varying current would be different from that produced by a steady one; this is shown by the existence of the term $(xy)\ddot{y}$ in equation (7), which now represents the force of type x, *i.e.*, the mechanical force. If $[xy]$ were a function of y, the term $\frac{d}{dy}[xy]\dot{y}^2$, which occurs in the expression for the mechanical force, shows that a current would produce a mechanical force proportional to the square of the intensity of the current, and which therefore would not be reversed if the direction of the current were reversed. Again, if we consider the expression for the force of type y, that is, the electromotive force, we see that the existence of this term implies the production of an electromotive force by a body whose velocity is changing depending on the acceleration of the body; this is indicated by the term $[xy]\ddot{x}$: if $[xy]$ were a function of x, equation (8) shows that a moving body would produce an electromotive force proportional to the square of its velocity. As none of these effects have been observed we conclude that this term does not exist.

Another point worthy of attention is that the existence of a force of any type ξ say, in any region, implies the existence in the expression for the kinetic energy of that region of terms involving ξ or $\dot{\xi}$, so that if we alter the values of ξ or $\dot{\xi}$ by external means there will be either an absorption or an emission of energy in the region which is the seat of the force of type ξ. This is one method of determining the seat of a force. The kind of energy absorbed or emitted will depend on the coordinates with which ξ and $\dot{\xi}$ are associated in the expression for the kinetic energy.

§ 3. We shall now go through the various types of the terms which involve the

products of the differential coefficients of coordinates of different kinds, and see whether they exist or not.

1. Terms of the type $\{xy\}\dot{x}\dot{y}$: we have just seen that these terms do not exist. See also MAXWELL'S 'Electricity and Magnetism,' vol. ii., chap. 7.

2. Terms of the type $\{xz\}\dot{x}\dot{z}$: z is a coordinate fixing the magnetic configuration; we can see just as in the last case that the existence of this term would require the production of a magnetic force by a body whose velocity is changing depending on the rate of change of the velocity, as well as other similar effects. As these have not been observed, we conclude that this term does not exist.

3. Terms of the type $\{xu\}\dot{x}\dot{u}$: u is a coordinate helping to fix the temperature; as the "u's" only enter the expression for the kinetic energy through the temperature, *i.e.*, through terms of the type $\{uu\}\dot{u}^2$, we see that these terms do not exist.

4. Terms of the type $\{xw\}\dot{x}\dot{w}$: w is a coordinate helping to fix the strain configuration of the system; these may exist in a vibrating solid which has also got a motion of translation, for the velocity of any point in a small element of the solid equals the velocity of the centre of gravity of that element plus the velocity of the point relatively to the centre of gravity. This latter velocity will involve $\dot{w}$, so that the square of the velocity, and therefore the kinetic energy, may involve the product $\dot{x}\dot{w}$.

5. Terms of the type $\{yz\}\dot{y}\dot{z}$: the production of a magnetic field by an electric current such that the direction of magnetic force is reversed when the direction of the current is reversed shows that terms of this type must exist.

6. Terms of the type $\{yu\}\dot{y}\dot{u}$: the reasoning given when we were discussing (3) terms of the type $\{xu\}\dot{x}\dot{u}$ shows that terms of this type do not exist.

7. Terms of the type $\{yw\}\dot{y}\dot{w}$: by reasoning in the same way as we did about the general term $[\lambda\mu]\dot{\lambda}\dot{\mu}$, we can see that if this term exists a varying current will produce strains differing from those produced by a steady current, and also an electromotive force could be produced by merely altering the state of strain of a body. As these effects are not known to occur, we conclude that terms of this type do not exist.

8. Terms of the type $\{zu\}\dot{z}\dot{u}$: the reasoning given when we considered the term (3) shows that terms of this type do not exist.

9. Terms of the type $\{zw\}\dot{z}\dot{w}$: terms of this type would correspond to effects analogous in character to those due to the terms of the type $\{yw\}\dot{y}\dot{w}$, and as these have not been observed, we conclude that terms of this type do not exist.

10. Terms of the type $\{uw\}\dot{u}\dot{w}$: the reasoning given when we considered the term (3) shows that terms of this type do not exist.

We thus see that all the terms which occur in the expression for the kinetic energy of any real material system are of one or other of the following types :—

$$\{xx\}\dot{x}^2;$$

$$\{yy\}\dot{y}^2;$$

$$\{zz\}\dot{z}^2;$$

$$\{uu\}\dot{u}^2;$$

$$\{ww\}\dot{w}^2;$$

$$\{xw\}\dot{x}\dot{w};$$

$$\{yz\}\dot{y}\dot{z}.$$

§ 4. We must now proceed to examine the terms of these types in greater detail, and see what coordinates the coefficients $\{xx\}$, $\{yy\}$, &c., involve.

Let us commence with the term $\{xx\}\dot{x}^2$, which corresponds to the expression for the kinetic energy in the ordinary dynamics of a rigid body. We have to consider what coordinates the quantity $\{xx\}$ can be a function of. We know that it may be a function of the geometrical coordinates x, but we need not consider here the consequences of this, as they are fully investigated in treatises on ordinary dynamics. Next, $\{xx\}$ may be a function of the electrical coordinates y, for in a paper published in the Philosophical Magazine for April, 1881, I have shown that the kinetic energy of a small sphere of mass m, charged with a quantity e of electricity, and moving with a velocity v, is

$$\left\{\tfrac{1}{2}m+\frac{2}{15}\frac{\mu e^2}{a}\right\}v^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where a is the radius of the sphere, and μ the magnetic permeability of the medium surrounding the sphere. Thus $\{xx\}$ may be a function of the electrical coordinates. The easiest way of finding the effects of the electrification on the motion of the body is to notice that by equation (9) these effects are exactly the same as those due to an increase $4\mu e^2/15a$ in the mass of the sphere, so that whenever an electric charge is communicated to a moving sphere its velocity will be impulsively changed. If the sphere is in air there will, of course, be a limit to the quantity of electricity which can be accumulated on it, and so a limit to the apparent increase in mass. Taking Dr. MACFARLANE'S value for the electric strength of air (Phil. Mag., Dec., 1880), viz., 75, as the intensity in electrostatic measure in C.G.S. units of the greatest force which a fairly thick layer of air can bear, it is easy to see from equation (9) that the ratio of the greatest apparent increase in mass to the mass of the sphere is of the order $3\cdot10^{-19}/\rho$ where ρ is the density, measured in C.G.S. units, of the sphere which is supposed to be solid. If the charge is on the surface of a thin spherical shell, the ratio will be greater in the proportion of the radius of the shell to three times its thickness.

Let us now go on to consider the electrical effects of this term. Since the potential

energy of a charged sphere is $e^2/2c$, where c is the capacity of the sphere, we see from equation (9) that the motion of the sphere will alter the coefficient of e^2 in the expression for the energy, and will therefore alter the capacity. To find this alteration let us suppose that the charge on the sphere is increased by δe, and that Q is the external electromotive force acting on the sphere, the energy of the system is

$$\left(\frac{m}{2}+\frac{2}{15}\frac{\mu e^2}{a}\right)v^2+\frac{1}{2}\frac{e^2}{Ka}, \quad \ldots\ldots\ldots \quad (10)$$

if K be the specific inductive capacity of the medium surrounding the sphere, if the increment in v be δv, and in e δe, the increment in the energy

$$=\left(m+\frac{4}{15}\frac{\mu e^2}{a}\right)v\delta v+\frac{4}{15}\frac{\mu v^2}{a}e\delta e+\frac{e\delta e}{Ka} \quad \ldots\ldots\ldots \quad (11)$$

but by the conservation of energy this must equal

$$Q\delta e$$

Now since the momentum of the sphere is not altered

$$\left(m+\frac{4}{15}\frac{\mu e^2}{a}\right)\delta v+\frac{8}{15}\frac{\mu e}{a}v\delta e=0 \quad \ldots\ldots\ldots \quad (12)$$

eliminating δv between these equations we find

$$\frac{e}{Ka}\left\{1-\frac{4}{15}\mu K v^2\right\}=Q \quad \ldots\ldots\ldots \quad (13)$$

so that the capacity of the sphere is increased in the ratio of 1 to $1-\frac{4}{15}\mu K v^2$; or since by the electromagnetic theory of light $\mu K=1/u^2$ where u is the velocity of light, the capacity of the sphere is increased in the ratio of 1 to $1-\frac{4}{15}\frac{v^2}{u^2}$, and thus the alteration depends on the square of the ratio of the velocity of the sphere to the velocity of light, and will consequently be very small. If the earth does not carry the ether with it, the alteration in velocity which occurs at a point on the earth's surface will produce a diurnal variation in the capacity of a condenser at that point. If the condenser is a sphere, the maximum diurnal variation in its capacity, which will be when the direction of motion of the solar system, is parallel to the direction of motion of the earth in its orbit: is about 4×10^{-8} per cent. of the capacity of the condenser. This is much too small to observe, but it is remarkable that the capacity of a condenser should depend upon its velocity.

There seems nothing to show that $\{xx\}$ is a function of the magnetic coordinates z,

and it cannot be a function of the temperature coordinates u because these coordinates are kinosthenic, *i.e.*, they only enter the expression for the kinetic energy through their differential coefficients.

In order to see that $\{xx\}$ may be a function of the strain coordinates w, it is convenient to notice that the kinetic energy of a system in ordinary dynamics may be written in the form

$$M\dot{\bar{x}}^2+Mk^2\dot{\theta}^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where M is the mass, $\bar{x}$ a coordinate fixing the position of the centre of gravity, k a radius of gyration, and θ the angle made by a line fixed in the body with a line fixed in space. Terms of the type $M\dot{\bar{x}}^2$ evidently cannot involve the w coordinates, but terms of the type $Mk^2\dot{\theta}^2$ may; for take the simple case of a bar which is compressed at its middle and extended at its ends, rotating about an axis through its centre, it is easy to see that the moment of inertia of this rod about the axis of rotation is less than it would be if the rod were unstrained, and thus Mk^2 may be a function of the strain components. These components will in general only enter through the expression for the alteration in the density, *i.e.*, using THOMSON and TAIT'S notation through the expression $d\alpha/dx+d\beta/dy+d\gamma/dz$, and this expression will only occur raised to the first power; if we employ the energy method of forming the equations of elasticity, we easily find that the presence of this term leads to the introduction of the so-called "centrifugal force" into the equations of elasticity for a rotating elastic solid.

Collecting our results we see that $\{xx\}$ may be a function of the coordinates x, y and w, but not of u and probably not of z.

§ 5. The next terms which we have to consider are the terms of the type $\{yy\}\dot{y}^2$, which is supposed to include terms of the form $\{y_1y_2\}\dot{y}_1\dot{y}_2$.

Now since the kinetic energy of a circuit carrying a current $\dot{y}$ is $\frac{1}{2}L\dot{y}^2$ if L be the coefficient of self-induction of the circuit, we see that $\{yy\}$ is a coefficient of self-induction; and since the coefficient of self-induction depends on the shape of the circuit $\{yy\}$ will be a function of the geometrical coordinates which fix the shape of the circuit: since there is a mechanical force between two circuits conveying electric currents, $\{y_1y_2\}$ must be a function of the geometrical coordinates which fix the relative position of the circuits, for if it were not so, we see by LAGRANGE'S equations that there would be no mechanical action between the circuits. Let us suppose that we have two circuits, and that the current in one circuit is $\dot{y}_1$ and the current in the other $\dot{y}_2$. Let the kinetic energy of the system be

$$\tfrac{1}{2}L\dot{y}_1^2+M\dot{y}_1\dot{y}_2+\tfrac{1}{2}N\dot{y}_2^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

then if x be any coordinate fixing the position of one circuit with respect to the other, LAGRANGE'S equation for x shows that there will be a force tending to increase x equal to

$$\tfrac{1}{2}\frac{dL}{dx}\dot{y}_1^2+\frac{dM}{dx}\dot{y}_1\dot{y}_2+\tfrac{1}{2}\frac{dN}{dx}\dot{y}_2^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Now we see that dL/dx and dN/dx must vanish, otherwise there would be a force between the two circuits when the current in one of them vanished, thus we see that the coefficients of self-induction are independent of the position of the other circuits in the neighbourhood. Thus the force tending to increase x reduces to

$$\frac{dM}{dx}\dot{y}_1\dot{y}_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

We see also from LAGRANGE'S equation for the coordinate y_1 that

$$\frac{d}{dt}\frac{dT}{d\dot{y}_1}-\frac{dT}{dy_1}=\text{external force of type } y_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

But $\frac{dT}{dy_1}=0$, as we shall see directly, and therefore

$$\frac{d}{dt}(L\dot{y}_1+M\dot{y}_2)=\text{external force of type } y_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

so that the term $d(L\dot{y}_1+M\dot{y}_2)/dt$ will produce the same effect as an external electromotive force equal to

$$-\frac{d}{dt}(L\dot{y}_1+M\dot{y}_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

and thus there is an electromotive force of this magnitude acting round the circuit through which the current $\dot{y}_1$ flows. This expression expresses the law of the induction of currents, either by the motion of neighbouring circuits which convey currents or by the alteration in the magnitude of the currents flowing through these circuits. This example is given in MAXWELL'S 'Electricity and Magnetism,' vol. ii., chap. 7; it illustrates the power of the method very well, as the existence of a mechanical force between two circuits showed that there was a term of the form $M\dot{y}_1\dot{y}_2$ in the expression for the kinetic energy, and from this the law of induction followed at once by LAGRANGE'S equations. There is no experimental evidence to show that $\{yy\}$ is ever a function of y the electrical coordinates, and when the system consists of a lot of conducting circuits it certainly is not, for if it were the coefficients of self and mutual induction in a system of circuits would depend upon the length of time the current had been flowing through the circuits; in any case it would involve the existence of electromotive forces which would not be reversed if the directions of all the electric displacements in the field were reversed.

There seems also good reason for believing that $\{yy\}$ is not a function of the

magnetic coordinates z, for if it were, a current $\dot{y}_1$ would produce a magnetic force proportional to $\dot{y}_1^2$, and thus the force would not be reversed if the direction of the current were reversed; as no such forces have been observed we conclude that $\{yy\}$ is not a function of z. It cannot be a function of the temperature coordinates u because these are kinosthenic coordinates.

The question as to whether $\{yy\}$ is a function of the elastic coordinates w or not is one where the experimental evidence is somewhat conflicting. Both WERTHEIM (Ann. de Chim. et de Phys. [3], 12, p. 160, or WIEDEMANN'S 'Lehre von der Elektricität,' vol. ii., p. 403) and TOMLINSON (Phil. Trans., 1882) have observed that the elasticity of a wire is less when a current is passing along the wire than when it is not, and that this diminution in the elasticity is not due to the heat generated by the current. STREINTZ (Wien. Ber. [2], 67, p. 323, or WIEDEMANN'S 'Lehre von der Elektricität,' vol. ii., p. 404), on the other hand, was unable to detect any such effect. Supposing that the passage of a current of electricity along a wire does alter the elasticity of it there must be terms in the kinetic energy of the form

$$\dot{y}^2\{A(e+f+g)^2+B(e^2+f^2+g^2-2ef-2ge-2fg+a^2+b^2+c^2)\} \quad . \quad . \quad (21)$$

where e, f, g, a, b, c denote as before the six strains; comparing this expression with that for the potential energy of a strained solid, and remembering that this energy is kinetic and not potential, we see that the rigidity is *diminished* by $B\dot{y}^2$ and the bulk modulus by $(A-B/3)\dot{y}^2$. Let us consider what electrical effects this term will indicate; since half the coefficient of $\dot{y}^2$ in the expression for the kinetic energy is the coefficient of self induction of the circuit conveying the current $\dot{y}$: we see that if this term exists the coefficient of self-induction of a circuit will be increased by straining the wire which forms the circuit; it will be increased because WERTHEIM'S experiments show that the elasticity was diminished by the passage of a current, and therefore that B is positive; so that if a current be flowing along a wire it will be momentarily diminished if the wire be twisted. If the coefficients of induction are altered by straining the wire the force between two currents or two elements of current will be altered by straining the wires along which they flow, even although the intensity of the currents is kept constant; this is contrary to AMPÈRE'S hypothesis that the force between two elements of a circuit conveying a current depends upon nothing but the strength of the currents and the position of the elements. If this term exists we shall see if we make a numerical calculation that the alteration in the coefficient of self-induction of a coil due to the straining of the wire of which it is made is likely to be much more easily detected than the corresponding alteration in the elasticity produced by the passage of a current of electricity. For using the C.G.S. system of units the coefficients of elasticity are quantities of the order 10^{11}; thus, taking copper as an example, YOUNG'S modulus is $1{\cdot}234\times10^{12}$ and the coefficient of rigidity is $4{\cdot}47\times10^{11}$, so that we shall be under the mark if we take 10^{11} as the value of a coefficient of

elasticity for copper. Thus, supposing that we can measure a coefficient of elasticity correct to one part in a thousand, in order that we may be able to detect the alteration in the coefficient of elasticity produced by a current of 10 ampères—whose value in absolute measure is unity—the quantities A and B must be at least $10^{11}/1000$ or 10^8. Now we can produce strains in a copper wire greater than 1/10000 without breaking the wire, and if we produce strains of this magnitude the alteration in the coefficient of self-induction will be between 1 and 10 per centimetre of wire in the coil; supposing A to be 10^8; this is a very large change in the coefficient of self-induction, and ought to be easily detected, much more easily than the corresponding changes in the elasticity of a copper wire produced by the passage of a current of electricity.

To sum up, we see that $\{yy\}$ is a function of the coordinates of the type x, but not a function of the y, z, or u coordinates, while it is doubtful whether it is a function of the strain coordinates w or not.

§ 6. Let us now consider the terms of the type $\{xz\}\dot{x}^3$, these are the terms which express the magnetic energy of the system. In § 1 we discussed a method of fixing the magnetic configuration by means of coordinates. To fix the intensity of magnetisation we use two coordinates, η and ζ, of which η is a coordinate of a geometrical kind, and ζ a kinosthenic coordinate, and they are chosen so that the intensity of magnetisation is $\eta\xi$, where ξ is the momentum of the type ζ. Since ζ is a kinosthenic coordinate ξ is constant, it is therefore convenient to use ξ instead of $\dot{\zeta}$ in the expression for the kinetic energy. Now Routh ('Stability of Motion,' p. 62) has shown that if θ be a coordinate and $d\mathrm{T}/d\dot{\theta}=a$, then if we eliminate θ' from the expression

$$\mathrm{T}'=\mathrm{T}-a\theta' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

we may use Lagrange's equations for all the other coordinates if we use the function T′ instead of T. Let us apply this theorem to the case we are considering: suppose we have a term of the form $\mathrm{A}\dot{\zeta}^2$ in the expression for the kinetic energy T, we must eliminate $\dot{\zeta}$ by the use of the equation

$$\frac{d\mathrm{T}}{a\dot{\zeta}}=\xi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

and use T′ instead of T where

$$\mathrm{T}'=\mathrm{T}-\dot{\zeta}\xi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

thus the term containing ξ^2 in T′ is

$$-\tfrac{1}{2}\frac{\xi^2}{\mathrm{A}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

this is of the same magnitude as the corresponding term in T but of opposite sign. [This change of sign is instructive if, as we have done in this paper, we regard potential energy as kinetic energy due to kinosthenic coordinates, for it explains at once why the Lagrangian function is T−V and not T+V; for as we saw above, the

kinetic energy due to the kinosthenic coordinates enters with the negative sign into the equations of motion.] As we suppose that $\xi\eta$ is the intensity of magnetisation, it is convenient to write (25) as

$$-\tfrac{1}{2}\frac{\eta^2\xi^2}{B} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

where $B=A\eta^2$.

If H be the external magnetic force, the work done when the intensity of magnetisation is increased by $\delta(\eta\xi)$ is $-H\delta(\eta\xi)$ or as ξ remains constant $-H\xi\delta\eta$, and thus LAGRANGE's equations as modified by ROUTH give

$$\frac{d}{dt}\frac{dT'}{d\dot{\eta}}-\frac{dT'}{d\eta}=H\xi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

or when things have settled down to a steady state

$$-\frac{dT'}{d\eta}=H\xi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

applying this equation to the term $-\frac{1}{2}\eta^2\xi^2/B$ we get

$$\frac{\xi\eta}{B}\left\{1-\tfrac{1}{2}\eta\frac{\frac{dB}{d\eta}}{B}\right\}=H \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

Thus, since $\xi\eta$ is the intensity of magnetisation and H the magnetising force, the coefficient of magnetic induction, which we shall denote by k, will be given by the equation

$$k=\frac{B}{1-\frac{1}{2}\frac{\eta}{B}\frac{dB}{d\eta}}=\frac{B}{1-\frac{1}{2}\frac{d\log B}{d\log\eta}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

At present we shall only consider magnetic induction, and suppose that there is no permanent magnetisation in the part of the system which we are considering.

[When a piece of soft iron is placed in a magnetic field where the force is, for the sake of simplicity, supposed to be parallel to x and equal to X, the energy is $-IX$ where I is the intensity of magnetisation, since $I=\xi\eta$, the energy $=-\xi\eta X$.

Now when the energy is expressed in terms of the velocities, LAGRANGE's equation gives

$$\frac{d}{dt}\frac{dT}{d\dot{x}}-\frac{dT}{dx}=\text{external force parallel to } x \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

so that the existence of T implies a force parallel to x equal to $d\mathrm{T}/dx$, but if T' is the energy expressed in terms of the momenta instead of the velocities,

$$\frac{d\mathrm{T}'}{dx} = -\frac{d\mathrm{T}}{dx},$$

so that the force parallel to $x = -d\mathrm{T}'/dx$. As in our case the energy is expressed in terms of the momenta, and not of the velocities, we see that the force parallel to x

$$\left.\begin{aligned} &= \xi\eta \frac{d\mathrm{X}}{dx} \\ &= \mathrm{I} \frac{d\mathrm{X}}{dx} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (32)$$

or since $\mathrm{I} = \mathrm{KX}$

$$= \tfrac{1}{2} \frac{d}{dx} \{\mathrm{KX}^2\} \quad \ldots\ldots\ldots\ldots \quad (33)$$

the usual expression, though (32) is preferable.*]

$\{zz\}$ cannot be a function of the electrical coordinates y, because if it were electromotive forces due to magnetised bodies would exist which would not be reversed when the magnetism of all the bodies in the field was reversed.

$\{zz\}$ cannot be a function of the quantity ξ which helps to fix the intensity of magnetisation, for if it were the kinetic energy would no longer be a quadratic function of ξ, but would involve higher powers. It may, however, be a function of η, and will be so if the coefficient of magnetic induction depends upon the intensity of magnetisation. As experiments show that this is the case we conclude that $\{zz\}$ is a function of η.

A large number of experiments have been made in order to find the value of the coefficient of magnetic induction for all values of the intensity of magnetisation; all these experiments agree in showing that the coefficient diminishes as the intensity of magnetisation increases, so much so indeed that there is a maximum value of the intensity of magnetisation which cannot be exceeded however intense the magnetising force may be. As the experiments are not sufficiently consistent to lead to the determination of the law connecting the intensity of magnetisation with the coefficient of magnetic induction, all that we can do is to take some empirical law which agrees with one set of experiments and deduce its consequences. It is probable that the results will in their main features agree with those which could be deduced from the true law. For our purpose we shall take the empirical law proposed by STEFAN (WIEDEMANN'S 'Lehre von der Elektricität,' vol. 3, p. 432), which is expressed by the equation

* This paragraph has been re-written since the paper was sent in to the Royal Society.

$$\frac{1}{k}=\frac{aI_0}{I^{1-n}(I_0-I)^n}-b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

where k is the coefficient of magnetisation, I the intensity of magnetisation, I_0 the maximum value of this intensity, and a, b, n constants, n being less than unity. If we express the intensity of magnetisation in terms of ξ and η the corresponding law will be

$$\frac{1}{k}=\frac{a'}{\eta^{1-n}(\eta_0-\eta)^n}-b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

where a' is a new constant and η_0 the maximum value of η.

But by equation (32)

$$\frac{1}{k}=\frac{1}{2\eta}\frac{d}{d\eta}\left(\frac{\eta^2}{B}\right)$$

so that

$$-\frac{1}{2\eta}\frac{d}{d\eta}\left(\frac{\eta^2}{B}\right)=b-\frac{a'}{\eta^{1-n}(\eta_0-\eta)^n}$$

therefore

$$-\frac{\eta^2}{B}=b\eta^2-2a'\int_0^\eta\frac{d\eta}{\eta^{-n}(\eta_0-\eta)^n}$$

or

$$-\frac{1}{B}=b-\frac{2a'}{\eta^2}\int_0^\eta\frac{\eta^n}{(\eta_0-\eta)^n}d\eta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

When $\eta=\eta_0$, *i.e.*, when η has its maximum value

$$-\frac{1}{B}=b-\frac{2a'}{\eta_0}\frac{\Gamma(1+n)\Gamma(1-n)}{\Gamma(2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

where Γ denotes the ordinary gamma function. But

$$\Gamma(1+n)\Gamma(1-n)=\frac{n\pi}{\sin n\pi}$$

and

$$\Gamma(2)=1$$

so that in this case

$$-\frac{1}{B}=b-\frac{2a'}{\eta_0}\frac{n\pi}{\sin n\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

this expression shows that $1/B$ is finite when $\eta=\eta_0$, but equation (34) shows that $1/k$ is infinite when $I=I_0$, which corresponds to $\eta=\eta_0$, so that when the magnetic force is so great that the iron or other magnetisable substance is magnetised to saturation

$-B$ and k will differ widely. According to equation (32) the force acting on a piece of soft iron placed in a magnetic field is

$$\tfrac{1}{2}\frac{d}{dx}\{IX\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

if I be the intensity of magnetisation, but according to the ordinary formula it is equal to the differential coefficient with respect to X of the energy per unit volume

$$=\frac{d}{dx}\left\{\frac{I^2}{B}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

Now we have just seen that B and k differ widely when I approaches the value which it has when the iron is magnetised to saturation, so that if our view be correct the ordinary formula will not give correct results in this case.

$\{zz\}$ cannot be a function of the heat coordinates u, as these are kinosthenic.

JOULE'S discovery that a bar of soft iron lengthens when magnetised in the longitudinal direction, and that the increment in length is proportional to the square of the intensity of the magnetisation, shows that $\{zz\}$ is a function of the strain coordinates w. We shall consider in some detail a few of the consequences of this result. Let us suppose that we longitudinally magnetise a soft iron bar, whose length is in the direction of the axis of x; let e, f, g be the dilatations at any point of the bar parallel to the axes of x, y, z respectively.

Then neglecting for the present any torsion there may be in the bar the potential energy V of the strained bar will in the usual notation be

$$\tfrac{1}{2}m(e+f+g)^2+\tfrac{1}{2}n(e^2+f^2+g^2-2ef-2eg-2fg) \quad . \quad . \quad . \quad . \quad . \quad (41)$$

But if T′ has the same meaning as in equation (22)

$$T'=\text{terms not depending on the magnetic coordinate}-\tfrac{1}{2}\frac{\eta^2\xi^2}{B}$$

$$= \quad ,, \quad ,, \quad ,, \quad ,, \quad -\tfrac{1}{2}\frac{I^2}{B} \quad . \quad . \quad . \quad (42)$$

if I be the intensity of magnetisation. JOULE'S discovery shows that B is a function of the strain coordinates e, f, g. If there is no external force tending to strain the bar, the modified Lagrangian equation gives, when things are in a steady state,

$$-\frac{dT'}{de}+\frac{dV}{de}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

with similar equations for f and g.

or considering only the term $-I^2/2B$ in the kinetic energy we have

$$\tfrac{1}{2}I^2\frac{d}{de}\frac{1}{B}+m(e+f+g)+n(e-f-g)=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

Now JOULE found that the volume of the bar was not altered, so that $e+f+g=0$ and the last equation becomes

$$\tfrac{1}{2}I^2\frac{d}{de}\left(\frac{1}{B}\right)+2ne=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

$$\text{or } ne=\tfrac{1}{4}\frac{d}{de}\left(-\frac{I^2}{B}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

as the bar lengthens when it is magnetised e is positive, and thus $-1/B$ increases with e.

Going back to equation (29) we have, if H be the external magnetic force,

$$\tfrac{1}{2}\frac{d}{d\eta}\left(\frac{\eta^2\xi}{B}\right)=H \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

or as it may be written

$$I\frac{d}{dI^2}\left(\frac{I^2}{B}\right)=H \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

Let us suppose that the magnetising force H remains constant, then I will increase if $-d(I^2/B)/dI^2$ diminishes, and diminish if this quantity increases. If the change in this quantity is due to an increase δe in the elongation, equation (48) shows that the corresponding increase δI in the intensity of magnetisation will be given by the equation

$$\delta I=-I\frac{\dfrac{d}{de}\dfrac{d}{dI^2}\left(\dfrac{I^2}{B}\right)}{\dfrac{d}{dI^2}\left(\dfrac{I^2}{B}\right)}\delta e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (49)$$

but if k be the coefficient of magnetic induction

$$1/k=\frac{d}{dI^2}\left(\frac{I^2}{B}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

by equation (48); so that

$$\delta I=-\frac{I}{k}\frac{dk}{de}\delta e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51)$$

but if E be the elongation of the bar produced when the bar is magnetised so that the intensity of magnetisation is I, equation (46) shows that

$$n\frac{dE}{dI^2} = -\tfrac{1}{2}\frac{d}{dI^2}\frac{d}{de}\left(\frac{I^2}{B}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

$$= \frac{1}{4k^2}\frac{dk}{de} \text{ by equation (50)}$$

so that by equation (51)

$$\delta I = -\tfrac{1}{4}Ink\frac{dE}{dI^2}\delta e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

Now, since the elongation increases with the intensity of magnetisation—at any rate when the latter is strong—dE/dI^2 is positive, so that from equation (53) δe and δI will have the opposite sign, so that if the strain of a strongly magnetised soft iron bar be increased the intensity of its magnetisation will be diminished. The experiments of VILLARI and Sir WILLIAM THOMSON confirm this result, but each of these physicists found that if the intensity of magnetisation was below a certain critical value, an increase in the strain was accompanied by an increase in the intensity of magnetisation; equation (53) shows that when this is the case dE/dI^2 must be negative, so that a soft iron bar will contract on magnetisation when the magnetising force is small. I have not been able to find any experiments on the extension of soft iron bars under the action of small magnetising forces. Sir WILLIAM THOMSON found that the critical force was about thirty times the earth's vertical force at Glasgow, so that to test this point the magnetising force ought to be less than this value. The critical value of the magnetisation, i.e., the intensity of magnetisation when it is not altered by slightly straining the bar, is given by the equation

$$\frac{d}{de}\frac{d}{dI^2}\frac{I^2}{B} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54)$$

Sir WILLIAM THOMSON'S experiments show that this value of I depends upon the state of strain of the bar; hence we see that equation (54) must involve e, so that if 1/B be expanded in powers of e it must contain powers of e about the first, but if 1/B involves the square of e there will be a term in the expression for the kinetic energy of the form AI^2e^2, and as the coefficients of elasticity depend upon the coefficients of e^2 in the expressions for the kinetic and potential energies, we see that in this case the elasticity of a soft iron wire will be altered by magnetisation. Many experiments have been made to detect this effect, but with negative results; an investigation similar to that on page 313 will show however that the effect on the critical value of magnetisation will be more easily detected than the effect on the elasticity. Since

1/B contains powers of e higher than the first, the terms which involve e may be written in the form

$$\alpha e+\beta e^2+\ldots$$

Thus, if we neglect the terms containing higher powers of e than the second, $d(1/B)de$, which equals $\alpha+2\beta e$, will change sign when $e=-\alpha/2\beta$. Equation (46) shows that a soft iron bar will expand or contract on magnetisation according as the strain e in it makes the expression $\alpha+2\beta e$ negative or positive; this changes sign when e passes through the critical value $-\alpha/2\beta$, so that a soft iron bar when strained beyond a certain limit will contract instead of expanding when it is magnetised. Some experiments which JOULE made on the effect of magnetising soft iron bars which were under great tension confirm this result. JOULE found (STURGEON'S "Annals of Electricity, 1842," Phil. Mag., 1847) that when soft iron wires were stretched beyond a certain limit they became shorter instead of longer when they were magnetised; since the limiting strain is an extension, the limiting value of e, $-\alpha/2\beta$ must be positive, so that α and β must be of opposite signs; and as α is negative when the magnetisation is intense, β must in this case be positive, and if β be positive the coefficients of elasticity will be increased as we can see from equation (41), so that we conclude that the elasticity of a soft iron bar will be diminished by strong magnetisation, *i.e.*, the same force will not stretch it so much.

A great many relations exist between torsion and magnetism, but many of these are relations between permanent magnetism and permanent set, both of which are outside the scope of this paper. We shall confine ourselves to the relations existing between temporary magnetisation and twists which are not large enough to give the body any permanent set. Since the magnetisation of a soft iron bar is altered by twisting it, 1/B (using the same notation as before) must be a function of the torsion coordinates a, b, c: let us suppose as before that the length of the bar is parallel to the axis of x, then the coordinate a will fix the rotation of the bar round this axis.

The couple tending to twist the bar is by the modified Lagrangian equation for a, equal to $d\mathrm{T}'/da$, and this equals

$$-\tfrac{1}{2}\frac{d}{da}\left(\frac{\mathrm{I}^2}{\mathrm{B}}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

if ω be the angle through which this couple twists the bar and n the coefficient of rigidity,

$$\omega=-\frac{1}{2n}\frac{d}{da}\left(\frac{\mathrm{I}^2}{\mathrm{B}}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (56)$$

When a twisted bar is magnetised it untwists to a certain extent (WIEDEMANN'S 'Lehre von der Elektricität,' vol. 3, p. 692), but if an untwisted bar be magnetised it does not twist at all; this shows that if 1/B be expanded in ascending powers of a, the first power must be absent, for if it were present an untwisted rod would become twisted when it was magnetised. By a similar investigation to that by

which we established equation (53), we find that if δI be the change in the intensity of magnetisation when the twist in the bar is increased by an angle $\delta\omega$, and k the coefficient of magnetisation, then

$$\delta I = -2nIk\frac{d\omega}{dI^2}\delta\omega \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (57)$$

Now since a soft iron wire untwists when the magnetisation is increased, $d\omega/dI^2$ is negative, so that δI and ϖ have the same sign if k be positive; hence we see that the magnetisation is increased by twisting the rod: this agrees with WIEDEMANN'S result. He found, however, that when the twist exceeded a certain value the magnetisation was diminished instead of being increased by twisting. Equation (57) shows that in this case $d\omega/dI^2$ must be positive, or when the original twist is great the bar will no longer be untwisted when it is magnetised but twisted. Professor G. WIEDEMANN'S experiments show that this is the case.* There are many other relations between magnetism and torsion which can be investigated in this way, but we have no space to consider them here.

§ 7. We must now go on to consider the part of the kinetic energy which depends upon the temperature, and which we denote by $\{uu\}\dot{u}^2$. We considered before a way of fixing the temperature by means of coordinates: we supposed that it was fixed by the coordinates $u_1, u_2, \ldots u_n$. Since the temperature depends upon the whole of the terms involving u, and not in a special way upon any one term in particular, we see that if one of the u's enters into any term it must be because the whole of that part of the kinetic energy which depends upon the differential coefficients of the u's enters into the term. Let us, as before, call this part of the energy Θ, then if $v_1, v_2, v_n, \ldots$ be the momenta of the type $u_1, u_2, \ldots u_n$ we have by equation (8)

$$\Theta = \tfrac{1}{2}\left\{\frac{v_1^2}{\{u_1, u_1\}} + \frac{v_2^2}{\{u_2, u_2\}} + \ldots\right\}$$

As Θ is the energy in unit of volume the total amount of energy of this kind is

$$\iiint \Theta dxdydz \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58)$$

As the consideration of the terms of the type $\{uu\}\dot{u}^2$ is somewhat difficult, in order to simplify it we shall neglect all the effects due to radiation, which is equivalent to supposing that the bodies are incapable of radiating or absorbing heat.

If this is so $\{uu\}$ cannot be a function of the geometrical coordinates x, for if it were it would be possible to alter the temperature of a body by merely moving it about.

* 'Die Lehre von der Elektricität.' Dritter Band. S. 688.

The phenomena of thermo-electricity show that $\{u, u\}$ must be a function of the electrical coordinates y, but it is difficult to determine the form of this function with certainty. The two most striking phenomena in thermo-electricity are (1) the PELTIER effect which shows that electromotive forces exist at the junction of different metals in the thermoelectric circuit, and (2) the THOMSON effect which shows that electromotive forces exist throughout an unequally heated conductor. We shall try to find a term whose presence in the expression for the kinetic energy would correspond to these effects. Let us consider a circuit formed by two metallic wires each parallel to the axis of x, let p, q, r denote the components of the electric displacement parallel to the axes of x, y, z respectively, let us suppose that we have in the expression for the kinetic energy per unit of volume the term

$$\kappa\left(\frac{dp}{dx}+\frac{dq}{dy}+\frac{dr}{dz}\right)\Theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

where κ is a quantity which depends upon the material of which the wire is made. Thus the expression for the energy of the wires will contain the term

$$\iiint\kappa\left(\frac{dp}{dx}+\frac{dq}{dy}+\frac{dr}{dz}\right)\Theta dxdydz \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

In order to avoid considerations about discontinuity let us suppose that κ instead of changing abruptly as we go from one wire to another changes very rapidly but continuously throughout a small region enclosing the junction of the wires, when we wish to pass to the actual case we shall suppose the rate of variation to increase and the size of the region to diminish indefinitely.

Now

$$\iiint\kappa\left(\frac{dp}{dx}+\frac{dq}{dy}+\frac{dr}{dz}\right)\Theta dxdydz$$

$$=-\iint\kappa\Theta(lp+mq+nr)dS-\iiint(p\frac{d}{dx}(\kappa\Theta)+q\frac{d}{dy}(\kappa\Theta)+r\frac{d}{dz}(\kappa\Theta))dxdydz \quad . \quad (61)$$

where dS is an element of the surface bounding the region through which we integrate, and l, m, n are the direction cosines of the normal to this surface drawn inwards. The case we are considering is that of two wires parallel to the axis of x, for by far the greater part of their length the bending which is necessary to make their ends join being supposed to occupy an indefinitely small space. Let us suppose that the electric displacement is parallel to the axis of x, then the surface integral vanishes and the term under consideration

$$=-\iiint(p\frac{d}{dx}(\kappa\Theta)+q\frac{d}{dy}(\kappa\Theta)+r\frac{d}{dz}(\kappa\Theta))dxdydz \quad . \quad . \quad . \quad . \quad . \quad (62)$$

or since q and r both vanish

$$=-\iiint p\frac{d}{dx}(\kappa\Theta)dxdydz \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63)$$

We see by LAGRANGE's equation for p that this corresponds to an electromotive force at each point of the bar equal to

$$-\frac{d}{dx}(\kappa\Theta) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64)$$

and if we remember that $d\kappa/dx$ becomes infinite in the limit at the junction of the wires we see by integrating over this region that the potential close to the junction in the wire (2) will exceed that close to the junction in wire (1) by $-(\kappa_2-\kappa_1)\Theta$, where κ_2 and κ_1 are the values of κ in the wires (2) and (1) respectively.

The force $-d(\kappa\Theta)/dx$ is that which corresponds to the THOMSON effect; the difference of potentials at the junction is analogous to the PELTIER effect; it is, however, probable that other terms of the form

$$\iiint\rho\kappa\Theta dxdydz$$

may occur in the expression for the kinetic energy near the junction, and the effects due to these terms may add on to the others; indeed, if the other terms explain the THOMSON effect, there must be some additional terms of this kind required to explain the PELTIER effect, for otherwise the total electromotive force round the circuit would vanish.

Since

$$\frac{dp}{dx}+\frac{dq}{dy}+\frac{dr}{dz}=\rho$$

where ρ is the volume density of the free electricity, we may write

$$\iiint\kappa\left(\frac{dp}{dx}+\frac{dq}{dy}+\frac{dr}{dz}\right)\Theta dxdydz$$

as

$$\iiint\kappa\rho\Theta dxdydz$$

If we keep the term in this form, and consider the temperature effects to which it corresponds, we are led to suspect the existence of some very remarkable phenomena.

Writing Θ in full we see that in the expression for the energy per unit volume there is the term

$$\kappa e\{\{u_1u_1\}\dot{u}_1^2+\{u_2u_2\}\dot{u}_2^2+\ldots\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

if e be the charge of electricity per unit volume, or if we write v_1, v_2 ... for the momenta corresponding to u_1, u_2 ... we see that the energy may be expressed as

$$\frac{1}{(1+\kappa e)}\left\{\frac{v_1^2}{\{u_1u_1\}}+\frac{v_2^2}{\{u_2u_2\}}+\ldots\right\} \quad \ldots\ldots\ldots \quad (66)$$

Then if e be increased by δe this part of the energy, which is measured by the temperature, will be diminished by

$$\left\{\frac{v_1^2}{\{u_1u_1\}}+\frac{v_2^2}{\{u_2u_2\}}+\ldots\right\}\frac{\kappa\delta e}{(1+\kappa e)^2} \quad \ldots\ldots\ldots \quad (67)$$

or if κe be small we may put $1/(1+\kappa e)^2$ equal to 1, and write Θ where Θ is the temperature for

$$\left\{\frac{v_1^2}{\{u_1u_1\}}+\frac{v_2^2}{\{u_2u_2\}}+\ldots\right\}$$

so that (67) becomes

$$\delta\Theta=-\Theta\kappa\delta e \quad \ldots\ldots\ldots\ldots \quad (68)$$

Thus if κ be positive the temperature of the body will be diminished by communicating to it a positive charge of electricity, or electricity will behave like a substance with real specific heat. Since the electromotive force in an unequally heated conductor $=-d(\kappa\theta)/dx$, when κ is positive the current goes from the hot to the cold parts of the wire, but when this is the case, what Sir WILLIAM THOMSON calls the "specific heat of electricity in the conductor," is negative. In this case we see that the temperature of the body will fall or rise according as a charge of positive or negative electricity is communicated to it; when κ is negative or the "specific heat of electricity in the conductor" positive, the temperature will rise when a positive and fall when a negative charge of electricity is communicated to the body. These effects have, as far as I know, never been observed.

Another relation between heat and electricity is afforded by the phenomenon of pyroelectricity. This phenomenon is well illustrated by a crystal of tourmaline which when warmed becomes positively electrified at one end, which we shall call the positive end, and negatively electrified at the other, which we shall call the negative end. Sir WILLIAM THOMSON has shown that this phenomenon can be explained by supposing that there is an electric displacement in the tourmaline depending upon the temperature. Now if $\{uu\}$ involves the coordinates which fix the electric displacement we can easily see that there must be such an effect. For take the case of a tourmaline crystal whose axis is taken as the axis of x, the positive direction of the axis being that drawn from the negative to the positive ends of the crystal; let p be the electric displacement parallel to this axis, and suppose that in the expression for the energy per unit volume there is the term

$$\phi(p)\Theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (69)$$

where $\phi(p)$ merely denotes some function of p and Θ denotes the temperature as before. By LAGRANGE's equation we see that there will be an electromotive force parallel to the axis of x and equal to

$$\phi'(p)\Theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (70)$$

so that if K be the specific inductive capacity of the tourmaline the electric displacement will be

$$\frac{K}{4\pi}\phi'(p)\Theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

and this will depend upon the temperature, which is all that is required for Sir WILLIAM THOMSON's explanation.[1] Let us now consider the effect of the term $\phi(p)\Theta$ upon the temperature. We see, as in the last case, that the part of the energy on which the temperature depends may be expressed in the form

$$\left\{\frac{v_1^2}{\{u_1u_1\}}+\frac{v_2^2}{\{u_2u_2\}}+\ldots\right\}\frac{1}{1+\phi(p)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

where $v_1, v_2, \ldots v_n$ will remain constant, so that if p be increased by δp the temperature will be diminished by

$$\left\{\frac{v_1^2}{\{u_1u_1\}}+\ldots\right\}\frac{\phi'(p)\delta p}{\{1+\phi(p)\}^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

and we may write this as

$$\delta\Theta=-\Theta\frac{\phi'(p)}{1+\phi(p)}.\delta p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (74)$$

As the positive direction of x is that drawn from the negative to the positive ends of the crystal the electric displacement in this direction must be positive, hence we see from equation (71) that $\phi'(p)$ is positive; but if $\phi'(p)$ be positive we see from equation (74) that when p is increased the temperature will fall. Thus if we take a tourmaline crystal and place it in an electric field so that the line joining the negative end of the crystal to the positive is in the direction of the electric force, the temperature of the tourmaline crystal will fall when the strength of the field is increased and rise when it is diminished.

§ 8. We know that when there are inequalities both of strain and temperature in a circuit made of one kind of substance there will be an electromotive force acting round the circuit, but if the strain be uniform this E.M.F. will vanish however the temperature may vary provided it remains continuous, while if the temperature be uniform the E.M.F. will vanish however the strain may vary from point to point.

Let us suppose that the circuit consists of a wire of which ds is an element of arc,

and that e denotes the longitudinal extension; then if in the expression for the kinetic energy there is a term of the form

$$\mathrm{A}y\theta\frac{df(e)}{ds} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (75)$$

where θ denotes the temperature, y the quantity of electricity which has crossed any section of the wire, and $f(e)$ denotes any function of e, LAGRANGE'S equation shows that the E.M.F. round the circuit equals

$$\int \mathrm{A}\theta\frac{df(e)}{ds}ds \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$

this vanishes when θ is constant and also when e is constant, so that the theory does not indicate the existence of a current when experiment shows that there is none. If we suppose that the circuit consists of two pieces of the same kind of wire, in one of which the longitudinal strain is constant and equal to e while the other piece is unstrained, and that one of the junctions of the strained and unstrained pieces is at a temperature θ_1 and the other at a lower temperature θ_2, then the E.M.F. round the circuit tending to make the current flow from the strained to the unstrained wire across the hot junction is

$$\mathrm{A}(\theta_1-\theta_2)\{f(o)-f(e)\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (77)$$

where $f(o)$ denotes the value of $f(e)$ when (e) is zero. VON TUNZELMANN (Phil. Mag. [5], 5, p. 339) has proved that the current is reversed when the strain exceeds a certain limit, this shows that $f(e)$ cannot be a linear function of e, but must involve powers of e above the first.

Since the term

$$\int \mathrm{A}y\theta\frac{d}{ds}f(e)ds$$

involves the coordinates which fix the state of strain of the wire, it indicates the existence of certain stresses in it. Let α be the displacement of a particle parallel to ds an element of the arc of the wire, then if α becomes $\alpha+\delta\alpha$, e is increased by $d.\delta\alpha/ds$, when this change occurs in e, the change in the term

$$\int \mathrm{A}y\theta\frac{d}{ds}f(e)ds$$

is

$$\int \mathrm{A}y\theta\left\{f'(e)\frac{d^2\delta\alpha}{ds^2}+f''(e)\frac{d.\delta\alpha}{ds}\frac{d\alpha}{ds}\right\}ds,$$

if we integrate this by parts we see that the coefficient of $\delta\alpha$ is

$$\int \mathrm{A}y\left\{\frac{d^2}{ds^2}\theta f'(e)-\frac{d}{ds}\left(\theta f''(e)\frac{d\alpha}{ds}\right)\right\}ds$$

since y is constant along the wire, the expression may be written

$$\int A y \frac{d}{ds}\left\{\frac{d\theta}{ds} f'(e)\right\} ds \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (78)$$

so that at each point of the rod there is a force tending to increase a equal to

$$A y \frac{d}{ds}\left\{\frac{d\theta}{ds} f'(e)\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

From the form of this expression we see that the force should increase with y, which increases indefinitely with the time the current has been flowing, so that theoretically this expression, and therefore the stress, should increase indefinitely with the time; in this case, however, other considerations will come in to modify this result. If we imagine that the strain is constant along one of the wires the expression (79) becomes

$$A y f'(e) \frac{d^2\theta}{ds^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80)$$

but by the theory of the conduction of heat $d^2\theta/ds^2$ varies as $d\theta/dt$ if there is no loss by radiation, so that in this case the force per unit length of the wire varies as

$$y f'(e) \frac{d\theta}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (81)$$

so that if we have a current flowing through a wire, one half of which is heated to redness and if necessary strained, if the temperature of the wire be allowed to equalise itself by conduction there will be forces along the wire tending either to stretch or compress it, and if the parts which are losing heat are compressed, the parts where the temperature is rising will be extended.

§ 9. Effects of Heat upon Magnetism.

The magnetic properties of bodies are very much affected by heat, thus a very high temperature seems to destroy altogether both the magnetic susceptibility and the power of retaining magnetism. This shows that $\{uu\}$ the type of the coefficient of the squares of the differential coefficient of the u's must involve the coordinates which fix the magnetic configuration; these coordinates are of two kinds: one, which is of a geometrical type, we shall denote as before by η, the other, which is a kinosthenic coordinate, will, as before, be denoted by ζ, and the momentum corresponding to it by ξ; the coefficient of $\dot{u}^2$ cannot contain the kinosthenic coordinate, because if it did it would not be of the dimensions of kinetic energy, it must therefore contain the geometrical coordinate η only. Suppose that in the expression for the kinetic energy there is, using the same notation as before, a term of the form

$$\tfrac{1}{2}f(\eta)\left\{\frac{v_1^2}{\{u_1u_1\}}+\frac{v_2^2}{\{u_2u_2\}}+\dots\right\} \quad (82)$$

then T′ the function which, in the modified Lagrangian equations, takes the place of T will contain the term

$$-\tfrac{1}{2}f(\eta)\left\{\frac{v_1^2}{\{u_1u_1\}}+\frac{v_2^2}{\{u_2u_2\}}+\dots\right\} \quad (83)$$

but we saw before in equation (25) that T′ also contained the term

$$-\tfrac{1}{2}\frac{\eta^2\xi^2}{\mathrm{B}}$$

so that if H be the external magnetising force we have, by the modified Lagrangian equations,

$$\tfrac{1}{2}\frac{d}{d\eta}\left(\frac{\eta^2\xi^2}{\mathrm{B}}\right)+\tfrac{1}{2}f'(\eta)\left\{\frac{v_1^2}{\{u_1u_1\}}+\frac{v_2^2}{\{u_2u_2\}}+\dots\right\}=\mathrm{H}\xi \quad (84)$$

so that if k be the coefficient of magnetic induction

$$\frac{1}{k}=\frac{1}{\xi^2}\frac{d}{d\eta^2}\left(\frac{\eta^2\xi^2}{\mathrm{B}}\right)+\frac{1}{\xi^2}\frac{d}{d\eta^2}f(\eta)\Theta \quad (85)$$

where Θ is the temperature. It is convenient to write the terms in this way, because $f(\eta)$ must be an even function of η, otherwise the magnetic susceptibility would be altered by reversing the direction of magnetisation. If k_0 be the part of the magnetic induction which does not depend upon the temperature

$$\frac{1}{k}=\frac{1}{k_0}+\frac{1}{\xi^2}\frac{d}{d\eta^2}f(\eta)\Theta \quad (86)$$

so that, approximately

$$k=k_0\left\{1-\frac{k_0}{\xi^2}\frac{d}{d\eta^2}f(\eta)\Theta\right\} \quad (87)$$

if we suppose that the second term in the bracket is small compared with the first, thus

$$\frac{dk}{d\Theta}=-\frac{k_0^2}{\xi^2}\frac{d}{d\eta^2}f(\eta) \quad (88)$$

Now let us consider the effect of this term on the temperature. There is in the expression for the kinetic energy the term

$$\tfrac{1}{2}f(\eta)\left\{\frac{v_1^2}{\{u_1u_1\}}+\dots\right\} \quad (89)$$

Now, suppose that no heat is supplied to the body, but that η is increased by $d\eta$, the v's will remain constant, so that

$$\delta\Theta=\tfrac{1}{2}f'(\eta)\left\{\frac{v_1^2}{\{u_1u_1\}}+\dots\right\}\delta\eta \quad \dots\dots\dots \quad (90)$$

or if $f(\eta)$ be small compared with unity

$$=\tfrac{1}{2}f'(\eta)\Theta\delta\eta \quad \dots\dots\dots\dots \quad (91)$$

since in this case we may write

$$\Theta=\frac{v_1^2}{\{u_1u_1\}}+\dots \text{ nearly}$$

thus

$$\frac{d\Theta}{d\eta}=\tfrac{1}{2}f'(\eta)\Theta \quad \dots\dots\dots\dots \quad (92)$$

$$=-\frac{\eta\xi^2}{k_0^2}\frac{dk}{d\Theta}\Theta \text{ by equation (87)}$$

or

$$\frac{d\Theta}{dI}=-\frac{\Theta}{k_0}\frac{dI}{d\Theta} \quad \dots\dots\dots\dots \quad (93)$$

if I be the intensity of magnetisation; if H be the magnetising force we may write this equation, since $I=kH$, as

$$\frac{d\Theta}{dH}=-\Theta\frac{dI}{d\Theta}. \quad \dots\dots\dots\dots \quad (94)$$

where the magnetising force is supposed to remain constant in the differential coefficient on the right hand side of this equation and the coefficient of magnetic induction on the left.

Thus if the coefficient of magnetic induction increases with the temperature, the temperature of a soft iron bar will be lowered by magnetisation. It is difficult to tell from the experiments which have been made what the effect of temperature on the coefficient of magnetic induction really is; according to WIEDEMANN, when the substance has been repeatedly heated and cooled down again to its initial temperature, the coefficient of magnetic induction of soft iron diminishes as the temperature increases, while for hard steel it increases with the temperature provided the temperature does not exceed a certain limit. Thus the temperature of hard steel ought to fall when it is magnetised and the temperature of soft iron to rise. As far as is known the temperature of all bodies rises on magnetisation, but this may be due to the electric currents induced by the sudden starting of the magnetic field; unless the heat generated by these currents is allowed for the results tell us nothing as to whether the temperature is raised or lowered when the magnetisation is increased.

The effect of temperature depends, too, upon the magnitude of the magnetising force; thus BAUR (WIEDEMANN'S Annalen, xi., 1880) finds that for soft iron the coefficient of magnetic induction increases with the temperature if the magnetising force be less than about 3·6, while if the magnetising force be greater than this the coefficient of magnetic induction diminishes as the temperature increases. Equation (88) shows that if this is the case $df(\eta)/d\eta^2$ must be a function of η; it is probably of the form $\alpha\{1-\kappa\eta^2\}$, where the critical value of η is $1/\sqrt{\kappa}$.

Since the u's are kinosthenic coordinates $\{uu\}$ cannot be a function of them.

As the coefficients of elasticity depend upon the temperature $\{uu\}$ will be a function of the coordinates which fix the strain configuration of the system; now the coefficients of elasticity are the coefficients of the squares of the strain coordinates in the expression for the kinetic energy; so that using the ordinary notation for the strain coordinates we may suppose that the term

$$\tfrac{1}{2}\{\alpha(e+f+g)^2+\beta(e^2+f^2+g^2-2ef-2eg-2fg+a^2+b^2+c^2)\}\left\{\frac{v_1^2}{\{u_1u_1\}}+\ldots\right\} \quad (95)$$

occurs in the expression for the kinetic energy T; in T′, the function which takes the place of T in the modified Lagrangian equations, this term will occur with the opposite sign.

Since the equation for any of the coordinates a, b, c, e, f, g is of the type

$$-\frac{d\mathrm{T}'}{de}+\frac{d\mathrm{V}}{de}=\text{external force of type } e \quad . \quad . \quad . \quad . \quad . \quad . \quad (96)$$

we see that the effect of the presence of the term (94) is the same as if the coefficients of elasticity denoted by THOMSON and TAIT in their treatise on Natural Philosophy by the letters m and n were increased by

$$\left.\begin{aligned}&\alpha\left\{\frac{v_1^2}{\{u_1u_1\}}+\frac{v_2^2}{\{u_2u_2\}}+\ldots\right\}\text{ or }\alpha\theta\\&\beta\left\{\frac{v_1^2}{\{u_1u_1\}}+\frac{v_2^2}{\{u_2u_2\}}+\ldots\right\}\text{ or }\beta\theta\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (97)$$

respectively, if θ be the temperature, since we may neglect the small part of it due to the terms we are considering. Let us now consider the effect of this term upon the temperature, and to illustrate the point in the simplest way let us suppose that all the strains but e vanish, then if e^2 be increased by δe^2 the corresponding increase $\delta\Theta$ in the temperature is by equation (95)

$$\tfrac{1}{2}(\alpha+\beta)\left(\frac{v_1^2}{\{u_1u_1\}}+\ldots\right)\delta e^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (98)$$

or

$$\delta\theta=\tfrac{1}{2}(\alpha+\beta)\theta\delta e^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (99)$$

but by equation (97) we see that

$$\alpha=\frac{dm}{d\theta};\ \beta=\frac{dn}{d\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (100)$$

so that

$$\delta\theta=\tfrac{1}{2}\theta\frac{d}{d\theta}(m+n)\delta e^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (101)$$

so that if the coefficient of elasticity diminish with the temperature, as is the case for most substances, an increase in the strain will produce a lowering of temperature which can be calculated by equation (101); if, on the contrary, the coefficient of elasticity increase with the temperature so that $d(m+n)/d\theta$ is positive, an increase in the strain will cause the temperature to rise. These results were given by Sir WILLIAM THOMSON in his paper on the "Dynamical Theory of Heat" ('Mathematical and Physical Papers,' xlviii., § 202).

§ 10. The next term in order is the coefficient of the squares of the differential coefficients of the coordinates which fix the strain configuration. From the way this coefficient arises we can see that, in the present state of our knowledge, there is no reason to believe that these involve any of the coordinates which we are considering. For if ξ, η, ζ, are the coordinates of the centre of an element $dxdydz$ of an elastic solid, and if ρ be the density of the element, which is supposed to be so small that its density may be taken as uniform, then the kinetic energy of the strained solid equals

$$\iiint\rho(\dot{\xi}^2+\dot{\eta}^2+\dot{\zeta}^2)dxdydz \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (102)$$

Now $\rho dxdydz$ is not altered by moving the body about, so that when the integration is completed the coefficient of $\dot{w}_1^2$ will not involve the geometrical coordinates α. It is conceivable that it might depend upon the electrical coordinates, just as the coefficient of $\dot{x}^2$ may depend upon these quantities; but if this were so, the capacity of a condenser would be altered by making the air between the plates vibrate, and the time of vibration of a bar made of some dielectric would be altered by communicating a charge of electricity to it. This latter phenomenon actually takes place, but it is probably due to an alteration in the elasticity of the bar produced by the electricity, and not to an effect of the kind we are considering. As there is no experimental evidence that the coefficient of $\dot{w}^2$ does involve the electrical coordinates, we shall not consider what the effects of its doing so would be. The same thing applies to the magnetic coordinate η. The coefficient of $\dot{w}_1^2$ is evidently not a function of the temperature or strain coordinates.

§ 11. The most important term containing the product of differential coefficients of coordinates of different kinds is the term containing the product of the rate of changes of the electric and magnetic coordinates. In considering this term we shall use the same notation as before, but it will be convenient to resolve η into three

components, ηl, ηm, ηn, parallel to the axes of x, y, z respectively. We are at liberty to do this, as η is evidently a vector. If we take AMPÈRE'S theory of magnetism, ηl, ηm, ηn will be proportional to the excess of the number of normals to the planes of the currents round the molecules which point in the positive direction of the axes of x, y, z respectively over those which point in the corresponding negative directions. If ξ has the same meaning as before, $\eta l\xi$, $\eta m\xi$, $\eta n\xi$ will be the components of magnetisation in the directions x, y, z respectively. It is proved in MAXWELL'S 'Electricity and Magnetism,' vol. ii., art. 634, that if we have currents whose components parallel to the axes of x, y, z respectively are u, v, w, placed in a magnetic field, the kinetic energy of the two

$$=\iiint(\mathrm{F}u+\mathrm{G}v+\mathrm{H}w)dxdydz \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (103)$$

where ('Electricity and Magnetism,' vol. ii., art. 405)

$$\left.\begin{aligned}\mathrm{F}&=\iiint\eta\xi\left(m\frac{dp}{dz}-n\frac{dp}{dy}\right)dxdydz\\ \mathrm{G}&=\iiint\eta\xi\left(n\frac{dp}{dx}-l\frac{dp}{dz}\right)dxdydz\\ \mathrm{H}&=\iiint\eta\xi\left(l\frac{dp}{dy}-m\frac{dp}{dx}\right)dxdydz\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (104)$$

where p is the reciprocal of the distance of the point where the components of magnetisation are $\eta\xi l$, $\eta\xi m$, $\eta\xi n$ from the point where the components of current are u, v, w. To apply LAGRANGE'S equations we must see what T′, the function which takes the place of T in the modified Lagrangian equation, becomes in this case, as we saw before that some of the terms occurred with opposite signs in T and T′.

Suppose that in the expression for the kinetic energy we have the terms

$$\tfrac{1}{2}\mathrm{A}\dot{\phi}^2+\mathrm{B}\dot{\phi}\dot{\psi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (105)$$

and that ϕ is a gyroscopic coordinate, then

$$\mathrm{T}'=\mathrm{T}-\dot{\phi}\Phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (106)$$

where Φ is the momentum corresponding to ϕ, so that, considering these terms alone,

$$\Phi=\mathrm{A}\dot{\phi}+\mathrm{B}\dot{\psi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (107)$$

Substituting for $\dot{\phi}$ we find

$$\mathrm{T}'=-\tfrac{1}{2}\frac{\Phi^2}{\mathrm{A}}-\tfrac{1}{2}\frac{\dot{\psi}^2\mathrm{B}^2}{\mathrm{A}}+\frac{\mathrm{B}\Phi\dot{\psi}}{\mathrm{A}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (108)$$

so that though the term in Φ^2 and $\dot{\psi}^2$ occur with opposite signs in T and T', the term involving the product $\Phi.\dot{\psi}$ occurs with the same sign in both. On this account we may apply LAGRANGE'S equation to the term

$$\iiint(Fu+Gv+Hw)dxdydz$$

as it stands.

If we apply LAGRANGE'S equation to the x coordinate we see that the force per unit volume parallel to x on the element conveying the current u, v, w is

$$u\frac{dF}{dx}+v\frac{dG}{dx}+w\frac{dH}{dx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (109)$$

or

$$v\left\{\frac{dG}{dx}-\frac{dF}{dy}\right\}-w\left\{\frac{dF}{dz}-\frac{dH}{dx}\right\}+u\frac{dF}{dx}+v\frac{dF}{dy}+w\frac{dF}{dz}$$

this differs from MAXWELL'S expression for the force by the term $u\frac{dF}{dx}+v\frac{dF}{dy}+w\frac{dF}{dz}$. This term vanishes when integrated over a closed circuit. There will be corresponding expressions for the forces parallel to y and z respectively. We have thus the forces given in MAXWELL plus the forces

$$u\frac{dF}{dx}+v\frac{dF}{dy}+w\frac{dF}{dz}$$

$$u\frac{dG}{dx}+v\frac{dG}{dy}+w\frac{dG}{dz}$$

$$u\frac{dH}{dx}+v\frac{dH}{dy}+w\frac{dH}{dz}$$

parallel to the axes of x, y, z respectively. We can prove that if the circuits are closed these form a system of forces in equilibrium, so that the force on any closed circuit is the same as that given by MAXWELL'S theory.

If we apply LAGRANGE'S equations to the electrical coordinates we find that the electromotive forces parallel to the axes of x, y, z are respectively $-dF/dt$, $-dG/dt$, $-dH/dt$. And if we apply them to the magnetic coordinates ηl, ηm, ηn, writing in the expression

$$\iiint(Fu+Gv+Hw)dxdydz$$

the values of F, G, H given in equations (104) we find that the magnetic forces parallel to the axes of x, y, z respectively are

$$\left.\begin{aligned}&\iiint\left(w\frac{d}{dy}\frac{1}{r}-v\frac{d}{dz}\frac{1}{r}\right)dxdydz\\&\iiint\left(u\frac{d}{dz}\frac{1}{r}-w\frac{d}{dx}\frac{1}{r}\right)dxdydz\\&\iiint\left(v\frac{d}{dx}\frac{1}{r}-u\frac{d}{dy}\frac{1}{r}\right)dxdydz\end{aligned}\right\}\quad\ldots\ldots\quad(110)$$

where r is the distance between the point where the magnetic force is required and the point where the components of the electric current are u, v, w. The expressions (110) agree with the ordinary expressions for these forces.

WIEDEMANN has shown that an electric current flowing through a longitudinally magnetised iron wire twists the wire. This shows that the coefficient of the term we are considering is a function of the strain coordinates. Let us suppose that we have a current flowing through a straight longitudinally magnetised wire coinciding with the axis of x; let a denote the twist of the wire round the axis of x, $\dot{y}$ the strength of the current, and $\eta\xi$ or I the intensity of magnetisation, then we have a term in the expression for the kinetic energy of the form

$$f(a)\eta\xi\dot{y}\quad\ldots\ldots\quad(111)$$

where $f(a)$ denotes some function of a.

We see by LAGRANGE's equation for the coordinate a that the couple Ω tending to twist the wire is given by the equation

$$\Omega=f'(a)\eta\xi\dot{y}=f'(a)\mathrm{I}\dot{y}\quad\ldots\ldots\quad(112)$$

If we apply LAGRANGE's equation to the electrical coordinate we see that this term corresponds to an electromotive force E given by the equation

$$\mathrm{E}=-\frac{d}{dt}\{f(a)\mathrm{I}\}\quad\ldots\ldots\quad(113)$$

so that if we twist a magnetised iron wire we shall get an electric current. This phenomenon has been observed. If we suppose that the intensity of magnetisation remains constant, equation (113) becomes

$$\mathrm{E}=-f'(a)\frac{da}{dt}\mathrm{T}\quad\ldots\ldots\quad(114)$$

or from (112)

$$\mathrm{E}\dot{y}=-\Omega\dot{a}\quad\ldots\ldots\quad(115)$$

If we apply LAGRANGE's equation to the magnetic coordinate η we see that there is a magnetising force

$$=f(a)\dot{y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (116)$$

so that when an electric current flows along a twisted wire it magnetises it. This phenomenon has also been observed.

§ 12. We may complete our survey of the terms in the kinetic energy with this example as the term which contains the product of the velocities of the geometrical and strain coordinates does not give rise to questions of much interest, and we saw that this was the only other product term in the expression for the kinetic energy.

We have hitherto made the very important restriction that the cases we considered were those where there were no resistances, frictional forces, or such things as electric resistance, &c. If we give a wide enough meaning to the term material system, we ought to be able to deduce such forces from the dynamics of such systems by the use of the ordinary dynamical methods. Forces of this kind are assumed to be proportional to the velocities of the corresponding coordinates, and a steady transformation from one kind of energy into another, generally heat, is supposed to go on without any reverse transformation taking place. It can, however, I think, be proved that such forces cannot be deduced from the dynamics of an ordinary system, supposing the arrangement of the system to remain continuous. What I believe the equations of motion with the frictional forces inserted in the usual way give, is a result which is true on the average taken over a time which depends upon the nature of the problem, but is not true at any particular instant. Thus, to take the case of electrical resistance as an example, we may look upon an electrical current flowing along a wire as the limit of what happens when we produce a succession of sparks across an air space by means of an electrical machine; if we suppose the interval between the sparks to diminish indefinitely, the discharge will behave like a continuous current, and we may suppose a continuous current to be a succession of discharges following each other at very short intervals. The ordinary electrical equations with the resistances inserted in the usual way will give the mean state of things when the mean is taken over a time which includes a good many discharges, but it does not represent the state of things at any particular instant. I hope in a future paper to return to the theory of this kind of average motion of dynamical systems.

V. *On the Constant of Magnetic Rotation of Light in Bisulphide of Carbon.*

By Lord RAYLEIGH, *M.A., D.C.L., F.R.S.*

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1. THE phenomenon, to which the present investigation relates, is FARADAY'S discovery of the "Magnetisation of Light," or in more usual language the rotation of the plane of polarisation of light in traversing certain media exposed to powerful magnetic force. One of the characteristics of this rotation is that it takes place in the same absolute direction which ever way the light may be travelling, differing in this respect from the rotation which occurs without the operation of magnetic force in quartz and many organic liquids. Advantage of this property has been taken by FARADAY and others in order to magnify the effect. By reflecting the light backwards and forwards it is possible to make it traverse several times a field of force whose length is limited.

A consequence remarkable from the theoretical point of view is the possibility of an arrangement in which the otherwise general optical law of reciprocity shall be violated. Consider, for example, a column of diamagnetic medium exposed to such a force that the rotation is 45°, and situated between two NICOLS, whose principal planes are inclined to one another at 45°. Under these circumstances light passing one way is completely stopped by the second NICOL, but light passing the other way is completely transmitted. A source of light at one point A would thus be visible at a second point B, when a source at B would be invisible at A; a state of things *at first sight* inconsistent with the second law of thermodynamics.

2. It is known that the rotation may be considered to be due to the propagation at slightly different velocities of the two circularly polarised components, into which plane polarised light may be resolved; and it is interesting to consider what difference of velocity our instrumental appliances enable us to detect. A retardation amounting to one wave length (λ), of one circularly polarised component relatively to the other would correspond to a rotation of the plane of polarisation through 180°. If we can observe a rotation of one minute, we are in a position to detect a retardation of $\lambda/10800$. If l be the thickness traversed, v and $v+\delta v$ the two velocities of propagation, the relative retardation is $l\,\delta v/v$. To take an example, suppose that $l=20$ inches, $\lambda=\frac{1}{40,000}$th inch; so that if $\delta v/v$ exceed 10^{-8}, the fact might be detected.* It appears therefore that we are able to observe extraordinarily minute relative differences in the velocities of propagation of the two circularly polarised rays.

* Camb. Nat. Sci. Trip. Ex., 1883.

3. The laws of the phenomenon were investigated in detail by VERDET, who proved experimentally that in a given medium the rotation between any two points on a ray of light of given kind is proportional to the difference of magnetic potential at those points. When the path of the ray is singly or doubly curved, the rotation is to be estimated upon principles similar to those applicable to *twist* * in curved rods.†

4. Absolute determinations of magnetic rotation in bisulphide of carbon have been made by GORDON,‡ and by H. BECQUEREL,§ whose results differ, however, by about 9 per cent. The former obtained his magnetic force by means of an electric current circulating a great many times round the column of CS_2. This column being a good deal longer than the coil, the electromagnetic effect is approximately determined by the strength of the current and the number of turns. Of these data the first was found by a comparison with H (the horizontal component of terrestrial magnetism). The number of windings in the coil was determined, not by a simple counting, but *à posteriori* by an electrical process.

In M. BECQUEREL's experiments the magnetic force was that of the earth acting on a column of CS_2 more than 3 metres in length. The very small effect (obtained by reversal of the apparatus in azimuth) was augmented by causing the light to pass the tube 3 or 5 times, but even with 5 passages the double rotation amounted to only about 30 minutes. M. BECQUEREL regards his determination for sodium light as accurate to within 1 per cent., which would be indeed a wonderful result considering the smallness of the rotation.

5. It is important to observe that great care is required in order to define with sufficient accuracy the kind of light employed. Since the rotation is approximately proportional to λ^{-2}, a change from one sodium line to the other would make a difference of two parts per thousand. Both of the above-mentioned experimenters started with white light. GORDON threw a spectrum upon a screen, perforated with a slit, the position of which was adjusted to correspond with the thallium line; while BECQUEREL corrected his results indirectly by a subsequent comparison between the effects of the more mixed light used by him and that emitted by sodium.

Considering that the employment of white light involved very elaborate arrangements for analysis (according to wave length), in order to avoid errors exceeding in magnitude those likely to be encountered in the polarimetric or electric determinations, I decided to use light actually emitted from sodium vapour. The sodium chloride was held by a spoon of platinum gauze in the flame of a small ordinary

* THOMSON and TAIT's 'Natural Philosophy,' §§ 119–123.

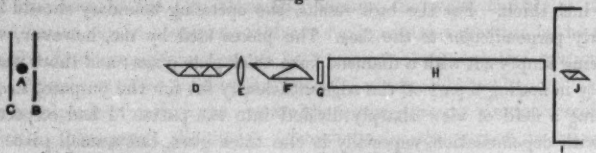
† When polarised light passes from one medium to another, *e.g.*, from air to glass, the plane of polarisation is in general twisted without the operation of any magnetic force. This effect, however, depends upon a part of the light being diverted by reflection, and would disappear if the transition from one medium to the other were gradual, *i.e.*, occupied a stratum a few wave-lengths thick. (See Proc. Math. Soc., vol. xi., No. 159).

‡ Phil. Trans. 1877, p. 1.

§ Ann. d. Chimie, 1882.

Bunsen burner (fig. 1, A). As in Mr. Glazebrook's optical investigations, the evaporation of the salt and the temperature of the flame were stimulated by a jet of oxygen gas, brought in laterally and caused to play round the gauze.

Fig. 1.



A. Bunsen burner.
B. Mirror with slit.
C. Back mirror.
D. Direct vision prism.
E. Collimating lens.
F. Polarising Nicol.
G. Sugar cell.
H. Tube of bisulphide of carbon.
I. Screen (blackened inside).
J. Analysing Nicol.

At the close of the experiments I examined the light thus obtained with a powerful spectroscope, and found that under the influence of the oxygen the originally narrow bright lines dilate almost to the point of contact, thus forming a bright field upon which the dark D-lines are seen with beautiful definition. Although the distribution of light appeared to be tolerably symmetrical, it is a question to what degree of accuracy the mean quality of this light can be identified with that coming from midway between the D-lines. Probably we shall be safe in estimating that the error from this cause is well below $\frac{1}{1000}$.

The bright part of the flame being much larger than is required, a screen (B), perforated with a slit, may conveniently be interposed. In this course there are two advantages. It allows us to purify the light from rays of other refrangibilities (of which there is always a sensible accompaniment, both red and blue), by use of a direct-vision prism (D). Again, by making this screen of looking-glass, from which a narrow strip of silvering is removed, and by backing the flame with a parallel mirror (C), we gain by repeated reflections to and fro, an important increase of illumination. The success of the polarimetry is very dependent upon the intensity of the light, but there must be also a reasonable steadiness. Several arrangements of flame which at first promised well failed in the latter requirement.

6. The rays from the slit, after purification by the direct vision prism, are rendered parallel by a collimating lens (E) and pass into the polarising Nicol (F). The polarimeter employed is on the principle of Laurent, but according to a suggestion of Poynting* the half-wave plate of quartz is replaced by a cell (G) containing syrop, so arranged that the two halves of the field of view are subjected to small rotations differing by about 2°. The difference of thicknesses necessary is best obtained by introducing into the cell a piece of thick glass, the upper edge of which divides the

* Phil. Mag., July, 1880.

field into two parts. The upper half of the field is thus rotated by a thickness of syrop equal to the entire width of the cell (say $\frac{1}{2}$ inch), but in the lower half of the field part of the thickness of syrop is replaced by glass, and the rotation is correspondingly less. With a pretty strong syrop a difference of 2° may be obtained with a glass $\frac{3}{16}$ inch thick. For the best results the operating boundary should be a true plane nearly perpendicular to the face. The pieces used by me, however, were not worked, being simply cut with a diamond from thick plate glass; and there was usually no difficulty in finding a part of the edge sufficiently flat for the purpose, *i.e.*, capable of exhibiting a field of view sharply divided into two parts. I had expected to be troubled with depolarisation, especially in the thick glass, but a small piece thus cut out of a large plate is relieved from most of the strain to which it was originally subject. Probably more care would be required in experiments where a strong white light could be used; but by previously testing the rather thin plates used for the sugar cell and for closing the CS_2 tube, I was able to secure a field of view either half of which under the actual circumstances could be made quite dark by suitable orientation of the analysing NICOL.

By this use of sugar half-shade polarimeters may be made of large dimensions at short notice and at very little cost. The syrop should be filtered (hot) through paper, and the cell must be closed to prevent evaporation.

7. On leaving the sugar cell the light entered the column of bisulphide of carbon (H). To contain the liquid two tubes of brass were employed at various times, the ends being closed with plates of worked glass cemented to the metal with a mixture of glue and treacle. Near one end these tubes were provided with a lateral (vertical) branch, closed with a cork, through which passed the stem of the thermometer used for observing the temperature of the CS_2. The length of the larger tube (used in Series I. and II.) was 31·591 inches, and the diameter about $1\frac{1}{2}$ inch. The length of the smaller tube (used in Series III.) was 29·765 inches, and the diameter 1 inch.

When, as in Series I., it was wished to cause the light to traverse the tube more than once, mirrors were necessary at the ends of the tube. They consisted of plates of thin looking-glass, from which part of the silvering was removed, and by means of a little glycerine they were brought into optical contact with the plates by which the tube was closed. This arrangement was simple, and had the further advantage of practically annulling some troublesome reflections; but the want of means of adjustment rendered it necessary that the closing plates should themselves be pretty accurately parallel.

8. The internal diameter of the ebonite tube, upon which the helix was wound (§ 13), was about $1\frac{7}{8}$ inch, and it was intended to utilise the annular space between the ebonite and the brass as a jacket, through which water at the temperature of the room might be made to circulate. This arrangement, however, failed utterly. Within about 10 minutes of the closing of the circuit of the helix, the definition was lost, and nothing further could be done until after a long interval of repose. The water-jacket

was then abolished, and the available space filled with paper wrapped pretty tightly round the tube. This effected a great improvement, enhanced still further in the later experiments of Series III., in which, by reduction of the diameter of the tube, a wider space became available for heat insulation. The disturbance by conduction of heat from the wire to the CS_2 remained, however, the worst feature of the experiments, and could not be obviated without a fundamental alteration in the apparatus. Probably the best arrangement would be a water-jacket next the wire, and a good thickness of paper or other insulator between the water and the CS_2.

9. The bisulphide of carbon was purified by treatment with corrosive sublimate and grease with subsequent distillation (according to the procedure advocated by BECQUERREL), until most of the unpleasant odour had disappeared. The transparency is much greater than is readily (if at all) obtainable with water, provided proper precautions are taken to avoid exposure to light. After being acted upon by light, the CS_2 attacks brass and becomes rapidly opaque. In this respect it would be an advantage to replace the metal tube by one of glass.

10. The analyser consisted, in some experiments, of a NICOL (J), and in others of a double image prism, and was mounted in a circle made by the Cambridge Scientific Instrument Company. In order that a rotation of the plane of polarisation may be correctly indicated by the difference of the two circle readings, it is necessary that the axis of rotation should coincide with the direction of the light. This requirement is, however, not very easily satisfied. At the commencement of a series of experiments the adjustment was made with the aid of a telescope and cross wires temporarily substituted for the NICOL, but during the course of a set of readings the passage of heat into the liquid tended to make the upper strata warmer than the lower, and thus to bend the rays into a different direction. It is known* that the error arising from maladjustment in this respect is in great part eliminated by reading the NICOL always in both the positions (differing by about 180°) which give extinction or (in the half-shade arrangement) equality of illumination. This plan was constantly followed, but it is not clear that the whole error can be thus got rid of. It occurred to me that another term in the harmonic expansion of the error would be destroyed by use of a double image prism read in *four* positions distant about 90°. Experiment showed that in spite of the glare of the unextinguished image, good readings could be obtained after a little practice, and the comparison of the results arrived at in this way tends to show that the error is not wholly eliminated in the mean of two readings taken in positions differing by 180°. But the matter could be much better investigated with a simplified apparatus and the use of a strong white light.

In Series II. and III., when the light traversed the tube but once, no magnification was necessary, and the eye was applied immediately behind the analyser. In

* "Zur Theorie des Polaristrobometer und des drehenden NICOLS." V. D. SANDE BAKHUYZEN. POGG. Ann., cxlv., 259, 1872.

"Notes on NICOL's Prism," GLAZEBROOK, Phil. Mag., October, 1880.

Series I., the apparent magnitude of the field was much less, and an opera-glass, magnifying about twice, was employed between the analyser and the eye.

11. The setting of the NICOL (or double-image prism) by adjustment of the match between the two parts of the field presented by the half-shade apparatus was facilitated by a device that may be found useful. "In addition to the principal helix, the tube was embraced by an auxiliary coil of insulated wire, through which could be led the current from a LECLANCHÉ cell. This current was controlled by a reversing key under the hand of the observer, who was thus able to *rock* the plane of polarisation backwards and forwards through a small angle about its normal position. The amount of the rocking being suitably chosen, the comparison of the three appearances (two with auxiliary current, and one without) serves to exclude some imperfect matches that might otherwise have been allowed to pass." *

12. Apart from the effect of heat upon the CS_2, the working of the optical parts was fairly satisfactory. The following zero readings taken without the current on June 4, 1884, will give an idea of the sort of accuracy attained. The analyser was a double image prism, and was read in all four positions, the circuit being made three times.

TABLE I.

	° ′	° ′	° ′	° ′
	103 2	193 4	283 0	13 2
	102 55	193 5	283 2	12 59
	102 58	193 3	283 2	13 4
Mean	102 58	193 4	283 1	13 2
Subtract	90	180	270	
	12 58	13 4	13 1	13 2

It appears that an error of 3 or 4 minutes may occur in a single setting.

13. I now pass to the description of the electrical arrangements. The magnetic force depends upon the helix and upon the strength of the current, and we will take these elements in order.

The helix.

The wire is wound upon an ebonite tube, the outside surface of which was turned true in the lathe, and is kept in its place laterally by ebonite flanges screwed upon the tube. The distance between the flanges, equal to the length of the helix, is 9·990 inches; but the tube itself projects some inches beyond the flanges, and when it was desired to use an internal water-jacket, could be further prolonged by additional lengths of brass tube.

In order to give better opportunity for testing the insulation, on which the correctness of the final results is entirely dependent, it was decided to wind on two wires simultaneously, which should be in contact with one another throughout their entire

* "Preliminary Note on the Constant of Electro-magnetic Rotation of Light in Bisulphide of Carbon." Proc. Roy. Soc., vol. 37, p. 146 (June 19, 1884).

length. The operation was performed on December 14–15, 1883, with triply-covered wire of diameter about $\frac{1}{70}$ inch, and no particular difficulty was experienced. The revolutions of the ebonite tube, mounted in the lathe, were taken with all care by an engine counter, and amounted to 1842, so that the total number of windings is 3684. The internal diameter of the helix is 2·188 inches, and the external diameter is 4·13 inches.

By endeavouring to force a current from one wire to the other we obtain a very severe, though of course not absolutely complete, test of the insulation. The resistance between the two wires varied with the hygrometric condition of the silk, which was not impregnated with paraffin. At first it was not much over 2 megohms, but latterly reached 6 or 8 megohms, and was thus abundantly sufficient.

14. As a further test observations were made of the external effect of the helix upon a suspended magnet, when a powerful current was passed in one direction through the first wire, and in the opposite direction through the second. If the positions of the two wires could be treated as identical, the external effect ought everywhere to vanish. In consequence, however, of the fact that one wire lies throughout on the same side of the other, the compensation could not be expected to be complete, except when the suspended magnet is equidistant from the two ends. Experiment with the magnet of a reflecting galvanometer showed that the effect, in fact, varied as the magnet was displaced, but even in the symmetrical position there was a perceptible outstanding differential effect. In order to eliminate the influence of other parts of the circuit, the readings referred only to the deflection of the needle as the current was reversed in the *helix*; and the scale of sensitiveness was obtained by repeating the observations after altering the connexions of the two wires, so that the current circulated the same way round both, and after insertion of a high resistance by which the intensity of the current was reduced in a known proportion. From this it appeared that the differential effect of the two wires (with a given current) was $\frac{1}{3500}$ of the combined effect.

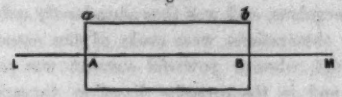
This fraction is tolerably small, but I had expected to find it smaller still. It seems probable that the incompleteness of compensation is due to a small difference ($\frac{1}{5000}$) in the mean diameter of the windings in the two cases. To throw light upon this I took careful measures of the resistances of the two wires. Although they had originally formed one length, their resistances differed by as much as $\frac{1}{70}$th part, that of the wire which had shown itself *least* effective being 7·075 B.A., and of the other 6·965. If, as it seems plausible to do, we attribute the difference of resistance to difference of diameter, this actual difference must amount to $\frac{1}{3800}$ inch. The mean diameter of the windings is about three inches; and if the two wires were wound upon a smooth cylinder of this diameter, the difference in the diameter of the windings would be $\frac{1}{3400}$ of the whole. As this estimate would be increased were we to take into account the fact that each winding really sits upon two windings of the layer underneath, and that these cannot be practically in actual contact, we may perhaps

consider the small anomalous differential effect upon the external magnet to be sufficiently explained by the observed difference of resistances.

Correction for finite length.

15. If the tube were infinitely long the difference of potentials at its ends due to the unit current in one winding would be 4π. But on account of the finiteness of the length a correction is required, whose approximate amount is given in GORDON's paper.

Fig. 2.



Considering, in the first place, one layer of windings of radius Aa, we know that the external effect is the same as would be produced by a uniform distribution of imaginary magnetic matter over the ends, positive (say) over Aa and negative over Bb, the superficial density being equal to the number (m) of windings per unit length. The potential at L of the matter on Aa is

$$2\pi m\,(\mathrm{L}a-\mathrm{LA}),$$

or approximately

$$\pi m\left(\frac{\mathrm{A}a^2}{\mathrm{LA}}-\tfrac{1}{4}\frac{\mathrm{A}a^4}{\mathrm{LA}^3}\right).$$

Similarly the potential at L for the matter on Bb is

$$-\pi m\left(\frac{\mathrm{A}a^2}{\mathrm{LB}}-\tfrac{1}{4}\frac{\mathrm{A}a^4}{\mathrm{LB}^3}\right),$$

so that altogether the potential at L for this layer of windings is

$$\pi m\ \mathrm{AB}\left(\frac{\mathrm{A}a^2}{\mathrm{LA.LB}}-\frac{\mathrm{A}a^4}{4}\,\frac{\mathrm{LB}^2+\mathrm{LA.LB}+\mathrm{LA}^2}{\mathrm{LA}^3.\mathrm{LB}^3}\right),$$

in which mAB denotes the whole number of windings in the layer. This result has now to be integrated so as to represent the effect of the helix, whose inner and outer radii we may call Aa_1 and Aa_2. The mean value of Aa^2 is

$$\frac{\mathrm{A}a_2^3-\mathrm{A}a_1^3}{3(\mathrm{A}a_2-\mathrm{A}a_1)},$$

and that of Aa^4 is

$$\frac{\mathrm{A}a_2^5-\mathrm{A}a_1^5}{5(\mathrm{A}a_2-\mathrm{A}a_1)}.$$

Thus if n be the whole number of windings on the helix, the difference of potential from L to M corresponding to the unit current is

$$4n\pi - 4n\pi\left[\frac{Aa_2^3 - Aa_1^3}{12\,a_2a_1}\left(\frac{1}{LA.LB}+\frac{1}{MA.MB}\right) - \frac{Aa_2^5 - Aa_1^5}{80\,a_2a_1}\left(\frac{LB^3+LA.LB+LA^3}{LA^3.LB^3}+\frac{MA^3+MA.MB+MB^3}{MB^3.MA^3}\right)\right]$$

In the present case

$$Aa_2 = 2{\cdot}065 \text{ (inches)}, \quad Aa_1 = 1{\cdot}094, \quad a_2a_1 = {\cdot}971,$$

from which we get

$$\frac{Aa_2^3 - Aa_1^3}{12\,a_2a_1} = {\cdot}6433, \quad \frac{Aa_2^5 - Aa_1^5}{80\,a_2a_1} = {\cdot}4632.$$

In the remainder of the calculation we have to distinguish the two tubes. For the first

$$LA = MB = 10{\cdot}800 \text{ inches}$$
$$LB = MA = 20{\cdot}790 \text{ inches};$$

and for the second

$$LA = MB = 9{\cdot}887 \text{ inches}$$
$$LB = MA = 19{\cdot}877 \text{ inches}.$$

Hence for the first tube we have

$$4n\pi(1 - {\cdot}00573 + {\cdot}00006) = 4n\pi \times {\cdot}99433\ ;*$$

and for the second

$$4n\pi(1 - {\cdot}00655 + {\cdot}00008) = 4n\pi \times {\cdot}99353,$$

the correction for finite length thus somewhat exceeding one-half per cent.

16. We have now obtained the difference of potential at the ends of the column of CS_2 due to the passage through the helix of unit current. It yet remains to describe the means adopted for the measurement of the actual current in absolute measure.

In a former paper "On the Electro-chemical Equivalent of Silver, and on the Absolute Electromotive Force of Clark Cells,"† it was shown how the E.M.F. of a Clark cell was obtained by comparison with the difference of potentials at the extremities of a wire of known resistance, due to the passage of a current known either directly from its effect upon a current measuring apparatus, or indirectly through the deposition of silver. For the purposes of the present investigation this process was reversed, the

* In the Preliminary Note the reducing factor for this tube was given as ·99449. The alteration is due to the use of more precise data in place of some quite rough measurements in round numbers on which, by an oversight, the first calculation was founded.

† Phil. Trans., 1884, Part II., §§ 35, 36, 38.

CLARK cell itself being treated as a standard of E.M.F., by which to determine the value of the current, which traversed the known resistance, and also the helix by which the magnetic rotation was produced. The arrangements differed so little from those elaborately described in the paper referred to, that it seems unnecessary to enter into the matter at length. If the reader will refer to Plate 17, fig. 1, he will understand the electrical connexions, and he may suppose the current-measuring apparatus EGF, replaced by the magnetising helix. In point of fact this helix was situated in another room at a distance from the E.M.F. compensator and its galvanometer T. The direction of the current in the helix was reversed by a mercury key of the rocker pattern, and care had to be taken that at this moment the galvanometer contact Q was open. The general nature of the arrangement will be sufficiently understood when it is said that the want of balance between the E.M.F. of the CLARK and that at the terminals of the resistance R was made up by E.M.F., taken from an auxiliary circuit, the value of which was afterwards expressed in terms of the CLARK. Denoting the force thus added or subtracted by r, upon a scale according to which the force of the CLARK was ρ, the actual difference of potential at the terminals of R may be written

$$\left(1 \pm \frac{r}{\rho}\right) \times \text{CLARK}.$$

17. As it was intended to use currents of about one ampère, the resistance R was made about [$1\frac{1}{2}$] ohms. The construction was somewhat similar to that of the [4] described in § 33 of the former paper, but on account of the increase in the current to be carried, three wires of German silver were used in parallel. The amount of heating was unimportant for the purposes of the present investigation.

The value of the [$1\frac{1}{2}$] was determined by comparison with a combination of three standard units, one (taking the whole current), and two in parallel (giving the $\frac{1}{2}$). At 13° the resistance is 1·4945 B.A. At 15°, which was adopted as the standard temperature for R and for the CLARK, we have

$$R = 1{\cdot}4958 \text{ B.A.}$$

18. In consequence of the heating of the copper wires, the current (usually obtained from secondary cells) fell off somewhat rapidly during a set of observations, and it was found convenient to take readings of the E.M.F. compensator simultaneously with the adjustment of the polarimeter. The former readings were taken by myself and the latter by Mrs. SIDGWICK, while the flame (at which the optical observer should not look) was regulated by an assistant, who also recorded the circle readings.

The procedure will be most easily explained by an example, for which purpose I take at random the observations of July 25, recorded in Table II.

TABLE II.—July 25, 1884.

Time.	r.	Circle reading. +	Time.	r.	Circle reading. −	Time.	r.	Circle reading. +	Time.	r.	Circle reading. −
h. m.		° ′	h. m.		° ′	h. m.		° ′	h. m.		° ′
6 3½	1525	261 44	6 5	1515	269 19	6 7½	1500	351 56	6 8½	1490	359 24
6 14½	1450	261 45	6 15½	1445	269 18	6 18½	1425	351 56	6 20½	1415	359 22
6 29½	1365	261 47	6 31½	1355	269 16	6 33	1350	351 57	6 35	1340	359 23
Mean	..	261 45·3	..	..	269 17·7	..	..	351 56·3	..	..	359 23·0
6 10	1480	81 48	6 11	1475	89 23	6 12½	1465	171 49	6 13½	1460	179 22
6 23	1400	81 48	6 24½	1395	89 19	6 26	1385	171 53	6 27	1380	179 20
6 37½	1325	81 53	6 39	1320	89 16	6 40	1315	171 55	6 41	1310	179 18
Mean	..	81 49·7	..	..	89 19·3	..	..	171 52·3	..	..	179 20·0

One passage. Double-image prism.

$\rho = 7017$.

Temperature of CLARK and R=17°·6. Mean temperature of CS_2=18°·3.

It will be seen that the cycle consisted of eight readings, four with positive and four with negative rotation of the plane of polarisation, and that this cycle is repeated three times.

The three readings under any one head vary in consequence of the diminution of the current as well as from errors of observation. The value of ρ was

at the beginning	$\rho=7018$
at the end	$\rho=7016$
Mean	$\rho=7017$

Thus in the first observation at $6^h\ 3\frac{1}{2}^m$, when the cycle reading was $261°\ 44'$, the difference of potentials at the extremities of the $[1\frac{1}{2}]$ was $\left(1+\frac{1525}{7018}\right)\times$ CLARK I., the temperature of CLARK I. and of the $[1\frac{1}{2}]$ being $17°{\cdot}6$.

For the mean double rotations in the four positions of the double image prism we have

269°	$17'{\cdot}7$	-261°	$45'{\cdot}3$	$=7^\circ$	$32'{\cdot}4$
359	23·0	−351	56·3	=7	26·7
89	19·3	− 81	49·7	=7	29·6
179	20·0	−171	52·3	=7	27·7
Mean . . .				7	29·1

Since all the effects are proportional to the current, it is sufficient to compare the mean rotation with the mean value of r, viz., 1413; so that the double rotation $7°\ 29'{\cdot}1$, or $449'{\cdot}1$, corresponds to a difference of potentials equal to

$$\left(1+\frac{1413}{7017}\right)\times \text{CLARK I.}=\frac{8430}{7017}\times \text{CLARK I.}$$

The double rotation that would have been found if the current had been just strong enough to balance CLARK I. (at the actual temperature) is

$$\frac{7017}{8430}\times 449'{\cdot}1=373'{\cdot}8.$$

19. This result is a function of the temperatures of the cell and of R as well as of the CS_2; and it is rather unfortunate that all three temperature corrections tell in the same direction. A rise of the thermometer involves a rise in R and a fall in the force of the standard cell, so that on both accounts the current giving the balance is diminished. At the same time the smaller current acts less advantageously in

producing rotation in consequence of the properties of the CS_2. It will be convenient to postpone the last correction, and take first the corrections for temperature in R and the E.M.F. of CLARK, which relate rather to the machinery for measuring the current, and which can be made from data obtained in previous investigations. For this purpose 15° C. is adopted as the standard temperature; and the proportional corrections per degree are ·00082 for the E.M.F. of CLARK and ·00044 for the R, making altogether ·00126 per degree. For the observations of July 25, the correction is therefore

$$+2{\cdot}6\times{\cdot}00126\times373'{\cdot}8=+2{\cdot}6\times{\cdot}471=+1'{\cdot}2.$$

If we take as a standard current that which in traversing R at 15° would balance CLARK I. at 15°, the double rotation of July 25 reduced so as to correspond with the standard current will be

$$373'{\cdot}8+1'{\cdot}2=375'{\cdot}0.$$

This rotation corresponds to the temperature 18°·3 of the CS_2. To obtain comparable results we must reduce to a standard temperature, for which purpose we will select 18°. According to BICHAT the rotation at $t°$ may be expressed by

$$1-{\cdot}00104t-{\cdot}000014t^2,$$

the rotation at 0° being taken as unity. To obtain a more convenient formula, applicable in the neighbourhood of 18°, we may write $t=18+t'$. Thus

$$1-{\cdot}00104t-{\cdot}000014t^2={\cdot}9767-{\cdot}00154t'={\cdot}9767(1-{\cdot}00158t');$$

so that the coefficient for the correction is ·00158. Hence, if the CS_2 on July 25 had been at 18°, we should have had

$$375'{\cdot}0+375'{\cdot}0\times{\cdot}00158\times{\cdot}3=375'{\cdot}0+{\cdot}592\times{\cdot}3=375{\cdot}0+{\cdot}2=375'{\cdot}2.$$

Thus reduced the results for the observations of different days should agree together.

Table III.

Series I.

Date, 1884.	Actual mean rotation (2α).	Mean value of r.	ρ.	$2\alpha\frac{\rho}{\rho \pm r}$	Temperature of Clark and R.	Correction to 15°.	Rotation corrected to standard current.	Temperature of CS_2.	Correction to 15°.	Rotation corrected to 15°.	Deviation from mean.
	′			′	°	′	′	°	′	′	′
May 5 . . .	1123·0	44	5032	1132·9	15·0	+0·0	1132·9	16·8	−2·2	1130·7	+2·2
„ 6 . . .	1086·7	205	5040	1132·8	15·6	+0·8	1133·6	17·2	−1·4	1132·2	+3·7
„ 7 . . .	..	..	..	1132·1	14·8	−0·3	1131·8	17·2	−1·4	1130·4	+1·9
„ 9 . . .	1035·0	402	5046	1124·7	16·5	+2·1	1126·8	18·0	0·0	1126·8	−1·7
„ 10 . . .	1008·0	515	5033	1122·9	17·0	+2·8	1125·7	19·8	+3·2	1128·9	+0·4
„ 13 . . .	1009·0	502	5019	1121·1	17·8	+3·9	1125·0	20·0	+3·6	1128·6	+0·1
„ 15 . . .	1040·5	359	5028	1120·5	18·2	+4·5	1125·0	19·3	+2·3	1127·3	−1·2
„ 16 . . .	543·6	2582	5026	1117·9	18·8	+5·3	1123·2	20·3	+4·1	1127·3	−1·2
„ 19 . . .	589·0	2416	5050	1129·3	16·2	+1·7	1131·0	17·5	−0·9	1130·1	+1·6
„ 21 . . .	681·2	1998	5065	1125·0	16·4	+2·0	1127·0	17·2	−1·4	1125·6	−2·9
„ 23 . . .	664·8	2070	5062	1124·7	17·0	+2·8	1127·5	18·1	+0·2	1127·7	−0·8
„ 26 . . .	755·0	1667	5066	1125·3	17·1	+3·0	1128·3	18·0	0·0	1128·3	−0·2
„ 29 . . .	..	..	..	1128·6	14·9	−0·1	1128·5	15·5	−4·5	1124·0	−4·5
„ 31 . . .	685·4	2850	7194	1135·1	14·4	−0·8	1134·3	15·0	−5·4	1128·9	+0·4
June 2 . . .	640·7	3116	7164	1133·9	15·7	+1·0	1134·9	15·9	−3·8	1131·1	+2·6
Mean . . .	..	..	..	..	..	..	1129·0	17·7	..	1128·5	..

Series II.

June 3 . . .	518·6	2719	7121	375·3	17·5	+1·2	376·5	16·5	−0·9	375·6	−0·1
„ 5 . . .	543·0	3168	7122	375·8	15·8	+0·4	376·2	16·3	−1·0	375·2	−0·5
„ 6 . . .	393·5	323	7118	376·4	15·1	+0·0	376·4	15·6	−1·4	375·0	−0·7
„ 9 . . .	552·9	3275	7110	378·5	14·3	−0·3	378·2	15·9	−1·3	376·9	+1·2
Mean . . .	..	..	..	..	..	..	376·8	16·1	..	375·7	..

TABLE IV.

Series III.

Date, 1884.	Actual mean rotation (2α).	Mean value of r.	ρ.	$\frac{2\alpha\rho}{\rho \pm r}$	Temperature of CLARK and R.	Correction to 15°.	Rotation corrected to standard current.	Temperature of CS_2.	Correction to 18°.	Rotation corrected to 18°.	Deviation from mean.
July 23	447·2	1358	7017	374·7	18·4	+1·6	376·3	19·2	+0·7	377·0	+1·2
„ 24	449·6	1410	7019	374·4	18·0	+1·4	375·8	18·9	+0·5	376·3	+0·5
„ 25	449·1	1413	7017	373·8	17·6	+1·2	375·0	18·3	+0·2	375·2	−0·4
„ 28	448·2	1390	7017	374·1	17·1	+1·0	375·1	17·9	−0·1	375·0	−0·8
Aug. 2	389·7	319	6998	372·7	19·1	+1·9	374·6	20·7	+1·6	376·2	+0·4
„ 4	343·9	516	7001	371·3	19·0	+1·9	373·2	20·1	+1·3	374·5	−1·3
„ 5	530·2	2963	7005	372·6	19·0	+1·9	374·5	20·8	+1·7	376·2	+0·4
Mean	..	..	..	..	..	..	374·9	19·4	..	375·8	..

20. The results of all the observations (other than preliminary) which were thought worthy of reduction are exhibited in the accompanying tables, grouped in three series. In Series I., II. the first tube was employed; the principal difference between them being that in Series I. the light traversed the tube *three times*, while in Series II. the light passed but *once*. It will be seen that in Series I. the actual double rotation varied from about 9° to 19°, and the currents from about $\frac{1}{2}$ ampère to 1 ampère. In Series II. stronger currents were usually passed, amounting to about $1\frac{1}{2}$ ampère, but the rotation was only about 9°. The extreme deviation from the mean is only about ·4 per cent., if we exclude the observations of May 29, which owing to interruptions and other causes were marked as unsatisfactory before reduction.

The NICOL was used as analyser in Series I., and on June 3 of Series II. The remaining observations of Series II. and the whole of Series III. were taken with a double-image prism, read in all four positions as already explained by the example of July 25.

For the observations of Series III. the second tube was employed, with some improvements in the provision against the communication of heat. The diminished diameter of the tube was the inducement to pass the light but once, though it would have been possible to work with three passages. But when the rays skirt the walls of the tube, there is more disturbance from heat; and, indeed, generally the advantage of augmented rotation is in great measure paid for by greater sensitiveness to deviation from optical uniformity.

Not only does the communication of heat disturb the definition, but it tends also to render the actual temperature uncertain. During some of the more protracted sets of readings with the stronger currents there was a rise of nearly 2° in the temperature of the CS_2; and, although this rise was carefully watched, it is difficult to feel confident that the effective mean temperature can be determined with a less error than say $\frac{1}{3}$ of a degree. Such an error would correspond to about $\frac{1}{2000}$ in the final number. To avoid increasing the uncertainty under this head the readings were often concluded, although the definition still remained satisfactory.

If the apparatus were to be designed afresh I should endeavour to guard more adequately against these disturbances, and it might then be possible to use five passages with advantage, more especially if by increasing the weight of the coil it were practicable to bring the double rotation up to about 90°. The determination of such a rotation with the double image prism would be free in high degree from the polarimetric errors considered in § 10. But it is doubtful whether in the present state of science the additional accuracy would repay the labour involved.

21. It only remains now to work out the results in absolute measure. And first as to the value of the standard current, defined as that which, flowing through the $[1\frac{1}{2}]$ at 15°, balances CLARK I. at the same temperature. This value in ampères is expressed by dividing the E.M.F. of CLARK I. in B.A. volts (see Table XI. of former paper) by the resistance of the $[1\frac{1}{2}]$ in B.A. units. Hence the standard current is

$$\frac{1{\cdot}4542}{1{\cdot}4958} = {\cdot}9722 \text{ ampère} = {\cdot}09722 \text{ C.G.S.}$$

If the tube were infinitely long, the difference of magnetic potentials at its ends would be $4n\pi$; but in the case of the actual tubes we have to introduce the correcting factors ·99433 and ·99353 (§ 15). Thus for the first tube, if x be the (single) rotation in minutes corresponding to difference of potential 1 C.G.S., the whole actual double rotation for a single passage of the light will be

$$2 \times {\cdot}09722 \times 4\pi n \times {\cdot}99433 \times x.$$

From Series I. at 18° this quantity is found to be $\frac{1}{3} \times 1128{\cdot}5$, or 376·2, so that

$$x = \frac{376{\cdot}2}{2 \times {\cdot}09722 \times 4\pi n \times {\cdot}99433} = {\cdot}04203.$$

In like manner from Series II. we get

$$x = \frac{375{\cdot}7}{2 \times {\cdot}09722 \times 4\pi n \times {\cdot}99433} = {\cdot}04198.$$

For the second tube used in Series III. we have to employ a slightly different correction for finite length. We have

$$x = \frac{375{\cdot}8}{2 \times {\cdot}09722 \times 4\pi n \times {\cdot}99353} = {\cdot}04202.$$

The results of Series I. and III. are thus in precise agreement, while that of Series II. is about $\frac{1}{1000}$ lower. Ascribing a somewhat less importance to Series II. in consequence of the smaller number of sets of observations, we may take as the final result of the investigation

$$x = {\cdot}04202,$$

which gives the rotation in minutes in bisulphide of carbon at 18°, corresponding to a difference of potential equal to 1 C.G.S. It should be noticed that the mean temperature of the observations was so nearly 18° that the result as given depends scarcely at all upon BICHAT'S formula for the dependence of the rotation upon temperature.

22. M. BECQUEREL gives as his result for 0° C. ·0463 minute. To find the rotation at 18°, this must be multiplied by ·9767 according to BICHAT'S formula: and as BECQUEREL'S observations were in fact made at about 18°, this reduction does not introduce, but rather removes, an extraneous element. Thus according to BECQUEREL—

$$x = {\cdot}0452 \text{ minute},$$

differing by about 7 per cent. from the value found by me.

The comparison with GORDON is more uncertain, inasmuch as his observations were

made on light of the refrangibility of the thallium line. The corrected* result for this light is in circular measure $1{\cdot}5238\times10^{-5}$, or ·05238 minute. To pass to sodium we may use a formula given by BECQUEREL† and VERDET, according to which the rotation for different wave lengths (λ) is proportional to $\mu^2(\mu^2-1)\lambda^{-2}$, μ being the refractive index. At this rate the ·05238 minute for thallium would be ·04163 minute for sodium. The temperature was not directly observed by GORDON, but was estimated to be about 13° C. Assuming this to be correct, the value for 18° would be ·0413 minute, or about 2 per cent. *less* than according to my determinations.

APPENDIX.

Notes on Polarimetry in general.

The problem of the polarimeter is how best to render evident the rotation through a small angle θ of the plane of polarisation of light of brightness h. The effect of the rotation is to introduce light of amplitude $h\sin\theta$, or $h\theta$, polarised in the perpendicular plane, and it is this which must be made to produce a recognisable change. By the use of a NICOL, or double-image prism, adjusted to the original plane, the light of brightness $h\theta^2$ may be isolated, but, as will be proved presently, this is not the best method of rendering its existence evident.

From the preceding mode of statement it is clear that the accuracy obtainable in determining the plane of polarisation increases indefinitely with the brightness of the light, and is in fact proportional to the *square root* of that brightness.‡ Again we see that little is to be expected from such devices as that of FIZEAU, in which the rotation is magnified by causing the light to pass obliquely through a pile of glass plates. The brightness of the light polarised in the perpendicular plane ($h\theta^2$) can only be diminished by such treatment, and the increase of rotation, being due merely to weakening of the first component, is of no value.

The arrangements to be adopted depend for their justification upon the physiological law of the perception of differences of brightness. If $d\mathrm{E}$ denote the difference of sensations, corresponding to two degrees of brightness, H and $\mathrm{H}+d\mathrm{H}$, we have§

$$d\mathrm{E}=\mathrm{A}\frac{d\mathrm{H}}{\mathrm{H}+\mathrm{H}_0},$$

in which H_0 is a certain constant brightness, supposed to depend chiefly upon the proper or internal light of the eye, but to which may be added the effect of light

* Mr. GORDON's result was originally given at double its proper value.

† Ann. d. Chim., t. xii., 1877, p. 78.

‡ This point is insisted upon in an excellent paper by LIPPICH (Wien. Ber., 85, 9 Feb., 1882), which has lately come to my notice.

§ HELMHOLTZ: 'Physiologische Optik,' § 27.

diffused by imperfect translucency of the optical apparatus. If dE denote the smallest perceptible difference, the value of dE/A is in favourable circumstances as low as $\frac{1}{50}$ or $\frac{1}{100}$, which means that with a sufficient total brightness differences of this amount may be apparent to observation.

Let us now consider the values of dE corresponding to different methods of procedure. If the analysing NICOL be adjusted for extinction of the original light, the comparison is between the brightness which cannot be got rid of (H_0) and ($H_0+h\theta^2$).* Near the limit of discrimination, to which case we may confine our attention, $h\theta^4$ is small relatively to H_0, and thus we may take

$$dE=A\frac{h\theta^2}{H_0}.$$

The procedure just considered is that which would naturally be adopted to render evident a small quantity of light of given amount, viz., to isolate it and compare it with the best attainable darkness. But in the present problem the circumstances are peculiar in that we are able to deal with phases. Now if we regard the amplitude (α) of the feeble light as given, putting $\alpha^2=h\theta^2$, we may produce more effect from it by combining it with other light in the same phase of amplitude (β) than by isolating it. The comparison is then between brightnesses $(\alpha+\beta)^2$ and β^2, or as α is very small, between $\beta^2+2\alpha\beta$ and β^2. Thus

$$dE=A\frac{2\alpha\beta}{\beta^2+\beta_0^2},$$

in which β_0^2 is written H_0.

The light of amplitude β is obtained in the simplest possible manner by merely rotating the analysing NICOL through a small angle, and the only question is how to exhibit the comparison light which shall not be affected, when β is changed to $(\beta+\alpha)$. For this purpose we may divide the field of view into two halves with an oblique mirror in which is seen by reflection a feeble light, of the same colour and coming ultimately from the same source.

It is possible that an instrument upon this principle might be made to work satisfactorily,† but the half-shade polarimeters of JELLET and LAURENT seem to be in most respects preferable. In them the comparison is between $(\beta+\alpha)^2$ and $(\beta-\alpha)^2$, so that

$$dE=A\frac{4\alpha\beta}{\beta^2+\beta_0^2},$$

representing twice as great a sensibility. The only thing to be said upon the other side is that the division line in these instruments can hardly be made as invisible as the sharp edge of a mirror may be.

* We may imagine the presentation of the two brightnesses to be consecutive, or more favourably that both are seen at once, half the field of view being occupied by a black body seen after reflection in an oblique mirror, whose edge forms the dividing line.

† Readings would of course be taken in both the positions (one on either side of extinction) which give a match with the comparison light.

In these formulæ β may be chosen at pleasure by suitable adjustments of the polarising arrangements. In order to get the best result, $d\mathrm{E}$ must be made a maximum by variation of β, α and β_0 being treated as constants. The maximum occurs when $\beta=\beta_0$, and its value in the last case is

$$d\mathrm{E}=\mathrm{A}\frac{2\alpha}{\beta_0}.$$

Taking $d\mathrm{E}/\mathrm{A}=\frac{1}{25}$, which is probably about as small as can be expected in practice, we have for the least perceptible value of α

$$\alpha=\frac{1}{50}\beta_0$$

whereas without the half-shade arrangement, and with a Nicol simply set to extinction of the original light,

$$\frac{h\theta^2}{\mathrm{H}_0}=\frac{\alpha^2}{\beta_0^2}=\frac{1}{25},$$

so that

$$\alpha=\frac{1}{5}\beta_0.$$

According to these numbers the half-shade arrangement would have a tenfold superiority, a result not fully borne out in practice. In explanation of this it is important to notice that the procedure in the absence of a half-shade arrangement would in reality be very different from what we have tacitly supposed. The experienced operator, in setting a Nicol to the position of maximum extinction, does not judge merely by the degree of darkness attained in the final position, but displacing the analyser alternately in opposite directions, estimates the position which lies midway between those which give similar revivals of light on the two sides; or, endeavouring to retain in his memory a certain degree of brightness, he may take actual readings on both sides, of which the mean will correspond to the desired position. In this way the fundamental advantage of the half-shade method is in a sense attained, the only difference being that the brightnesses to be compared are seen consecutively after a short interval of time, instead of almost simultaneously; and even this difference becomes less important when the line dividing the field of view of the half-shade apparatus is so coarse that it cannot be rendered invisible.

The carrying out of this method is facilitated by a device which is worthy of trial. The Nicol may be mounted loosely, so as to be capable of turning through a small angle (2 or 3 degrees) between two stops. These stops are rigidly attached to a rotating piece carrying the vernier, and it is to the position of this piece (and not that of the Nicol) to which the readings relate. In taking an observation the piece is turned until the degree of brightness is unaltered, when the Nicol is put over from the one stop to the other. It is probable that under these advantageous conditions

more favourable results than hitherto would be obtained with an undivided field of view.

In the application of the polarimeter, with which the present paper is mainly concerned, the free play of the NICOL is advantageously replaced by an equivalent rocking of the plane of polarisation itself through a small angle on either side of its normal position, produced by the action of an auxiliary electric current, embracing the experimental tube a moderate number of times, and reversed at pleasure by a suitable key under the hand of the observer.

In these discussions it has been convenient to take as a basis the fractional difference of brightnesses which can be recognised on simple presentation to the eye,* but it must be remembered that if suitable precautions are taken to avoid asymmetry, there is no theoretical limit of final accuracy. Thus in ordinary photometry with a divided field (*e.g.*, BUNSEN'S grease-spot photometer), the match must not be approached from one side only. By combining a large number of observations in which the match is approached as much from one side as from the other, a degree of accuracy may be practically attained far beyond that corresponding to the difference of brightness which can be directly recognised by the eye. It is not necessary actually to take readings on the two sides, though it is sometimes desirable to do so; the essential point is to secure symmetry. Time may be saved by the plan of providing means for instantaneous displacements of given amount on either side, as was done in the experiments of the present paper by the auxiliary reversible current.

In practical applications of the polarimeter we have almost always to determine, not so much a particular plane of polarisation as the *rotation* of this plane, due to electromagnetic action, to the substitution of syrop for water, etc., and it appears that the measurement of this angle must be affected with a possible error, double of the error possible in the determination of a single plane. M. BECQUEREL, indeed, in his interesting memoir upon the rotation in bisulphide of carbon under the terrestrial magnetic force,† describes a procedure by which, as he considers, the error may be reduced. By the introduction of a half-wave plate, adjusted so that its principal section coincides nearly with the plane of first polarisation, the angle of rotation is, as it were, reflected by the former plane, and the difference of readings taken with and without the plate is the double of the real angle of rotation. If ϵ be the greatest angular error possible in determining a single plane, M. BECQUEREL shows that the error in setting the plate cannot exceed ϵ, from which he argues that the whole error possible in determining the double angle of rotation is only 3ϵ, or $\frac{3}{2}\epsilon$ upon the single angle. It appears, however, that the error of adjustment of the half-wave plate enters

* [August, 1885.—I find that the sensitiveness of the eye to small differences of brightness is subject to very rapid fatigue. Even a few seconds' gazing is often enough to obliterate a distinction quite apparent at first, and appreciable again after a little repose. This defect is a great obstacle to the further improvement of photometric methods.]

† Ann. d. Chimie, t. 201, p. 323, 1882.

doubly into the result, so that the whole error possible in determining the double angle of rotation rises to 4ϵ, and the use of the half-wave plate gives no advantage.

One other point may be considered in conclusion. In determinations of rotation by magnetic force, the effect to be measured may be multiplied (as FARADAY showed), by causing the light to be reflected backwards and forwards at the ends of the tube. Against this augmentation of the angle of rotation we must set the loss in the section of the beam, and the waste of light in reflection and by absorption. Putting out of sight for the moment the alteration in the section of the beam, we may easily determine the most advantageous number of passages as dependent upon magnitude of rotation and intensity of light. If r be the factor by which the original intensity must be multiplied, in order to express the intensity after a single passage and reflection, r^n will express the intensity after n such passages and reflections. The accuracy of the determination will thus be proportional to nr^n, which is a maximum when $r=e^{-1/n}$. The values of r corresponding to n equal to 1, 3, 5, 7, . . . , are ·135, ·514, ·670, ·752, . . . , so that 3 or 5 passages will usually give the best result.

The argument in favour of a moderate use only of the principle of reflection is strengthened when we take into account the diminution in the section of the beam. The already contracted aperture is seen at a greater distance (proportional to n), so that the apparent magnitude of the field of view is rapidly narrowed. Under these circumstances the comparisons cannot be made with the usual accuracy. If we have recourse to a telescope we can indeed restore the apparent magnitude, but (usually) only at expense of the illumination, since the aperture of the telescope is limited. If the available aperture do not exceed $\frac{1}{4}$ inch, *any* degree of magnification involves a loss of brightness. The importance of these considerations depends upon the length and diameter of the tube; but the tendency of the discussion is to show that more than five passages can rarely be desirable, and that in many cases three passages ought to be preferred to five. If there is any exception it will be when powerful white light (as from the sun) is available, or when it is possible by use of a larger number of passages to bring the whole rotation up to 90° or 180°, in which cases, as has already been noticed, the angle may be determined with peculiar advantage.

POSTSCRIPT.

(October, 1885.)

An important paper* has recently been communicated to the French Academy† by M. BECQUEREL, in which he abandons his former result (§ 4), obtained with the aid of terrestrial magnetic force, in favour of a number agreeing more nearly with that given by GORDON and myself. In the new experiments a long column of CS_2 was employed, encompassed by a spiral conveying a current, the effect of which is shown to depend upon the magnitude of the current and upon the number of turns, in approximate

* Ann. d. Chim., Oct., 1885. † C. R., June 2, 1885.

independence of other circumstances. M. BECQUEREL speaks of this method as new, but it is in reality that employed by GORDON in 1877.* Most of the complication in GORDON'S memoir relates to the determination of the current, and especially to the circumstance that the number of turns in the spiral was not ascertained (as it should have been) during construction, *but subsequently by electrical processes.* When the number of turns and the current are known, there is no difference between the procedure of GORDON and BECQUEREL and that of the present memoir.

There is a pretty close resemblance between M. BECQUEREL'S recent work and mine. In both a soda flame is used as the source of light, and in both the number of windings on the helices is ascertained during construction. In the current determinations, M. BECQUEREL used a galvanometer as an intermediate standard, while I employed for the same purpose a CLARK'S cell, the ultimate standard being a silver voltameter (and in my case a current-weighing apparatus). Inasmuch as M. BECQUEREL uses the same number as that which I obtained for the electro-chemical equivalent of silver, there should be no difference between us in the estimation of currents.

In M. BECQUEREL'S experiments the temperature of the CS_2 was usually about 0° C., and he reduces his results to that standard temperature. He regards BICHAT'S formula as confirmed by his observations. According to this my result for 18° would become

·04302′;

whereas M. BECQUEREL obtains

·04341′,

nearly 1 per cent. higher. I am at a loss to understand the cause of this discrepancy. M. BECQUEREL estimates that his result should be correct to $\frac{1}{800}$, about the same degree of accuracy which I also had hoped to have attained. So far as I can judge, I should consider that in respect of current measurement the advantage lay with me, but that on the optical side M. BECQUEREL'S arrangements were probably superior.

M. BECQUEREL repeats his proposal† to found upon his value of the constant a method for current measurement. I had considered this question at (I believe) an earlier date; and the less sanguine view expressed in the following paragraph seems to be justified by the discrepancies between the results of various observers at various times as to the value of the constant in bisulphide of carbon:—

"Another method, available with the strong currents which are now common, depends upon FARADAY'S discovery of the rotation of the plane of polarisation by magnetic force. GORDON found 15°‡ as the rotation due to the reversal of a current

* See his equation (24), p. 15.

† C. R., t. xcviii., p. 1253; 1884.

‡ Jan., 1884. In a note recently communicated to the Royal Society (Proceedings, Nov. 15, 1883), Mr. GORDON points out that, owing to an error in reduction, the number given by him for the value of VERDET'S constant is twice as great as it should be. The rotations above mentioned must therefore be halved, a correction which diminishes materially the prospect of constructing a useful instrument upon this principle.

of 4 ampères circulating about 1000 times round a column of bisulphide of carbon. With heavy glass, which is more convenient in ordinary use, the rotation is somewhat greater. With a coil of 100 windings we should obtain 15° with a current of 40 ampères; and this rotation may easily be tripled by causing the light to traverse the column three times, or what is desirable with so strong a current, the thickness of the wire may be increased and the number of windings reduced. With the best optical arrangements the rotation can be determined to one or two minutes, but in an instrument intended for practical use such a degree of delicacy is not available. One difficulty arises from the depolarising properties of most specimens of heavy glass. Arrangements are in progress for a redetermination of the rotation in bisulphide of carbon."*

* From the Proceedings of the Cambridge Philosophical Society for Nov. 26, 1883. See also 'Nature,' Dec. 13, 1883.

VI. *The Theory of Continuous Calculating Machines and of a Mechanism of this class on a New Principle.*

By Professor H. S. Hele Shaw.

Communicated by Professor Sir William Thomson, *F.R.S.*

Received and Read June 19, 1884.

There are an almost unlimited number of applications for a mechanism by means of which the velocity ratio between two rotating pieces could either be determined at any instant, or be made to vary in any required manner. Such a mechanism would enable two variable quantities to be dealt with numerically, for with it the operations of differentiation and integration could be mechanically performed.

For differentiation it would be necessary to cause two rotating bodies, which might be disks or rollers, to be turned at speeds which varied respectively according to the rates of change of the variable quantities, one quantity being a function of the other, when the velocity ratio, shown by a suitable index, would give their differential coefficient. This is evident, because velocity is simply the space passed over in a given time, and is, in the limit, proportional to the increment of this space. For integration, while one body is moved at a rate which changes with the independent variable, the velocity ratio of the two bodies (*i.e.*, the two rotating disks or rollers) must be made proportional to the dependent variable. The actual velocity of the second or driven rotating body then becomes a measure of the product of the latter into the rate of change of the former at the same instant. The motion of the driven body, as recorded for any period of time by a suitable index, is therefore a measure of the integration of their product for that time.

The primary or simple form of mechanism has but small value in comparison with the possibilities which a combination of such mechanisms seems to offer. The index, which shows the differential coefficient, or the driven disk which records the results of integration, can have respectively but one position or rate of motion at any instant. By employing, however, a suitable series or chain of such mechanisms, if need be of both kinds, the final index or the final disk could be made to either indicate or record, as the case might be, the result of any required number of conditions. It is difficult to say what limit there would be to the powers of the continuous calculator which such a combination would form.

The only hitherto known mechanism by which both the foregoing operations can be,

in theory, effected appears to be that commonly referred to as the "disk and roller," with its modification, the disk-globe and cylinder-mechanism, or apparatus such as cones and belting which when analysed are all found to depend for their action upon the same kinematic principles. The action of the mechanism is simple enough, the disk, which is usually the piece to which motion is primarily imparted, drives the roller with a speed proportional both to its own angular velocity, and to the distance of the roller from its centre. Pieces of apparatus, such as cones and belting, apart from their unavoidable inaccuracy, are only suitable for the latter of the two above operations, viz., integration, and will therefore not be further alluded to.

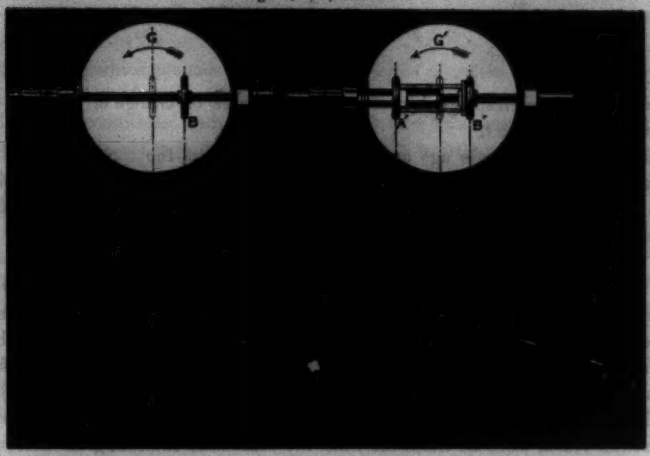
The disk and roller mechanism is chiefly known as a mechanical integrator. Its application for this purpose is due to PONCELET, who first suggested its use for ergometrical purposes as set forth in his 'Mécanique Industrielle,' and it was thus employed by General MORIN. In this case it was used to perform the operation $\int F ds$, where F was the force of traction measured by a spring, ds being the increment of space passed over at that instant by the cart. It was soon afterwards employed by Professor MOSELEY in a "constant steam engine indicator" designed for the use of a committee of the British Association.* In this instrument the disk was replaced by a cone, and F was the varying pressure of steam in the cylinder. Since then the disk and roller principle has been adopted by many inventors for purposes of integration in conjunction with dynamometers for measuring the power transmitted from a prime mover or to a machine. The cone and roller principle was employed by Professor MOSELEY in a calculating machine,† and by SANG and others as a Platometer or Planimeter. Messrs. ASHTON and STORY have revived the application of the disk and roller to a continuous Indicator, which should, however, really be called a steam-power *Integrator*. In the last-named instrument advantage is taken of the fact that as the steam pressure acts alternately on one side or the other of the piston, the roller may be made to take a corresponding position on either side of the centre of the disk, thus both the forward and backward stroke of the engine combine to effect the continuous rotation of the roller in one direction, and thereby integrate the total work done.

All the foregoing examples are applications of the disk and roller mechanism for purposes of integration. Differentiation may practically be effected by means of the disk and roller in the following manner:—In fig. 1 let G represent the disk, and B the movable roller. Instead of the roller B being fixed to its axis the latter is a screw (E E) of which B forms the nut. For convenience the disk is supposed to be driven from the hollow shaft F by means of the roller A, which is always at a constant distance from the centre of the disk, and may be therefore alluded to as having the same motion as the disk. Suppose this screw (EE) to be turned by one body whose rotary motion varies with the rate of increase of one of the two variable quantities, while the motion of the disk depends on the rate of increase of the other

* 11th Report of the British Association, p. 308.

† L. E. and D. Phil. Mag., vol. xxx., p. 171.

Figs. 1, 5, 6, and 7.



variable quantity. If now the roller B which is driven without circumferential slipping by the disk has a greater angular velocity than the screw, it will, by its action on the screw, be moved inwards towards the centre of the disk until its rate of motion is the same as that of the screw. If its rate of angular motion is less it will be moved by the screw outwards until in a similar way it is at rest. This may be simply expressed in symbols thus—

Let

ω=angular velocity of screw EE, *i.e.*, of roller B.

ω_1=angular velocity of shaft F, *i.e.*, of roller A.

R=radius of disk.

y=distance of B from centre of disk.

Then if, as is assumed to be the case, diameter of A=diameter of B

$$\frac{\text{angular velocity of B}}{\text{angular velocity of A}} = \frac{\omega}{\omega_1} = \frac{y}{R} = yK.$$

K being constant and equal to $\frac{1}{R}$.

That is, y is a direct measure of the ratio $\frac{\omega}{\omega_1}$.

Now there is no limit in theory to the speed of the screw (EE) or of the roller (B), consequently the latter may be caused to occupy its correct position with any required rapidity.

Theoretically, therefore, y may be made to measure at every instant the ratio of the angular velocities, *i.e.*

$$yK = \frac{\omega}{\omega_1} = \frac{\frac{dz}{dt}}{\frac{dx}{dt}} = \frac{dz}{dx}$$

where dz and dx correspond respectively to the rates of change of the two variable quantities at any time t.

The author three years ago applied this principle to the construction of a speed indicator. The disk was driven by a clock, and the screw by the body whose speed was to be indicated. In this case ω_1, the angular velocity of the shaft A, was constant.

Then

$$\omega = y\frac{\omega_1}{R} = yK.$$

There were in the above instrument two screws of opposite pitch working independently upon one axis, so that each half of the diameter of the disk, available upon either side of its centre, could be employed. On either screw was a roller, working as a nut and connected with a corresponding index upon a dial, so as to always indicate its position upon the disk. One dial was arranged to indicate the speed of the engines of a steam vessel, while at the same time the other was indicating its corresponding speed through the water, so as to simplify the work of progressive speed trials. Two practical advantages may be noticed in connexion with such an instrument. The first is that a speed indicator of this kind may be placed in any part of the ship and communication made electrically, as was done in the above case. The second is that the slight and irregular variations in velocity which in most speed indicators, such as the strophometer of Hearson and the tachymeter of Buss, necessitate special arrangements of springs to diminish the oscillations of the index hand, are not recorded with this kind of instrument. This latter fact is the result of the gradual action of the screw, which may, however, be so arranged as to cause an indication of any required degree of sensitiveness. It is not necessary to further describe the instrument, which suffers from the inherent defects of the disk and roller, and the principle of which the author found had previously been suggested by two correspondents in 'Engineering,'* and very possibly by others. Quite recently (May 24) a speed indicator of this kind was exhibited before the Physical Society by Mr. W. Golden and Sir A. Campbell, in which the disk was replaced by a cone, and on the same occasion Mr. N. Bailey described a speed indicator of his own which was no other than the simple disk and roller applied in this way.†

Enough has been said to show that a great many attempts have been made to employ for practical purposes the invaluable principle of the disk and roller in two distinct ways, one of which is the converse of the other.‡

* 'Engineering,' vol. xx., p. 314.

† 'Nature,' vol. xxx., p. 140.

‡ There is, it is true, yet a third way of using the disk and roller, which is most important and

It now, therefore, becomes necessary to examine the defects of the disk and roller in view of both the foregoing purposes. These defects may be said to be of two kinds—

(1.) Those which are the result of the principle of action itself.

(2.) The limited range of action of the instrument.

(1.) The very conditions under which the disk and roller works are contradictory. On the one hand the roller must slide sideways, that is, in a perpendicular direction to its plane of rotation, or the relative velocity cannot be changed. On the other hand no sliding or slipping must take place in the direction of its rotation, which must be only a motion of pure rolling contact. The roller has to work upon continuously-changing circles, and nothing in the nature of a toothed or serrated edge is admissible. Such a serrated edge has, indeed, been introduced by some inventors, and the surface of the cone or disk on which it works made, as of course it must be, of softer metal. This was so in the instrument of MOSELEY for integrating the work of a steam engine; but it is a significant fact that the committee speak of the 'slight furrows' caused in consequence upon the driving cone.* Now it is easy to see that these slight furrows must introduce an error, as the position of the roller continually changes, and quite vitiate the differential principle of action. The force of friction which must therefore be employed to ensure rolling contact leads to the three following defects:—

(i.) Grinding action between the edge of the roller and face of the disk.

(ii.) Necessity for the application of force in order to change the position of the roller.

(iii.) Error in numerical results.

(i.) The first of these is well known, and is in fact illustrated in an extreme case by every mortar or pug mill. As the edge, which must initially have *some* appreciable thickness, however slight, grows wider, the evil increases, and the size of the roller altering the accuracy of its record is destroyed. This must take place rapidly in the case of the steam engine integrator, where such a wide range of motion occurs at every stroke of the engine.

(ii.) The second defect is a serious one where the instrument is employed for ergometrical purposes, as even allowing that the friction may be always constant at one portion of the disk, it cannot be so where the conditions of rolling are different.

Where the disk and roller is employed as a screw for the converse process, the same actual side friction must also take place, though it is not so apparent. The objects for which it has been more frequently suggested or employed require the use of a clock, and the author has, in endeavouring to apply the principle to the above-mentioned

entirely distinct in principle from the two already discussed, but its application is only suitable for numerical and *discontinuous* calculation; and as, moreover, it involves no new mechanical arrangement, it is not alluded to until hereafter (p. 385), when discussing the *other* purposes of the new mechanism.

* 11th Report of the British Association, p. 321.

speed indicator, experienced no small difficulty in consequence of the powerful and expensive clockwork required to ensure a uniform speed of rotation.

It is with combinations of this mechanism that the evil results of this loss of power are most evident, not merely because of the limited extent to which it can only in consequence be applied, but because of the unavoidable introduction of errors.

(iii.) The third and last objection, first pointed out by Professor CLERK MAXWELL, is perhaps the most serious of all.

Fig. 2.

Let A (fig. 2) be a plan of the roller, O being the centre of the disk which is turning in the direction indicated by the arrow N. Let A Q be the reaction to the force required to drive the roller round, acting at its periphery at the point of contact with the disk. This force though small must exist, and the result is that the displacement of the roller when the larger force A P (which is that tending to alter its position *radially* on the disk) comes into operation is not radial but along A R. This line A R is the direction of the resultant of A P and A Q, and by moving along it the roller really *slips* through a circumferential distance R P, which represents the *actual error* thus *introduced into the result*. The total error is thus proportional to the distance moved sideways by the roller, that is, to its transverse displacement. It is this fact which is ignored in the tests of ergometers. The common method of procedure is to turn the instrument through a certain distance with a load deflecting the spring to a known and constant amount, and the roller consequently at one radial position on the disk. This is repeated for various other positions of the roller when the record of work done is in each case found to be correct, and it is hard to see why it should be otherwise. These are not, however, the practical conditions of working, which, as a rule, are totally different. In ergometers and steam engine integrators, for instance, a continual change takes place in the position of the roller due to fluctuations of power or steam pressure. This introduces an error in the way described which it does not appear possible to allow for even if the law of variation be known, without elaborate calculation. It certainly would be difficult to test an ergometer or similar instrument with a view to ascertaining and allowing for this

error since exactly similar conditions to those occurring in practice must be ensured, and it may be fairly asked, What check is there upon the records of the large number of instruments of this kind in use?

It was for the purpose of obviating the defects of combined sliding and rolling action* that Professor JAMES THOMSON invented the disk- globe- and cylinder-integrator. This mechanism was at once applied by Sir WILLIAM THOMSON for purposes which had previously been deemed incapable of mechanical treatment, viz., the analysis required in connexion with tide-calculating machines. But, more than this, it was shown by Sir WILLIAM THOMSON that the combination for purposes of integration, now possible from the reduced friction, were not only applicable for calculating $\int \phi(x)\psi(x)dx$, the integral of the product of two functions, but could in theory integrate linear differential equations of any order with variable coefficients.† In one application of the mechanism which has been devised by that gentleman for the solution of a differential equation two machines are employed. The fork of one sphere is connected with the cylinder of the second, and the fork of the second sphere with the cylinder of the first. Thus the motions g_1 and g_2 of the centre of the sphere are obtained, and by eliminating the latter, the former (g_1) is found to be the solution of the differential equation. This method requires the *measurement of the movement* of the fork, that is, of the centre of the sphere, to obtain the result. The use of the *position* of the fork indicated on a scale at every instant in order to obtain a differential coefficient does not appear to have been proposed, and the author therefore ventures to suggest the following arrangement of the disk- globe- and cylinder-mechanism for that purpose.

Figs. 3 and 4.

Figs. 3 and 4 represent this arrangement in front and side elevation. The disk, as in the corresponding disk and roller mechanism, is driven by the wheel A, at a speed which varies as one variable quantity, and communicates a corresponding motion to the globe, and thus to the cylinder. The axis of the cylinder is prolonged, and upon it is cut a screw which works through a nut N. This nut forms the axis of the second

* 'Proceedings of the Royal Society,' vol. 24, p. 263.

† *Ibid.*, pp. 266, 271.

wheel B, which is likewise driven at a speed which varies with the other variable quantity. The wheel B, though not rigidly connected with N, compels it by means of its square or polygonal section to rotate with the same angular velocity, while at the same time allowing it to slide freely in a longitudinal direction. If, now, the wheel A, or what is the same thing, the cylinder, has the same angular motion as the wheel B, then the nut N remains at rest. If the motion is not the same, the nut is moved by the action of the screw until it is so. The position of the centre of the globe, or of the index I on the scale S, indicates at a glance the ratio of the increments of the two quantities.

Just as in the case of the disk and roller, by causing the motion of the wheels A and B to be sufficiently rapid, the differential coefficient may be approached with any required degree of closeness according to the equation

$$yK=\frac{\omega}{\omega_1}=\frac{ds}{dx},$$

where y is the distance of the centre of the sphere from the centre of the disk. If this mechanism were applied to measure velocity, the wheel A would be driven directly by the clock, the wheel B by a machine whose velocity is to be measured.

Another mode of dealing with the resistance of the roller to sliding has been suggested by Mr. Vernon Boys,* in connexion with a different mechanism. This method consists of a very ingenious device which the inventor terms a "mechanical smoke ring," but though described by means of a detailed drawing it does not appear to have been actually constructed, and indeed it is not easy to see how this could be accomplished to ensure a piece of mechanism giving accurate results.

(2.) It is now necessary to consider the second kind of objection, viz., the limited range of measurement of the disk and roller. This does not affect the magnitude of the growth of either variable, but it does affect the measurement of their ratio, which is given by the radius of the circle on which the roller or sphere is turning on the disk. In theory it is only necessary to alter the units and reduce the scale to any required extent. This, in the integrating form, merely results in magnifying the errors near the zero position, that is near the centre of the disk. In the converse application it has a more serious result, which must be briefly considered.

Bearing in mind that it is the *ratio* of the rate of change of the two quantities which is being now considered, let it be assumed that this with each of them in turn becomes relatively very small. 1st. Let this take place with the one which regulates the motion of the screw; in which case the roller B, fig. 1, simply moves inwards towards the centre of the disk, and will register to the limiting value however small. That is in the notation already adopted, since

$$\omega=y\frac{\omega_1}{R}$$

* 'Philosophical Magazine,' vol. xxii., p. 80.

In the limit when

$$\omega=0 \quad y=0$$

2ndly. If the rate of change of the *other* quantity, that is, the one connected with the disk, becomes very small, then the roller B moves *outwards* and reaches the edge of the disk, at which point

$$y=R \text{ and } \omega=\omega_1$$

that is, with the equal rollers (A and B) it is impossible to indicate a ratio less than unity. It should be noted that any increase, however great, of the quantity now being considered can be (in theory) measured, for if ω_1 becomes very great, y becomes very small, and may approach as nearly as desired the limiting case when

$$y=0 \quad \omega_1=\infty .$$

Any advantage arising from this latter consideration does not counterbalance the disadvantage that when $\omega=\omega_1$ any further decrease in the speed of the roller A causes B to leave the disk and necessitates special arrangements, not so simple as it might at first be supposed, to bring B on the disk again.

In the cases in which this kind of mechanism would most probably be applied in practice, a clock would be used, so as to make one of the two speeds constant, and only introduce one variable quantity. It is evident from what has been said that the clock would always be employed to drive the disk by means of the roller A, for then the maximum rate change of the variable, which variable might for instance be velocity, would be previously ascertained. The dimensions could then be arranged so that the indicating roller B would never leave the disk, while the lowest velocities, down to the stopping of the body in motion, would be recorded. The author found that in designing the speed indicator previously alluded to, a suitable velocity of the disk was not easy to attain without unduly increasing its size and introducing consequent mechanical disadvantages.

In order to obtain unlimited range within a small compass, the author therefore designed the arrangement shown in fig. 5. The driving roller A′ is directly connected with the other roller B′, so that their radial positions on the disk change together and to an equal extent, their distance apart being equal to the radius of the disk (R). At the same time their angular motions are independent of each other.

Let z=distance of A′ from centre of disk, then

$$\frac{\omega}{\omega_1}=\frac{y}{z}$$

but

$$z=R-y$$

therefore

$$\omega=\frac{y}{R-y}\omega_1$$

Let

$y=0$ then $\omega=0$
$y=z=\mathrm{R}-y$,, $\omega=\omega_1$
$y=\mathrm{R}$,, $\omega=\infty$ (*i.e.*, may approach that value).

Though the greatest possible range is thus obtained the result is not really so convenient for practical purposes. For instance, with an ergometer or steam engine integrator one advantage of the former mechanism is lost, for now work is not registered in simple proportion to the deflection of the spring. For the converse purposes the scale would merely have to be graduated according to the equation betweed ω and y (ω_1 being constant).

There is, however, a connexion between the two forms of disk and roller which bears a resemblance of considerable interest to two corresponding forms of sphere and roller, and when followed out leads to important results.

In fig. 6 let O be the centre of a sphere which rotates on two fixed centres C and C_1 that is about the axis $\mathrm{C}\,\mathrm{C}_1$.

Let a be the point of contact of the roller A which works against, or rolls upon, the great circle of revolution (*i.e.*, the equator), and b the point of contact of the roller B which rolls on a small circle whose radius is bq.

Let angle of plane of rotation of B with axis $\mathrm{C}\,\mathrm{C}_1=\alpha$
then

$$\frac{\text{angular velocity of B}}{\text{angular velocity of A}}=\frac{\omega}{\omega_1}=\frac{bq}{\mathrm{O}a}$$

but

$$\frac{bq}{\mathrm{O}a}=\frac{bq}{\mathrm{O}b}=\sin\alpha$$

therefore

$$\frac{\omega}{\omega_1}=\sin\alpha$$

or

$$\omega=\omega_1\sin\alpha$$

In fig. 7 the roller A′ is movable as well as roller B′. They are both attached to the same movable frame—their planes of rotation being always perpendicular to each other.

Then

$$\frac{\text{angular velocity of B}'}{\text{angular velocity of A}'}=\frac{\omega'}{\omega_1'}=\frac{b'q'}{a'p'}$$

but by similar triangles $\mathrm{O}'q'b'$, $\mathrm{O}'p'a'$

$$a'p'=\mathrm{O}'q'$$

therefore

$$\frac{\omega}{\omega_1}=\frac{b'q'}{O'q'}=\tan\alpha$$

or

$$\omega=\omega_1\tan\alpha$$

These may be called the "sphere and roller" arrangements or mechanisms. The relation between the two forms of disk and roller is now clear. The first is derived from the more general sine or cosine (or secant, or cosecant) form of sphere and roller mechanism, the second may be in the limiting positions compared with the tangent or cotangent form.

For in the first case, when

$\alpha=0$ then $\omega=0$
$\alpha=90°$ „ $\omega=\omega_1$

or as before by changing the rollers

$\alpha=90°$ then $\omega=\omega_1$
$\alpha=0$ „ $\omega_1=\infty$

So that the range in the sine form is either from 0 to ω, *or* from ω_1 to ∞.

In the second case, when

$\alpha=0$ $\omega=0$
$\alpha=45$ $\omega=\omega_1$
$\alpha=90$ $\omega=\infty$

So that the range in the tangent form is (theoretically) from 0 to ∞.

These results were brought by the author before the Physical section of the Bristol Naturalist Society in November last, and illustrated by a model with a wooden sphere 6″ in diameter. It was, however, in endeavouring to apply them to practical purposes that the author was led to the investigation which has resulted in the present communication.

In fig. 6 take any point P in the axis of rotation of the sphere. Draw P F in a direction perpendicular to the plane of rotation of the roller B. Then by a suitable mechanical device consisting of a cross F, sliding upon a rod O T through which another rod turning about the centre P freely passes, it is evident that for any value of the angle α

$$\frac{PF}{OP}=\sin\alpha$$

but by previous reasoning it may be arranged that

$$\frac{\omega}{\omega_1}=\frac{dx}{ds}=\sin\alpha$$

also

$$OP = \text{a constant} = k$$

therefore

$$PF = k\frac{\omega}{\omega_1} = k\frac{dz}{dx} = y$$

So that P F is a direct measure of the ratio, as is the radial distance of the roller in the simple form of disk and roller. Moreover if s= distance turned through by a point in the circumference of B in any time

$$s = b\int_0^z dz = \frac{b}{k}\int_0^x y dx$$

where b is the radius of roller B.

In fig. 7 let P′ be again the fixed point, but now take P′ F′ perpendicular to the axis of rotation of the sphere instead of as previously to the plane of roller B, and fix it rigidly in this position.

Employ at F′ a sliding *swivel* instead of a cross as before, through which O′ F′ freely passes, so that as the angle α' changes in any way

$$\frac{P'F'}{O'P'} = \tan \alpha'$$

or as before from previous reasoning

$$\frac{P'F'}{O'P'} = \tan \alpha' = \frac{\omega}{\omega_1} = \frac{dz}{dx} = \frac{y}{k'}$$

or

$$P'F' = k'\frac{dz}{dx} = y$$

or

$$s' = b\int_0^{z'} dz = \frac{b}{k'}\int_0^{x'} y dx$$

The sphere and roller mechanism might, therefore, be at once employed to replace the disk and roller, but with the following important difference. The sine form has still the limited range of the corresponding form of disk and roller; but the tangent form, while having unlimited range, has not now the inconvenient relation between the variables of the corresponding form of disk and roller, but is as *simple to graduate and read as the other.*

For practical purposes the graduation along the fixed perpendicular bar of the tangent form will be shown to be much more convenient to read than those along the oscillating or swinging bar of the sine form, and therefore would probably be generally employed. For the present in what follows that form will be the only one treated of.

Although one of the practical objections, viz., that of grinding, has been obviated, there is still left that of side slipping of the roller on the sphere when change of velocity ratio is required to take place. This may be obviated in the following manner.

Suppose the fixed centres C and C_1 (figs. 6 and 7), each in contact always with the same point on the spherical surface to be replaced by rollers having horizontal planes of rotation.

It will be seen that as these rollers offer less resistance than the side friction of the other rollers, the sphere will be carried bodily round when the angle α' is changed, and the resistance will now only be that of the turning of the rollers C and C_1. To make the arrangement a practical one, and to hold the sphere in its place, idle rollers must be placed respectively opposite to A and B, and fixed to the same frame in order to counteract their pressure. Also a supporting roller must be placed underneath the sphere, with its plane of rotation always perpendicular to the axis of rotation (C C) of the sphere, or what is the same thing, its axis must be always parallel to that of the sphere, and consequently carried in the same frame as the centres.

It would not be convenient to actuate the rollers A′ and B′, or to employ the screw axis, if the frame carrying A′ and B′ actually moved in position, and there is no reason why this should be done.

There are clearly two distinct frames, and two only to be dealt with, one carrying the centres and supporting roller, the other carrying the two rollers A and B, and the idle ones opposite to them. The motion of the two frames being purely relative may be reversed, and the frame carrying the centres made the movable one.

Thus the sphere will now rotate about *movable centres*.

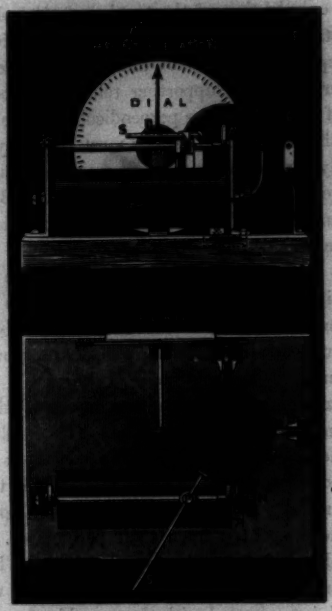
To put these ideas into practical shape, and to see if the rolling centres would answer, a model was made to integrate areas. This is shown in plan, and front elevation, in figs. 9 and 10. A sphere of boxwood (G) is held in a frame simply formed by bending round suitably shaped sheet brass, which thus caused the rollers to grip the sphere. The rollers, A A′ B B′, were disks of boxwood, and merely had common red elastic bands cemented upon their peripheries. The paper on which is the area to be integrated is folded round a cylinder (M), and held there by two india-rubber bands. The cylinder is turned by means of a milled wheel, N, with one hand, and the pointer P, which is connected with the movable frame of centres, is kept on the curve with the other. The roller A works in contact with the surface of the paper, and communicates its motion by frictional contact of the indiarubber to the sphere. The motion of the pointer connected with the axis of B is registered on a suitable dial.

The model not only worked entirely in obedience to the movable centres as far as its range of action permitted, and required only the application of an extremely small force to change the relative position of the frames (*i.e.*, the velocity ratio of A and B), but although only roughly made gave approximately correct results. It was found, however, that the weight of the sphere was not sufficient to keep it accurately in its place, and the more elaborate integrating machine shown in plan and elevation (figs. 11 and 12) has been constructed.

In this latter instrument rollers are placed both above and below the sphere, the

distribution of all the rollers being shown by figs. 13 and 14 (p. 17), which are respectively a diagrammatic plan and elevation of the arrangement. In fig. 11 the movable frame is for the sake of clearness omitted, but it is shown in plan in fig. 12.

Figs. 9 and 10.



The principle upon which the area of any curve is integrated when the pointer has passed round the periphery is perfectly simple. In fig. 15 (p. 383) let ON, the axis of x, be the trace of roller A on the cylinder, let P be the position of the pointer at any point on the curve for which the ordinate

$$PP' = y$$
$$PN = y_2$$
$$P'N = y_1$$

The motion of the roller B when the pointer passes along the line hPg gives in suitable units

$$\int y_2 dx = \text{area of curve } hPglm.$$

The motion of B when the pointer passes along $gP'h$ is

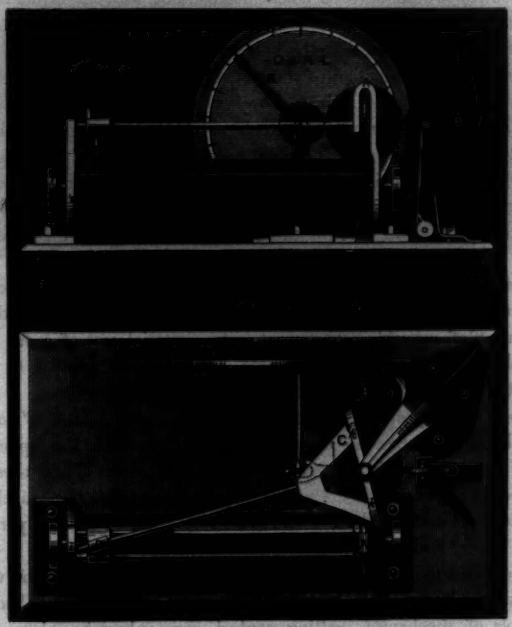
$$-\int y_1 dx = \text{area of curve } gP'hml$$

because the roller A is moving in the opposite direction, and dx is negative.

Finally, the result shown by the index is

$$\begin{aligned}\int y_2 dx - \int y_1 dx &= \int (y_2 - y_1) dx \\ &= \int PP' dx = \int y dx \\ &= \text{area } PgP'h\end{aligned}$$

Figs. 11 and 12.



The same result might have been obtained with the sine arrangement (fig. 16), but now either the top or bottom roller in the movable frame must travel along the line of ordinates, and the point N be the centre of rotation for the end of the drum on

which the area is wrapped. This is, of course, not so convenient, and its possibility is chiefly interesting from a theoretical point of view.

The satisfactory working of the first model proved that the principle of action was practicable, and some of the most important results were at once evident; one of these being the formation of a chain of such mechanisms in which the loss from friction would be inappreciable; another being the application of a rapidly moving screw with clock-work for such instruments as speed indicators. Moreover, the compound arrangement might be of very compact form. There was one objection which, though of no importance in most applications for purposes of integration, was a serious one for certain applications of the converse process. This was the fact that the movable centres, notwithstanding the great range of velocity ratio, could never take such a position as to give the limiting values in either direction. This is shown at once by fig. 12, where it is seen that to do this, that is, for α to become 0° or 90° the movable centres would have to come into contact with the other rollers A A′ or B B′. It must not be overlooked that although the roller centres are nominally in contact at a point, yet that really the sphere, in turning, twists upon the movable centres at its equator, while motion of the movable frame causes the supporting rollers to twist upon the sphere at its poles. The result is that a smooth, hard, and consequently expensive sphere is required, which in the integrating machine shown in figs. 11 and 12 is made of ivory. It should be noted that even if slight wear takes place at the centres it is distributed over the whole spherical surface, for directly the frame moves round, the former centre becomes a point which, by a sort of precessional action, is not likely to again become the centre, at any rate for a considerable period of time.

It was in endeavouring to overcome these difficulties, and also account for the satisfactory action of the rolling centres, that the author discovered that the foregoing arrangement of movable centres was only a special case of a far more general principle, which, like it, might, with suitable mechanism, be applied to many bodies but which, in the case of the sphere, leads to very practical results.

Let fig. 17 be the perspective view of a sphere. Suppose A C A′ C′ to be the great circle formed by the intersection of a horizontal plane with the spherical surface. Let D C′ D′ C be the great circle formed by a vertical plane. Then these two planes always intersect in a diameter C C′, which may have any direction in the horizontal plane. Suppose a series of rollers, whose planes of rotation are all perpendicular to the horizontal plane, and which are in contact with the great circle A C A′ C′, suppose also a second set of rollers, whose planes are perpendicular to the vertical plane D C D′ C′, and which are in contact with the circle D C D′ C′. Then, by a well-known principle of mechanism, rolling contact can only take place between the spherical surface and the former set of rollers, when the axis of rotation of the sphere is in the horizontal plane, for only then will all their axes intersect the axis of rotation of the sphere. Similarly the rollers round the vertical great circle can only roll on the

sphere when its axis of rotation is in the plane of that great circle. *The only axis fulfilling both these conditions is that coinciding with the intersection of the two diametral planes.*

Figs. 13, 14, 15, 16, 18, and 19.

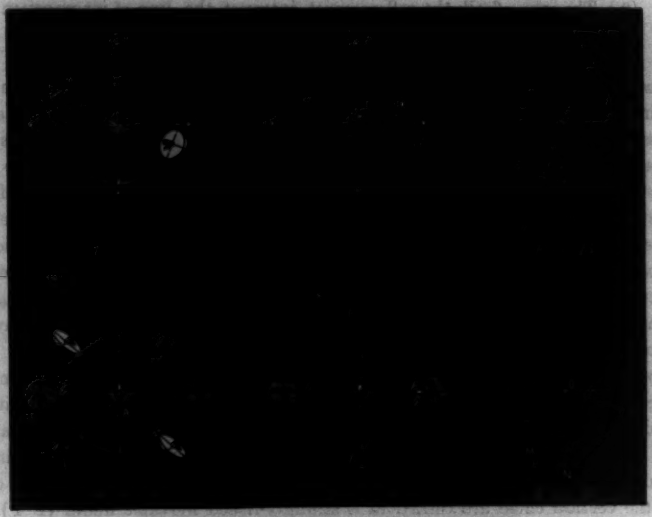


Fig. 17.

It was at the extremity of this axis C C′ (fig. 17) that the movable centres were always in contact; but their successful action was not due to this fact, but simply because they, together with the supporting roller, *fulfilled the other conditions.* These conditions, which can really be approximately, but never perfectly, fulfilled, may be briefly stated as three in number :—

(1.) No slipping must take place.

(2.) Each set of rollers must always be in contact round the great circles formed by two perpendicular (not necessarily horizontal and vertical) diametral planes.

(3.) The axes of all the rollers must intersect the line of intersection of the two planes.

It will be observed that nothing is said about the planes of rotation of the rollers containing the centre of the sphere in the last condition, and the fact that this is not necessary is taken advantage of in the last application which is described in this paper.

From the foregoing results it is evident that it is not the distribution of the rollers round the great circle that is important, but the direction of their axes. Therefore the rollers hitherto alluded to as movable centres do not require to be placed at the points of intersection of the diametral planes, and may be removed to such a position that they can never come into contact with the rollers round the horizontal great circle of contact. At the same time the top and bottom rollers which are not required to be in contact with the poles of the sphere may be removed to any other convenient position along their great circle of contact.

Figs. 18 and 19 (p. 383) show respectively a diagrammatic plan and elevation of the sphere and rollers thus arranged, and the views are lettered to correspond with figs. 11 and 12. Figs. 20 and 21 show the details of the mechanism designed to carry the above principles into operation. The rollers in contact with the horizontal circle are carried on a bracket, which is part of the fixed frame (I I). Those in contact with the vertical circle are carried in a strongly-ribbed movable frame (F F). The mode of attaining numerical results, if required, would be of course identical with that employed in the instruments already described, and would require the additional parts shown in connexion with them.

The two kinds of defects shown at the commencement of this investigation to exist in the disk and roller have thus been practically eliminated.

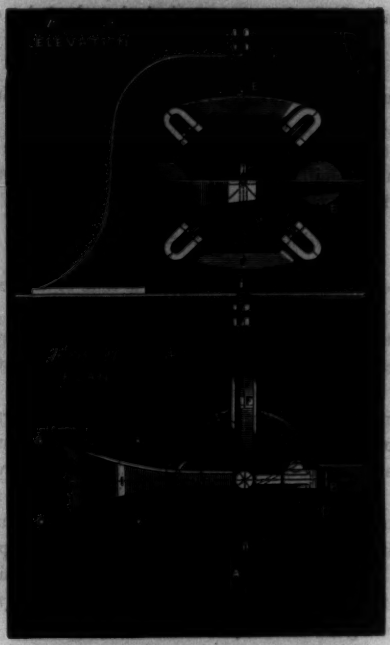
(1.) The results of friction are reduced to a minimum, for not only does the change of velocity ratio, *i.e.*, the relative position of the axes of rotation of the sphere to the fixed rollers, take place by rolling, but the sphere itself is entirely supported by rollers, and thus there is *absolutely nothing but rolling contact*. This was not the case with movable centres. The immediate result of this is that the hard, smooth surface of the sphere is no longer necessary, since no twisting now takes place, and the edges of the rollers transmitting the motion to be measured may now be serrated so as to form the envelopes of their teeth upon the elastic surface of the sphere.

This effects a great reduction of friction, for the pressure which was formerly necessary to ensure the transmission of rolling by frictional contact of the driving roller (A) in the fixed frame is now greatly reduced. The contact is now scarcely frictional contact at all, and the pressure on the bearings is so reduced that the most

delicate forces may be dealt with by the mechanism, and accuracy of numerical results ensured.

(2.) With regard to range of action it is clear that theoretically this is now infinitely great in both positive and negative direction, as may be seen by examining the travel of the movable frame (fig. 21).*

Figs. 20 and 21.



Applications for discontinuous calculation.

It has been already shown how, with the sphere and roller, areas may readily be integrated and the operation of differentiation (by employing a screw) be performed. Various other results may however be obtained in a third way which has already been alluded to (p. 371). Before discussing this it may be well to point out that a very simple numerical computator is at once formed by the sphere and roller integrator in the special case where the position of the movable frame is constant during any given motion of the driving roller. A product or quotient of two given numbers may be

* See note at end of the paper.

directly obtained in the following way. Suppose the rollers A and B (fig. 22) to be graduated and a pointer or index attached to each so as to enable the distance turned through by either to be read. Suppose each reading to be brought to zero and the pointer P (connected with the movable frame) moved into a position such that

$$\text{AP} = x = \text{one factor.}$$

Then turn the roller A through a distance such that the index on A reads

$$m = \text{the other factor.}$$

Then by taking suitable units, or what amounts to the same thing, by using suitably proportioned rollers,

Reading of B $= n = mx =$ product of the two given factors.

To obtain a quotient a similar method would be adopted, but the roller B would now be turned instead of A, then

$$\text{Reading of A} = m = \frac{n}{x} = \text{quotient.}$$

The practical application which suggests itself is that of the reduction of tables, in which case one factor is constant. The work might be rapidly performed by always turning A or B (as the case might be) through a constant distance from zero, the pointer P having been first set to the other factor, to an adjustable stop when the reading of the driven roller gives at once the required result.

Thus far the working is only a special case of that of the integrator, viz., where the area is a rectangle, but with the form of instrument for the converse process, that is with the roller A screwed upon its axis, a new principle can be brought into operation. Suppose the roller B (fig. 22) to be turned through a distance n, then the roller A will in consequence turn through a certain distance m, but inasmuch as it forms a nut upon the screw A P, it will at the same time *continually alter* the position of the pointer P, and consequently the value of x, thereby changing the position of the movable centres kk, or axis of rotation of the sphere. Thus the reading (m) of the index of A is a value which is no longer a simple product or quotient but of a nature which must be investigated.

Let θ, θ_1 be the angles turned through by A and B respectively.

Let

$l =$ pitch of the screw AP.
$a =$ radius of roller A.
$b =$ „ „ B.
$x =$ distance AP.
$k =$ „ AO.

Then

$$\frac{bd\theta_1}{ad\theta} = \tan\alpha = \frac{x}{k}$$

And

$$x=\frac{l\theta}{2\pi}$$

$$d\theta_1=\frac{al}{2\pi bk}\theta d\theta$$

Then by integration

$$\theta_1=\frac{la}{4\pi kb}\theta^2$$

or

$$\theta=\sqrt{\frac{4\pi kb}{la}}\sqrt{\theta_1}$$

Thus suppose m and n are the respective final readings on the recording dials connected with the rollers A and B, the initial readings having been zero—suppose also that the pitch of the screw is such that the constant quantity $\sqrt{\frac{4\pi kb}{la}}$ is equal to unity.

Then

$$m=\sqrt{n}$$

that is, by turning B through a distance recorded on its own dial as n, the reading (m) on the dial of A gives the square root of the former quantity.

Fig. 22.

Fig. 23.

Again, suppose P (fig. 23) to be taken on *the other side* of A. Then by similar reasoning since now

$$\frac{bd\theta_1}{ad\theta} = \tan\alpha = \frac{k}{x}$$

$$d\theta_1 = \frac{2\pi ak}{bl}\ \frac{d\theta}{\theta}$$

and by integration

$$\theta_1 = \frac{2\pi ak}{bl}\log_e\theta.$$

If, as before, m and n are the respective readings at the end of the operation, and the pitch of the screw be such that

$$\frac{2\pi ak}{bl} = \text{modulus of the Naperian system of logarithms.}$$

Then

$$n = \log_{10} m.$$

In order to graduate the dials in the latter application, since θ can never be zero the limiting position can never be reached; but when

$$m=1 \quad \text{then } n=0.$$

therefore the dials of A and B must be adjusted so that these two conditions are simultaneously fulfilled.

In this way it is possible to find the *logarithm* of a number m to any base by merely turning the roller A through that distance and reading the dial of B.

There is another mode of obtaining *any* root, which will be easily understood when it is remembered that by using n sets of spheres and rollers, the value of $(x_1 \times x_2 \times x_3 \times \ldots x_n)$ can be obtained. Make these values of x all equal; then with n frames x^n is given. By turning the last wheel through x^n, and keeping the frames in suitable positions, the first wheel of the series will turn through a distance x.

Thus, if the last wheel be turned through a distance N.

Reading of first wheel or roller

$$= m = \sqrt[n]{\mathrm{N}}.$$

The application of a mechanism to obtain the foregoing results had been previously suggested by Professor Moseley in a paper already alluded to (p. 368), but the foregoing investigation was made without any knowledge of this. The modification of Professor James Thomson's disk- globe- and cylinder-mechanism (figs. 3 and 4) may be at once applied to obtain similar results.

Applications for continuous calculation.

The foregoing applications are for the purposes of obtaining numerical results in which the mechanism is employed, rather as a discontinuous than a continuous

calculator. The effects to be obtained by using two or more sets of spheres and rollers in one or both the primary ways, that is for purposes of integration and differentiation, must now be considered.

First let two sets of integrating mechanism be employed. In fig. 24 let

$MN = a$.
$QN = b$.
$\Delta x =$ width of element NQ.
M = moment of area of MN about Ox where Ox is the trace of the roller A on the cylinder.

then

$$M = a\left(\frac{a}{2} + b\right)\Delta x.$$

Fig. 24.

Fig. 25.

Suppose, now, that as the pointer of the integrator travels along the upper portion of the curve, the wheel B, which in the simple integrator records the area, now drives another sphere which has its frame kept so that the value corresponding to y of the original proof is always for that frame equal to $\frac{y}{2}$. This result is easily effected by keeping the frames perpendicular to each other, as in fig. 25, the axes being $k_1\ k_1$ and $k_2\ k_2$ respectively, and using suitable proportions of wheels or rollers.

Then when the roller A travels a small distance Δx at M (fig. 24), the pointer

P describes upon the upper part of the curve say at M, a distance corresponding to upper boundary of element of width Δx. Then :

$$\text{Reading of roller C} = \frac{(a+b)^2}{2}\Delta x.$$

Similarly when the pointer m traverses the lower boundary of the element at N,

$$\text{Reading of roller C} = -\frac{b^2}{2}\Delta x$$

being negative because the roller A is now *returning*, and its motion is reversed.

Taking both results together

$$\text{Reading for element MN} = \frac{(a+b)^2 - b^2}{2}\Delta x$$

$$= a\left(\frac{a}{2}+b\right)\Delta x$$

$$= \text{moment of area of element MN about } Ox.$$

If now the whole distance moved through round the curve be taken where y_1 and y_2 represent respectively b and $(a+b)$ at any point of the curve, then

$$\text{Reading of C} = \int_{x_1}^{x_n}(y_2^2 - y_1^2)dx$$

$$= \text{moment of area of whole curve about } Ox.$$

Fig. 26.

Secondly, let three sets of spheres and rollers be employed, as in fig. 26,*

$$I = \text{moment of inertia of element MN about } Ox,$$

* The axis $k_3 k'_3$ should be perpendicular to its present direction in the figure.

then

$$I=\tfrac{1}{12}a^3+a\left(\frac{a}{2}+b\right)^2$$
$$=\tfrac{1}{3}(a^3+3a^2b+3ab^2).$$

Let now the *third* frame, and consequently the axis of rotation of the third sphere, be kept parallel to the first and perpendicular to the second, as in the former case, with the two sets, but let now the units be taken so that the reading of the distance of the pointer P_3 is

$$y_3=\frac{1}{3}y$$

then for the travel of the wheel A over the distance Ox, *i.e.*, the width of the element of area, both forwards and backwards, the reading of the recording wheel D of the third set is

$$=\frac{(a+b)^3}{3}-\frac{b^3}{3}$$
$$=\tfrac{1}{3}(a^3+3a^2b+3ab^2)$$
$$=\textit{moment of inertia}\text{ of element MN about axis } Ox \text{ (fig. 24).}$$

By taking, as in the previous case,

$$y_1=b$$
$$y_3=(a+b)$$

it is found that final reading of the dial of D for the whole travel of the curve

$$=\int_{x_1}^{x_2}\tfrac{1}{3}\{(y_3-y_1)^3+3(y_3-y_1)^2y_1+3(y_3-y_1)y_1^2\}dx$$
$$=\tfrac{1}{3}\int_{x_1}^{x_2}(y_3^3-y_1^3)dx$$
$$=\text{moment of inertia of whole area about } Ox.$$

Both the foregoing results may be at once proved in a more general way, thus it is evident that the wheel B is turning at the rate k_1dA where k_1 is some constant, and

$$A=\text{area of curve,}$$

then by means of the addition of the second sphere and roller the product is given as with the first, and C turns at the rate

$$=k_2y(k_1dA)$$

therefore, if

S_2= reading of index for travel of the whole area of curve .

$$S_2 = k_1 k_2 \int_{x_1}^{x_2} y dA$$

or with proper units

$$= k_1 k_2 \int_{x_1}^{x_2} (y_2^3 - y_1^3) dx$$

S_2= moment of area about Ox.

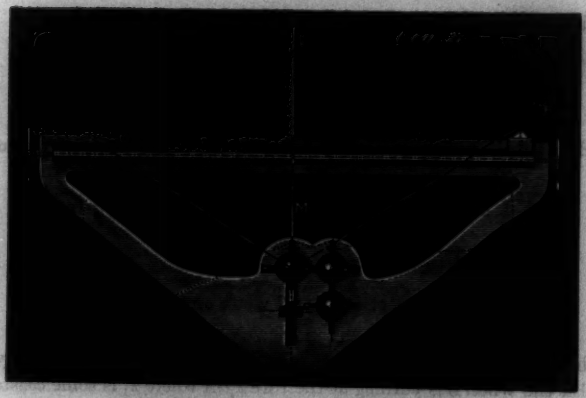
With the addition of the third set of sphere and rollers

S_3= reading of index D$= k_1 k_2 \int_{x_1}^{x_2} k_3 y (dM)$

$$= k_1 k_2 k_3 \int_{x_1}^{x_2} (y_2^3 - y_1^3) dx$$

= moment of inertia of area about Ox.

Fig. 27.



The three dials of B, C, and D may be arranged on the parallel ruler as shown in fig. 27, and thus a compact and inexpensive instrument formed, which will give on three dials *simultaneously* and without the *necessity of any calculation whatever*, the three quantities A, M, and I.

But there are certain practical points of particular interest in this instrument which

are worthy of consideration. In the first place its symmetrical proportions and wide range of action, as shown by fig. 27, are due to the use of the general form of the sphere and roller mechanism, by which the axis of rotation of the sphere can be made to change across through the centre of the recording roller (B), so as to enable the relative motion of this roller and that of the driving one (A) to be reversed, so that positive and negative values may be obtained.

Suppose the pointer to be on that portion of the curve to the right of the line L M, up which the driving roller is passing (*i.e.*, really the axis of x)—then it has already been proved that the area is integrated by the motion of B. But when the pointer passes the intersection of the line L M with the curve the relative motion of A and B is reversed; this causes B to turn in the opposite direction (unless the widest point of the area is accidentally reached at this point). But the pointer is really now travelling round the opposite way on the curve to the left of L M, and the dial of B is therefore only cutting out or subtracting *the negative portion of the curve.* When the highest point is reached then the motion of the roller A *will be reversed*, and thus along the rest of the curve to the left (since the pointer is still to the left of L M) the motion of the index of B is positive, *i.e.*, the area is *continuously integrated.*

Thus as far as the integration of areas is concerned it is *absolutely immaterial* how the parallel ruler or rolling integrator is applied to the paper, or along what line the roller A runs, whether within or without the curve.

Coming now to the moment of an area it is evident that the position of the line L M has everything to do with the result, but it is clear that mathematically there is an essential difference between this and the first case, for now a negative value of y does not as in the first case give a negative result—from the equation

$$M = \tfrac{1}{2}\int y^2 dx$$

and on examination it is found that the index of the wheel or roller C which records this result always moves in *the opposite direction* to the index of B when y is negative. Lastly the equation for moment of inertia

$$I = \tfrac{1}{3}\int y^3 dx$$

shows that in this case where y is negative the value of I is recorded in the same way as the area. It will be seen that the mechanism always effects this result, without adjustment or correction by the very principle of its action, and *adds* the moment of inertia for both sides of the line.

This peculiar relation between A, M and I, though evident upon a consideration of what these quantities mean in practical mechanics is thus clearly brought out.

There is no reason (in theory) why the number of spheres should be limited to three or the frames kept parallel or perpendicular to each other. Thus it is possible to obtain the integration of

$$\int F_1(x)F_2(x) \ldots . F_n(x)dx$$

where n is the number of sphere and roller mechanisms, each frame being made to assume a position depending on the respective function with which it deals.

The volumes of solids of revolution can be obtained (with two sets) by merely passing the roller A in direction of the axis of x, and keeping the pointer P on the surface, so that at all times

$$y = \text{radius of circle of revolution.}$$

Then

$$\text{volume} = \pi \int y^2 dx$$

Also with ruled surfaces the pointer of 1st set is kept on one surface, so that

$$y_1 = \text{one ordinate} = y = \phi(x)$$

and

$$y_2 = \text{the other} = z = \psi(x)$$

$$\text{Reading of index of C} = \int yz dx$$

$$= \text{volume} = \int \phi(x)\psi(x)dx$$

Again, in the integration of trigonometrical functions, which are of great importance in naval architecture, it is only necessary to keep the pointer of the index which is attached to the movable frame, and so controls the axis of rotation at the angle, the trigonometrical ratio of which is a function of x, as for instance, in the equation

$$y = \int_0^x \tan \alpha \, dx$$

which gives the area of a curve in which

$$\alpha = \text{angle of heel of vessel}$$
$$x = \text{corresponding ordinate.}$$

All the ratios could be in this way easily dealt with.

The converse application of the sphere and roller mechanism will easily be understood, as the principle has already been fully dealt with in the case of the disk and roller, though only the one example of the speed indicator has yet been suggested—in this case the position of the pointer indicates $v = \frac{ds}{dt}$.

Suppose, however, that a single sphere and roller arranged for the purpose be applied to a single integrating mechanism which is giving the work done in a steam

engine as already described (p. 368). The recording wheel of the latter is turning at a rate $=ydx$, where, if W be the measure of the work done,

$$ydx=d\mathrm{W}$$

Therefore if this wheel or roller *actuate* the former single sphere, the pointer of the movable frame of that set of sphere and rollers indicates the result

$$\mathrm{R}=\frac{d\mathrm{W}}{dt}=\textit{rate of working.}$$

In suitable units

$$=\textit{indicated HP.}$$

By causing the pointer in its motion to work the driving wheel of a second sphere whilst a clock drives the screw, the pointer indicates the ratio$=\alpha$.

Where

$$\alpha=\frac{d\left(\frac{ds}{dt}\right)}{dt}$$

$$=\frac{d^2s}{dt^2}$$

= acceleration, or *rate of change of velocity.*

So in theory any order of differential coefficient $\left(\frac{d^ny}{dx^n}\right)$ may be obtained, n being the number of sets of spheres and rollers used.

Fig. 28.

For integrating the volume of any irregular solid the following approximate method may be suggested. Let A C B, fig. 28, be the section of the solid by the coordinate plane parallel to which the ordinates (y) are measured. Let R Q be a screw the axis of which coincides with the axis of x. Suppose the pointer P to be passed round the solid in planes perpendicular to the above plane and at the same time carried along by the screw R Q—P making one revolution round the solid for one turn of R Q—then the final result of record of the index of roller

$$=\int ydx$$

Let

n= number of revolutions of RQ

l= pitch of its screw

Then approximately

$$\text{Volume of solid} = nl\int y dx$$

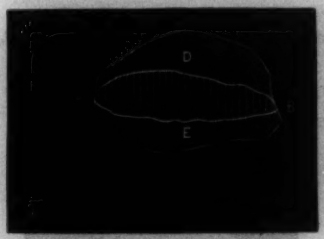
By making l very small the result may be made to approach the true value as nearly as desired.

The operation

$$\iint \phi(x, y) dy dx$$

may, however (in theory), be performed in the following way.

Fig. 29.



Let A C B, fig. 29, be as before the section of any solid. Then to find its volume the foregoing expression would (if possible) first be integrated with respect to y, and an expression of the form Volume$=\int\psi(x)dx$ obtained. The expression $\psi(x)$ may be called the ordinate of *a curve of areas* which may be represented by A D B E, the ordinate D E $=y=\psi(x)$ at any point of which gives the numerical value of the area for that value of the abscissa x. By passing the pointer of the simple integrator over the curve A D B E, the volume of the solid is given at once.

To apply this reasoning to effect a mechanical solution, the pointer of a simple integrator must be passed with sufficient rapidity round the solid as it moves along the axis of x, so that the differential coefficient $\frac{d\text{A}}{dt}$= average areas can be obtained, this gives at any instant the ordinate (D E), which can be used as above stated by means of a suitable mechanism.

Investigation of work lost in friction.

The work lost in friction in the sphere and roller mechanism occurs partly in rolling contact and partly upon the axles, which will be here investigated, is that upon the axles of the movable frame. There are two kinds of effect and two only upon these, viz., direct pressure upon the bearings and twisting of the plane of rotation. The combined effect of these results in

(1.) A direct pressure on the bearings,

(2.) End pressure resulting, according to the construction, in either pivot or collar friction.

Let

L = work lost in one revolution of the sphere
N = number of revolutions of rollers for one revolution of sphere
W_1 = pressure on bearings (direct)
W_2 = pressure on pivot or collar (end)
ϕ = coefficient of friction
R = radius of sphere
r = radius of rollers
ρ = radius of axles of rollers
n = number of rollers in movable frame.

Then

$$L = L_1 + L_2$$

where

L_1 = work lost in direct friction by direct pressure on bearing
$= 2\pi\rho N W_1 \sin\phi$
L_2 = work lost in pivot friction
$= \frac{1}{3}\pi\rho N W_2 \tan\phi$

$$L = 2\pi\rho N \sin\phi \left(W_1 + \tfrac{2}{3} \frac{W_2}{\cos\phi} \right)$$

Then it is required to find an expression from which N is eliminated and W_1, W_2 expressed in terms of the force to be transmitted.

Let figs. 30 and 31 be respectively a plan and elevation showing diagrammatic views of sphere G with rollers A A_1 B B_1, in contact respectively at points a a' b b'.

Let

P = force to be transmitted acting on periphery of driving roller (generally taken as A in the figures).
Q = reaction of the driver wheel (B).
α = angle of vertical diametral plane to plane of rollers B.

kk' being axis of rotation of the sphere (fig. 30).

Then must

$$Q=P\frac{ap}{bq}=P\cot\alpha$$

otherwise the motion will be accelerated.

Then, since the component of force applied on wheel A normal to the surface, in direction of arrow a', necessary to maintain adhesion, is balanced by the reaction at a, and also that at B by the reaction b', the effect of the two forces P and Q is the same as two forces at the axis of the sphere and two couples about the axis. The forces act respectively at p and q (fig 30). The couples $P\times ap$ and $-Q\times bq$. Their combined effect is

$$M=P.ap-Q.bq=P(ap-\cot\alpha.bq)$$

but

$$ap=R\cos\alpha$$
$$bq=R\sin\alpha$$

Resultant couple about axis

$$=P(R\cos\alpha-R\cot\alpha\sin\alpha)$$
$$=0.$$

Therefore the only effect is that due to parallel forces P and Q acting perpendicular to the axis at the points p and q. The effect of these is equivalent to the couples $(P.op+Q.oq)$ acting in the vertical plane of the axis, and the force $(P-Q)$ acting at the centre of the sphere.

Figs. 30 and 31.

In fig. 31 let

γ = angle of plane of rotation of the rollers with a vertical plane.

Then supposing the rollers to be symmetrically distributed.

(1.) Vertical force on point of contact of each

$$=\frac{P-Q}{2}$$
$$=\frac{P(1-\cot\alpha)}{2}$$

This may be resolved into

$$s=\frac{P(1-\cot\alpha)}{2}\cos\gamma=\text{ direct pressure on bearings}$$

$$r=\frac{P(1-\cot\alpha)}{2}\sin\gamma=\text{ force tending to twist the plane of rotation.}$$

(2.) The couple

$$\begin{aligned}&=P.op+Q.oq\\&=P(op+oq\cot\alpha)\\&=PR(\sin\alpha+\cot\alpha\cos\alpha)\\&=PR\left(\frac{\sin^2\alpha+\cos^2\alpha}{\sin\alpha}\right)\\&=\frac{PR}{\sin\alpha}\end{aligned}$$

This acts on the rim of the rollers, and if they were four in number would produce the effect upon each rim

$$\begin{aligned}T_2&=\frac{PR}{\sin\alpha}\times\frac{1}{2R}\\&=\frac{P}{2\sin\alpha}\end{aligned}$$

Summing, now, the total twist

$$T_1+T_2=\frac{P}{2}\left\{\frac{1}{\sin\alpha}+(1-\cot\alpha)\sin\gamma\right\}$$

Let d = half the length of roller axis—that is distance between centres of pressure on either side of roller.

Then the result of this twist is

Direct pressure on pivot or collar

$$=\frac{P}{2}\left\{\frac{1}{\sin\alpha}+(1-\cot\alpha)\sin\gamma\right\}$$

Pressure on bearing

$$=\frac{P}{2}\left\{\frac{1}{\sin\alpha}+(1-\cot\alpha)\sin\gamma\right\}\frac{r}{d}$$

Finally, since

$$N=\frac{2\pi R\cos\gamma}{2\pi r}=\frac{R}{r}\cos\gamma$$

and

$$W_1=\frac{P}{2}\left[\{(1-\cot\alpha)\cos\gamma\}+\left\{\frac{1}{\sin\alpha}+(1-\cot\alpha)\sin\gamma\right\}\frac{r}{d}\right]$$

$$W_2=\frac{P}{2}\left\{\frac{1}{\sin\alpha}+(1-\cot\alpha)\sin\gamma\right\}$$

$$L=2\pi\rho N\sin\phi\left(W_1+\frac{2}{3}\frac{W_2}{\cos\phi}\right)$$

$$=2\pi\rho N\sin\phi\frac{P}{2}\left[\left(\{(1-\cot\alpha)\cos\gamma\}+\left\{\frac{1}{\sin\alpha}+(1-\cot\alpha)\sin\gamma\right\}\frac{r}{d}\right)+\frac{2}{3\cos\rho}\left(\frac{1}{\sin\alpha}+(1-\cot\alpha)\sin\gamma\right)\right]$$

Differentiating with respect to γ and equating to zero gives the best angle for γ, therefore

$$\tan\gamma=\frac{r}{d}+\frac{2}{3}\cos\phi$$

which result can be at once applied in practice.

In the model made

$$\gamma=45°.$$

Then

$$\tan\gamma=1$$
$$\phi=5°$$
$$\cos\phi=\cdot996.$$

This gives

$$\frac{r}{d}=\frac{1}{3}\text{ (nearly).}$$

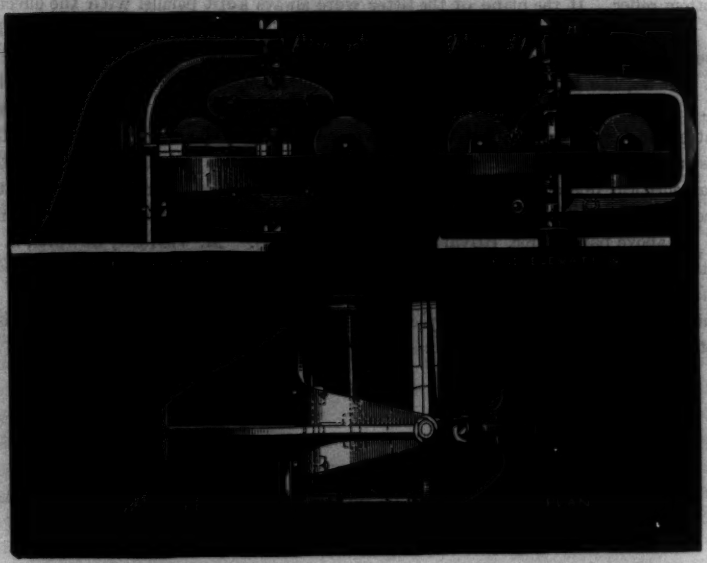
Power Form.

Figs. 32, 33, and 34 show three views of the "power form" of sphere and roller mechanism, which has been constructed to test the actual power transmitted. In this form an important modification, due to the author's brother, Mr. EDWARD SHAW, Whitworth Scholar, Stud. Inst. C.E., has been introduced. It was mentioned that there was no necessity for the planes of rotation of the bearings to contain the centres of the sphere so long as their axes *all* passed through its axis of rotation. It is, therefore, possible instead of merely having sharp-edged disks to act as roller bearings (as in figs. 20 and 21) to have conical rollers. The elastic spherical surface yields to the very slight extent necessary to allow the flat edge of these rollers to form a straight line (instead of a point) of rolling contact passing through the apex of either of the cones, the fustra of which are shown in fig. 32. *Every part of such a cone therefore, rolls upon the sphere.* The action of these cones, which have a number of fine grooves round their peripheries, is even better than was anticipated. There are various points about this design which might be noticed. One is the construction of the frame F (fig. 34), which allows the pressure put upon the centres to close in its two sides and so maintain the requisite pressure on the sphere by means of the locked centre screw (H) without the use of expensive arrangements for springs, which were employed in the movable frame of the second integrating machine. The frame, though ribbed and otherwise rigid, is of smaller cross-section at h h (fig. 34) to allow this springing in or out to take place. The convenient nature of the supporting bracket or frame, which is self-contained, and which can be placed anywhere or in any position, is evident, and also the important point of its general simplicity of construction.

It will be seen from the previous investigation that the loss of work in the trans-

mission of power is small and easily calculated, and, moreover, may be reduced by increasing the value of N, and therefore reducing both ρ and P. Practical defects from any possible yielding of the sphere from the effect of the last of these three quantities, viz., P, which cannot well be made a matter of calculation, are thus reduced to any required extent. If the mechanism should prove to be durable, there is, apart from the purely mathematical objects which necessitate accurate working, and which led to its discovery, no apparent limit to its practical applications. It could replace combinations of wheel-work for such purposes as are required in cotton-spinning or

Figs. 32, 33, and 34.



textile machinery, small lathes, electric, and other machines where rapid change of velocity ratio is required, and absolute accuracy is not essential.

The thanks of the author are due to his friends Mr. EDWARD BUCK, M.A., and Mr. C. D. SELMAN, of whose valuable opinions he has been glad to avail himself in several points in the paper. Also to his brother, Mr. EDWARD SHAW and to Mr. CHARLES BULLOCK, B.A., for the great skill they have shown in constructing the second form of Integrator and the "power" mechanism, and for the assistance of the former in the preparation of the drawings.

Note.

(Added February 4, 1886.)

Some months after the foregoing paper was communicated the author happened to read a letter, written Nov. 5, 1881, by M. Ventosa, of Madrid, and published in 'Nature' (Nov, 24), in which the writer describes a proposed method of obtaining the N., S., E., and W. components of the wind. The method was briefly this. A sphere resting at one pole on a point in the periphery of a roller or disk, and in frictional contact at four points of its equator with four other rollers or disks, would (the planes of all the rollers being vertical) freely revolve upon the turning of the bottom roller. If the plane of the latter were always kept parallel with the direction of the wind, and it were turned at a rate proportional to the wind velocity, and at the same time the planes of the four others were placed in pairs respectively N. and S., E. and W., the latter would record the four corresponding components of the wind. M. Ventosa was led to this idea in 1878, after reading an account of Von Oettingen's Integrating Anemometer, by the endeavour to obviate the sliding friction of that instrument. The arrangement he suggests is *per se* incapable of performing other mathematical operations than simple addition, and, in fact, is only one of the two necessary but distinct features of the author's mechanism mentioned in the abstract of the above paper (Proc. Royal Society, 1884, p. 191). M. Ventosa, however, in his letter, truly says of his proposal, "Cette transmission se fait ici par *roulement sans glissement*" (the italics are his own), and he must be regarded as having, prior to the author, suggested the use of a sphere and rollers for effecting a change in the axis of rotation of the sphere without necessitating other than rolling contact.—H. S. H. S.

VII. *On Beds of Sponge-remains in the Lower and Upper Greensand of the South of England.*

By George Jennings Hinde, *Ph.D., F.G.S.*

Communicated by Henry Woodward, *LL.D., F.R.S., F.G.S.*

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[Plates 40–45.]

In the course of a search for fossil sponges in the lower cretaceous deposits of the South of England, I have noticed the occurrence at different horizons of beds of rock, of not inconsiderable thickness, composed to a large extent of the spicular remains of sponges. The strata in which these remains occur are well known, and they have been described in numerous papers from the date of Dr. Fitton's celebrated memoir of "The Strata between the Chalk and Oxford Oolite," in 1827, to the present time, but owing probably to the microscopic dimensions of the remains, and the various changes in their fossilisation which have oftentimes rendered them unrecognisable to ordinary observation, the* organic character of the strata has been seldom recognised, and only in one or two cases have their contents been noticed in detail. The beds have usually been regarded as deposits of sandstone, chert, or siliceo-calcareous materials of an inorganic nature; but though in most cases they contain a varying amount of mechanically derived materials, yet the sponge-remains, and the siliceous deposits derived from their solution, constitute such an important part of their volume as to entitle the beds to be classed as of organic origin, in the same sense as, for example, the upper chalk of this country. In nearly all the outcrops of the lower and upper greensand of the south and south-west of England these sponge-beds are exposed, differing, however, considerably in their development, in the nature of the sponge-remains, and in their condition of fossilisation in different areas, and it is my purpose in this paper to treat (I.), of the general characters of the beds in the various localities; (II.), of the mineral condition of the sponge-remains and of the beds derived from them; and (III.), of the nature of the sponges present in the deposits.

I may premise that the sponge-beds to be referred to, do not, with one or two exceptions, consist of the skeletons of these organisms in a complete or partially complete condition; but they are made up of innumerable multitudes of the detached, minute, microscopic spicules of which the skeletons are composed. The deposits appear to have been produced by the disintegration of the skeletons of numerous generations of sponges, which have successively lived and died in the same areas in

* Mr. J. Arthur Phillips mentions the presence of numerous fossils, particularly sponge-spicules, in cherts of lower greensand age. "On the Constitution and History of Grits and Sandstones," Quart. Journ. Geol. Soc., vol. xxxvii., 1881, p. 16.

which the deposits occur. On the death of the animal, the soft tissues by which its skeleton was held together would rapidly decay, and its individual particles would thus become detached from each other and scattered over the surface of the sea-bottom. As a result of this, the fossil sponge-beds consist of a heterogeneous mixture of the skeletal spicules of various kinds of sponges. Under exceptionally favourable conditions of preservation the minute spicules are now met with in a loose, detached condition, not unlike that in which they may be supposed to have been present at the sea-bottom soon after the death of the animal, but, as a general rule, they are now amalgamated with the inorganic materials of the deposits into hard beds of rock, and they have suffered partial or complete solution and replacement by calcite, glauconite, and iron-peroxide.

I. The general Characters of the Sponge-Beds in different Localities.

It is generally known that the principal outcrops of the lower and upper greensand occur round the borders of the Wealden area in the counties of Surrey, Kent, Sussex, and Hampshire. Both divisions are well represented in the Isle of Wight, and, farther westward, the upper greensand is exposed in Dorsetshire, Wiltshire, Somersetshire, and Devonshire. I propose first to refer to the sponge-beds of the lower greensand, and to begin with those on the north-western margin of the Weald, in the county of Surrey.

Haslemere.—In the mostly unconsolidated sands of the district round this town the sponge-beds occur as thin layers of porous siliceous rock of a grayish or yellowish tint. The beds are from one to three inches (·025 to ·075 m.) in thickness. Their exposed surfaces are usually rough and harsh to the touch and uneven. When somewhat weathered the surfaces of the beds exhibit an agglomeration of sponge-spicules, which are thickly intermingled together, without arrangement of any kind. The spicules are cemented together by siliceous material, but the cement is in small proportion to the mass of spicules. The spicules are less clearly shown in the interior of the beds where they are concealed by the matrix, but it is evident from exposed fractured surfaces that the beds throughout are similarly composed of masses of spicules resembling those shown on their upper surfaces, and that there is but a slight admixture of the quartz-sand grains which compose the strata inclosing the sponge-beds. In a sand-pit a short distance to the north-west of Haslemere, a sponge-bed occurs in an unconsolidated condition, and the sponge-spicules are disseminated loosely in the sand, perfectly free from each other. This bed afforded abundant material for the study of the spicules, which are now partly in the state of chalcedony and partly of crystalline quartz. It is a curious circumstance that though loose in the bed, many of the larger spicules have been evenly fractured in three or four places, and the fractured portions have been re-cemented together, sometimes with a slight lateral displacement (Plate 42, figs. 6*a*, *b*).

The sponge-spicules present in these beds are exclusively those of siliceous sponges, and the large majority are acerate and trifid forms belonging to the Tetractinellidæ with a few of lithistid and hexactinellid type.

The exposed sections of the lower greensand round Haslemere are insufficient to determine the extent to which the sponge-beds are developed in it, but fragments of the beds are shown at or near the surface over a considerable district, and they cover the surface of Hindhead and some of the adjoining hills. On the slopes of one of the hills the sponge-beds are extensively worked for road materials. The sandy strata in which they occur have been mapped by the Geological Survey as belonging to the lower or Hythe division of the lower greensand, which is estimated by Mr. Topley* to attain a thickness of about 300 feet in this district. With one or two exceptions the sponge-beds in the lower greensand are limited to the lower division of it.

In a roadside cutting, about half-way between Haslemere and Godalming, sponge-beds are exposed in the form of layers of siliceous and cherty rock which, as presenting a very common type of fossil sponge-beds, may be mentioned in detail. The beds in question consist of a central layer of light or dark translucent chert with an upper and under layer of yellowish, porous siliceous rock. The upper and under crust of rock varies from half an inch to two inches in thickness (·012 to ·05 m.), whilst the central layer of chert is usually of greater thickness; the boundary between the chert and the outer crust is uneven and irregular, and the one kind of rock gradually passes into the other. The outer porous crust consists of a matrix of translucent chalcedonic silica, with, at times, an admixture of quartz-sand and glauconite grains. This matrix is filled with minute cylindrical or fusiform cavities which are the empty moulds of sponge-spicules. In the centre of these cavities there extends occasionally an elongated, very slender, cylindrical siliceous rod, which is really the solidified infilling of the canal of the spicule. Thus, by a peculiar sequence of conditions, the spicules in this portion of the rock have been first imbedded in a siliceous matrix, then their hollow canals have been filled with silica, and finally the spicules themselves have been completely dissolved away, leaving only the casts of their outer form and their solidified canals. In the central layer of massive chert, on the other hand, very frequently the spicules have disappeared; but that it was equally as filled with spicules as the porous crust is shown by the fact that in microscopic sections faint shadowy outlines of the spicules can be detected, and the solidified canals are distinguishable even when other traces of the spicules are wanting. The silica derived from the solution of the spicules of the outer crust appears, in part at least, to have been re-deposited in the central layer in the form of chalcedonic chert, and the spicules of this layer have been thus enveloped by the cherty matrix. As the refractive index of the spicules and of the matrix is similar, the shadowy appearance of their outlines can be accounted for. In some instances there are empty spicular cavities in the chert itself, as well as in the outer crust.

* "The Geology of the Weald." 'Memoirs of the Geological Survey of England and Wales,' 1875, p. 124.

Though sponge-beds are abundant in the Haslemere district I have not met with a single specimen of an entire fossil sponge. The chert beds have been described by Dr. Fitton,* but their organic nature does not appear to have been noticed.

Midhurst and *Petworth, Sussex.*—In the lower greensand strata on the southern side of the Wealden anticlinal, about two and a half miles north of Midhurst, there are exposed several beds of hard siliceous rock, from eight inches to three feet in thickness (·2 to ·9 m.), of a nearly similar character to the sponge-beds near Haslemere. The rock is of a yellowish tint, harsh to the touch and filled with minute cavities. The weathered, broken-up fragments which are plentifully distributed over the fields, clearly show that the beds consist of an aggregate of spicules resembling those in the Haslemere beds. Nearer Midhurst, and at a higher horizon, beds of porous siliceous rock, exhibiting numerous empty casts of spicules, are largely quarried for road materials. These beds consist partly also of sand grains which have been cemented together by the silica derived from the solution of the sponge-spicules, but the spicular casts are sufficiently numerous to show that sponges must have been very abundant. At Midhurst itself there are beds similarly filled with spicular casts and containing also glauconite grains which are cemented by a hard siliceous material. These beds are still higher in the series, and thus the presence of sponge-beds is shown throughout the lower or Hythe division of the lower greensand in this area.

Near Petworth, about five miles east of Midhurst, sponge-beds from four to seven inches in thickness (·1 to ·17 m.), are largely quarried for road material. The beds consist of an outer crust of porous siliceous rock with a central layer of very hard and brittle chert; which, according to Mr. Topley,† bears the local name of whinstone. The chert is filled with spicules, though they are only faintly shown. Both the spicules and the matrix exhibit, under crossed nicols, similar brilliant prismatic tints, indicating that their component silica is now approximately crystalline.

Godalming, Surrey.—On the surfaces of the fields on the summit of the plateau near this town there are frequently present small flat slabs or masses of hard, porous, siliceous rock, from three-quarters of an inch to four inches in thickness. The character of the rock resembles that of the sponge-beds of Haslemere, but the spicules have suffered greater alteration, and they do not weather out so clearly. They can, however, be recognised, not only in casts, but also as delicate white threads intercrossing each other on the weathered surfaces of the beds. Quartz-sand grains, and in some instances a reddish ferruginous material, are also included in the same siliceous cement with the spicules. These rock slabs are evidently portions of a continuous sponge-bed which have remained after the removal of the loose, incoherent sands which constitute the principal part of the strata immediately beneath the surface in this locality.

Tilburstow Hill.—To the south of the village of Godstone, Surrey, the beds of the Hythe division of the lower greensand are well exposed on the north slopes of the

* "Strata below the Chalk." Trans. Geol. Soc., ser. 2, vol. iv., p. 147.

† "Geology of the Weald," p. 125.

ridge known as Tilburstow Hill. They are here largely worked for road material, and, as we have already seen elsewhere, they consist principally of the débris of fossil sponges. To reach the hard beds in the quarries now opened it is necessary to remove the overlying beds of loam, fuller's earth, and dark-green calcareous rock, belonging to the Sandgate division, which are about thirty-eight feet in thickness. Beneath these are beds of siliceous and cherty rock, with thin intermediate layers of sandstone, with glauconite grains and a bed of limestone, in all about eleven feet in thickness (3·3 m.), which are so filled with spicular remains that they may be regarded as a series of sponge-beds. The chert occurs in layers, from 12 to 14 inches in thickness (·3 to ·35 m.), of a lenticular form, and it passes upwards and downwards into the white porous siliceous rock which has been already described as forming an outer crust to it. The sponge-spicules are shown in faint outline in thin microscopic sections of the chert, and the spicular canals, which have been infilled with glauconite, are very distinct. The siliceous matrix inclosing the spicules appears to have been partly deposited in a globular form; it is now either in the condition of chalcedony or of crystalline quartz.

The porous siliceous rock in these sponge-beds is more developed than the chert; there is one layer of it three feet in thickness. Spicular casts are not clearly shown in it, but the silica is of the same character as that in which the moulds of spicules are well preserved, and it also passes into true chert.

The thin intermediate beds of quartz-sand, and glauconite grains, also contain sponge-spicules or their casts, and thus indicate a continuous succession of sponge-life during their deposition.

Besides the chert and the porous siliceous rock, there is in this series a bed of compact grayish-blue limestone, exceedingly hard and tough, filled with sponge-spicules which are now of calcite. This bed is from 12 to 14 inches in thickness (·3 to ·35 m.). A thin microscopic section shows numerous spicules imbedded in a matrix of granular calcite (Plate 40, fig. 4). The spicules are nearly entirely composed of clear, crystalline calcite, but some are of silica; and in many the canals are preserved. Their forms and dimensions clearly prove that they belong to the same siliceous sponges as those of the chert and siliceous beds above and below the layer of limestone, and it is evident that in this latter bed the original silica of the spicules has been nearly entirely dissolved away, and the casts have been refilled with calcite. Exceptionally, an infilling of glauconite takes place instead of calcite. No other organic remains besides the spicules are distinguishable in the limestone.

Dr. Fitton has given a detailed section of these strata at Tilburstow Hill without however noticing the organic nature of the chert. The sections at present exposed vary to some extent from those described in the "Strata below the Chalk."

Nutfield, Surrey.—Beds of gray sandstone, filled with spicules are exposed at this place. A microscopic section of the rock, for which I am indebted to Professor Judd, F.R.S., shows the spicules partly replaced by glauconite and partly dissolved,

only their infilled canals remaining. The grains of quartz-sand have been cemented together by the silica derived from the spicules.

Sevenoaks, Kent.—On the summit of the lower greensand plateau a short distance to the south of Sevenoaks, there are beds of chert filled with sponge-remains which are extensively worked for road material. The beds belong to the Hythe division, and according to Mr. TOPLEY* they extend from Brastead Chart to some three miles beyond Sevenoaks Common, and further east the surface of the ground in Great Comp and Malling Woods is also covered with fragments of the same rock.

In a stone quarry near the high road, about a mile from Sevenoaks, the following section was exposed at the time of my visit.

	feet.	inches.
(1.) Chert (sponge-bed)	2	6
(2.) Yellow and reddish sand	2	0
(3.) Chert (sponge-bed)	2	0
(4.) Sand.	1	0
(5.) Chert (sponge-bed)	2	0
(6.) Sand.	0	8
(7.) Chert (sponge-bed) bottom of quarry . .	1	6
	11	8

The sponge remains appear to be limited to the beds of chert, of which there are four exposed, with a total thickness of eight feet (2·4 m.). The chert is of a gray or light-brown tint, very hard and brittle. In some cases there is an outer crust of light-coloured porous siliceous rock filled with the casts of spicules. In microscopic sections of the chert itself numerous spicules can be seen, embedded in a translucent matrix of chalcedony, which appears to have been deposited concentrically round them. The spicules are also of chalcedony, and their canals are infilled either with silica of a darker tint than the walls or with glauconite. Frequently all traces of the spicular walls have disappeared and only the infilled canals remain, and these might readily be mistaken for perfect spicules. A few small foraminifera and fragments of echinoderms are occasionally present in the chert; with these exceptions it seems to consist of the remains of tetractinellid and lithistid sponges.

In the higher beds of the Hythe division, shown in the railway cutting at the north end of the Sevenoaks tunnel, I did not recognise any well-defined sponge-beds, though in some of the upper beds in the section, both in the so-called hassock and in the rag, the rock is in places filled with casts of spicules.

Fossil sponges are also numerous in beds of loose, unconsolidated, yellowish quartz-sand, belonging to the Folkestone or uppermost division of the lower greensand, of which a section 25 feet in thickness is shown near the Gas Works at

* "Geology of the Weald," p. 119.

Sevenoaks. The sponges here, however, do not occur in definite beds as a rule, but are distributed irregularly through the deposit, in nodular masses of one or more entire sponges enveloped in the sand, which is lightly cemented round them. These sponges are lithistids; but while retaining their complete outer form, their spicular structures have been dissolved, and only casts in a siliceous matrix remain.

A single definite sponge-bed does, however, occur in the deposit, but it is not more than one or two inches in thickness (·025 to ·05 m.). It is made up principally of relatively large acerate and trifid spicules of tetractinellid sponges irregularly mingled together, and with these there are minute skeletal fragments of lithistid, and even of hexactinellid sponges. The spicules are but lightly cemented together with the quartz-sand by a siliceous cement, so that detached forms can be procured by breaking up the softer portions of the bed. The sponges in this deposit were first discovered by Professor PRESTWICH, F.R.S., who very kindly directed my attention to them and conducted me to the beds.

Maidstone, Kent.—There are no very clearly marked sponge-beds in the numerous exposures of the lower greensand near Maidstone. As a general rule sponge-remains are altogether absent in the rag or limestone beds, but in one or two layers of this material near the base of the section in Bensted's quarry, spicules were abundant, but they are now replaced by crystalline calcite or remain as empty moulds. The principal part of the sandstone or so-called hassock-beds, which intervene between the limestone or rag beds, consists of quartz and glauconite grains lightly cemented together by calcareous material, but there are also present other beds somewhat similar in appearance, but with a cementing material of chalcedonic silica, and inclosing lenticular deposits of chert. In these, sponge-remains are abundant, but usually as empty casts. The chert is largely filled with fragments of other organisms as well as sponge-spicules, and the sponges are not so exclusively present as to justify regarding the deposits as sponge-beds. In some of the hassock-beds, however, there are porous siliceous accretions nearly entirely composed of sponge remains, but they are not continuous in a single bed.

On the surface of the fields to the north-west of Maidstone there yet remain numerous slabs or fragments of a hard porous siliceous rock, from one to three inches in thickness (·025 to ·075 m.), in which spicules can be seen. These fragments are very similar to those already described from Godalming, and like them are evidently portions of sponge-beds.

Sponge-spicules appear to have been more abundant in some of the older quarries at Maidstone than they are in those which are now worked, since Sir RICHARD OWEN* mentions that they were sufficiently numerous to be detrimental to the hands of the workmen, and Mr. BENSTED† also states that there were large quantities of them in the lower beds of the series.

* 'Palæontology,' 1861, vol. i., p. 6.

† 'Proceedings of the Geologists' Association,' vol. i., 1859–60, p. 58.

Hythe, Kent.—Sections of about 50 feet in thickness (12·5 m.) of the basal division of the lower greensand are shown in the quarries at the back of the town. The strata consist of alternating beds of limestone, sand, and sandstone, approximately of the same character as those at Maidstone, but less regular as regards the thickness of the beds, and no chert is present. The only traces of sponge remains occur in some of the layers of limestone, and one bed in particular, in the lower part of the section, is so extensively filled with spicules that it may fairly be regarded as a sponge-bed. This bed is about 15 inches (·375 m.) in thickness; it is of a light-gray or bluish tint, hard and massive, except in small patches, where the spicules have been dissolved away and the rock is consequently porous. Thin microscopic sections show that the matrix is granular and the siliceous spicules have been replaced by crystalline calcite. In a few cases the replaced spicules yet show traces of their canals. In addition to the sponge-remains, the bed contains foraminifera and fragments of echinoderms.

The limestone sponge-bed is also shown in a quarry just below the Hythe railway station, and in this quarry there is also a thin shell-bed, principally consisting of the casts of *Trigonia*, in a calcitic matrix. The matrix of this shell-bed is completely filled with the empty casts of sponge-spicules.

Folkestone, Kent.—The uppermost division of the lower greensand, which is named the Folkestone division, from its prominent development in this locality, contains numerous sponge-beds, and in this respect is of a somewhat exceptional character, since sponge remains are of rare occurrence in the strata of this division in other localities.

The best exposure of the Folkestone beds is in the cliffs on the shore between the east side of the Harbour and Copt Point. About half-way between these places the following section was shown at the time of my visit. I here give it in detail, since it illustrates the manner in which the sponge-beds alternate with other beds.

		feet.	inches.
(1.)	Above, coarse sandstone and grit	1	0
(2.)	Brown and yellow sands	6	0
(3.)	Sandstone with spicules	1	0
(4.)	Unconsolidated sand	4	0
(5.)	Siliceous rock (sponge-bed)	1	10
(6.)	Unconsolidated sand	1	8
(7.)	Siliceous rock (sponge-bed)	1	5
(8.)	Sand	1	8
(9.)	Siliceous rock (sponge-bed)	0	5
(10.)	Sand	2	0
(11.)	Siliceous rock (sponge-bed)	1	4
(12.)	Sand	5	0
(13.)	Siliceous rock (sponge-bed)	1	0

	feet.	inches.
(14.) Sand and siliceous rock	1	6
(15.) Sand	3	6
(16.) Siliceous rock (sponge-bed)	0	11
(17.) Sand	1	0
(18.) Siliceous rock (sponge-bed)	0	6
(19.) Sand	1	6
(20.) Siliceous rock (sponge-bed)	1	0
	38	3

Thus, in this section of 38 feet 3 in. (11·475 m.) there are eight distinct zones or beds of sponge-remains, with a total thickness of 8 feet 5 in. (2·52 m.). These sponge-beds, being of a much harder nature than the intermediate layers of sand, stand out prominently in relief in the face of the cliff, and through the weathering away of the loose sand, large masses and blocks fall down, and are piled over each other on the beach. The sponge-beds consist of spicules in a matrix of coarse grains of quartz-sand and glauconite, cemented by siliceous material. The rock is of a porous or spongiose character, harsh and rough to the touch, and readily friable. There is in some of the beds a central layer of cherty rock, but this is usually porous, though in places it passes into true chert. The spicules in these beds are most clearly shown on weathered surfaces, and they appear to the unaided sight as minute white rods of a dull porcellanic aspect. They are often sufficiently numerous to give a distinct white aspect to the rock surface. In freshly fractured surfaces only empty casts of spicules can, as a rule, be detected. By the aid of a lens, gradual transitions can be detected between apparently perfect spicules and forms with indistinct outlines, in which the spicules are gradually breaking up to form the whitish siliceous cement which binds them and the sand grains into one mass. The cavities in the cherty portions of the sponge-beds are frequently lined with a smooth layer of chalcedonic silica, which shows under the microscope a radiately fibrous structure. When the cavities are infilled by successive layers of this silica, which there can hardly be a doubt is derived from the sponge-spicules, a true chert is produced.

Not infrequently the weathered surfaces of the sponge-beds are covered by a raised network of sub-cylindrical anastomosing branches, resembling the structures generally known as *Spongia paradoxica*. These branching bodies have been mistaken for actual sponges,* but they appear to me to be merely the burrows of marine animals into which the loose sponge spicules have been washed, and from the partial solution of these the burrows have been filled with a strongly cemented material, which resists weathering better than the inclosing rock.

The spicules of these sponge-beds are exclusively of siliceous sponges, and belong

* Mr. F. G. H. Price refers to the sponge-beds in this section as seams of coarse calcareous sandstone, and states that large white tabular masses of branching sponge are met with on them. 'Proceedings of the Geologists' Association,' vol. iv., p. 129.

principally to the tetractinellidæ and lithistidæ. I did not find in any of the beds a single example or even the cast of an entire sponge.

The sponge-beds appear not to continue with any regularity in a horizontal direction, and they vary also in their characters in short distances. Thus the beds in the cliff section to the west of the harbour, though only imperfectly shown, are thicker, and decidedly contain more chert than those of the section described. This fact has already been noticed by Dr. Fitton* in the detailed sections which he has given of the various exposures round Folkestone. The same author describes very faithfully the peculiar characters of the sponge-beds, but, strange to say, he does not seem to have noticed the sponge-spicules in them. Mention is indeed made of siliceous spongy or spongiform concretions, but it is clear from the context that reference is only made to the peculiar porous nature of the deposits. They are also compared to the concretionary whetstones in the upper greensand of the Blackdown Hills in Devonshire, which latter, as will be shown in the sequel, are likewise sponge-beds.

Isle of Wight.—Notwithstanding the thickness of the lower greensand strata in the Isle of Wight, sponge-remains do not appear to be present in them.

Faringdon, Berkshire.—Though somewhat beyond the area of which this paper treats, it is desirable to make a brief mention of the remarkable deposit at this place, which has long been known under the name of the "sponge gravel." The character of its deposition, and the nature of the sponges in it are altogether different from those of the beds of the same geological horizon in the Wealden area. It is a thick bed of unconsolidated quartz-sand and pebbles, with numerous sponges and other fossils. Sponges predominate, and they are present in astonishing profusion. They differ from those of the lower greensand of the Wealden area, not only in retaining their complete entire forms, but in the fact that they are exclusively calcisponges. Not a single siliceous sponge has as yet to my knowledge been met with in this gravel, though these calcisponges have been described by various authors as originally siliceous. They thus present a striking contrast to the sponge-beds in other areas, in which the sponges have been entirely disintegrated, and are also exclusively of siliceous origin.

Sponge-Beds of the Upper Greensand.

Accepting as a typical section that shown in the Undercliff of the Isle of Wight, I propose to include in this term the strata there present between the base of the so-called chloritic marl and the blue micaceous sands, which, in common with Captain Ibbetson† and Mr. Parkinson,‡ I regard as the upper portion of the gault. Thus

* "Strata below the Chalk," p. 116, *et seq.*

† 'Notes on the Geology and Chemical Composition of the Various Strata in the Isle of Wight,' London, 1849.

‡ Quart. Journ. Geol. Soc., vol. xxxvii., 1881, p. 375.

limited, the upper greensand may be regarded as comprising two broad divisions, an upper, in which beds of chert are largely developed, and a lower, principally consisting of micaceous and quartzitic sands, with occasional layers of chert. The upper division corresponds generally with the zones C and D or Warminster beds of Barrois,* and the lower with the Blackdown beds or zones B and part of A of the same author. Though these divisions are available for the upper greensand strata of the Isle of Wight, and the outcrops further westwards in the counties of Dorsetshire, Wiltshire, and Devonshire, they are not clearly recognisable in the beds of the same geological horizon to the north and west of the Wealden area. The upper greensand in this area has an altogether different facies; the beds are exposed at intervals in a narrow tract of country, at the base of the chalk escarpment between Godstone in Surrey and Selborne in Hampshire, where they occasionally form a distinct escarpment in advance of that of the chalk. They are known and have been described under the local names of malm, firestone, burrystone, and hearthstone. The peculiar character of these beds arises chiefly from the sponge-remains contained in them, and I propose to describe these before referring to the sponge-beds of the more normal deposits of the upper greensand in the Isle of Wight.

Merstham, Surrey.—There are several exposures of the upper greensand-beds in the escarpment running parallel to that of the chalk, both in road-cuttings and in quarries, the material being extensively employed for building. In a quarry about 1½ mile east of Merstham the following section was shown :—

	ft.	in.		ft.	in.
(1.) Siliceous and siliceo-calcareous rock in thin beds; those near the surface being partially broken up	4	0	to	5	0
(2.) Bed of same rock as No. 1, but in large compressed nodules which in places decompose to a reddish clay				0	8
(3.) Bed of siliceo-calcareous rock. Firestone or malm . .				10	0
				15	8

In an adjoining roadside cutting, where, however, the beds are not so distinctly shown as in the quarry, the bed marked 1 is overlaid by a layer of soft glauconitic marl, which is probably the equivalent of the chloritic marl.

The rocks of the above section are, when fresh, of a light brown tint and earthy aspect; they become white or grayish-white when dry, and are then considerably harder. The rock is minutely porous and largely absorbent of water, and when dry, of light specific gravity. It is soft to the touch and not adherent to the tongue. In some places the light brown rock gradually passes into one which is heavier, more compact, and of a light-bluish tint, and frequently becomes nodular. These nodules are locally known as flints, but they have an altogether different appearance to the

* 'Recherches sur le terrain crétacé supérieur de l'Angleterre et de l'Irlande,' p. 106.

normal flints of the upper chalk. There are also pockets of a reddish-yellow loamy clay which appear to be produced by an alteration of the normal rock. Some portions of the beds do not appear to contain calcareous material, in others calcite is present and the rock is then harder and denser. When a fractured surface is examined with a strong hand-lens it is seen to consist of minute silvery mica-scales, points and rods of glauconite, and white powdery siliceous granules. Fragments of sponge-spicules are also visible, but these are less numerous than the minute hollow cavities of spicules with which the rock is throughout interpenetrated, and to which it owes its porosity. These spicular casts are very minute, and it requires careful observation to detect them (Plate 40, fig. 7).

Thin sections of the harder portions of this rock, examined under the microscope, still further confirm its organic origin and its derivation from the spicules of siliceous sponges. The sections show spicules, mostly fragmentary, irregularly distributed in the siliceous and siliceo-calcareous matrix. The points and rods of glauconite can be recognised as portions of siliceous spicules replaced by this mineral. The spicules which yet remain siliceous are white by reflected, and transparent by transmitted light; they are negative to polarised light, and thus show that the silica still retains the amorphous colloid condition in which it is present in recent siliceous sponges. The silica of the matrix, and also that infilling the empty casts of the spicules, is partly in the form of minute discs or globules (the character of which is more fully referred to in the sequel), likewise of amorphous silica, and partly of transparent chalcedonic silica (Plate 40, fig. 5).

In the general mass of the rock, the spicules which yet remain intact are firmly imbedded in the siliceous matrix, but in the decomposed clayey portions some may be obtained by washing, free from matrix. They are principally acerate and trifid spicules belonging to tetractinellid sponges, mingled with others of lithistid and hexactinellid types. There is a general resemblance to the spicules of the sponge-beds from the lower greensand, and to those from the upper greensand of the Isle of Wight and the South-Western counties, but at the same time they are notably of smaller dimensions. Not only in this locality but generally in the beds of malm or firestone to the north and west of the Weald the same minute forms occur.

The spicular constituents of this rock are not shown with equal clearness in every portion of the beds exposed, but, as the nature of the rock is similar throughout, there is reason to suppose that it is all of similar origin, and may be regarded as a sponge-bed. I could not determine whether the quarry-section referred to, 15 feet 8 inches in thickness (4·7 m.), embraces the total* thickness of the malm at this place.

Notwithstanding the abundance of the detached spicules I did not meet with a single entire sponge or even a fragment of one in this bed; they all appear to have

* Mr. Topley, in the "Geology of the Weald," p. 154, states that the total thickness of the upper greensand at this place is 22 feet, but this includes the bed of chloritic marl; Dr. Fitton, in the "Strata below the Chalk," p. 140, places the total thickness at 25 feet, but he did not himself see the section.

been disintegrated and reduced to their individual elements. The organic nature of the rock does not appear to have been previously noticed.

Godstone, Surrey.—About three miles east of the quarries at Merstham, beds of the same age are worked by the side of the high road north of the village of Godstone. There is here an upper bed of soft rock 5 feet 8 inches in thickness (1·36 m.), which is largely quarried for hearthstones. This rock is of a more friable character, and contains more calcite than the malm or firestone; but empty spicular casts and spicules replaced by glauconite are present in it, and the silica is also in the amorphous globular form. Beneath the hearthstone-bed there are beds 14 feet in thickness of malm or firestone, of a similar character to those at Merstham, and similarly filled with sponge-remains. In these there are at intervals parallel layers of the hard blue nodular aggregations known as flints, which are from 3 to 4 inches (·075 to ·1 m.) in diameter. These nodules are not sharply delimitated from the inclosing rock, in the same manner as the flints in the chalk, but there is a gradual passage from one to the other. They are disposed, however, in definite planes of bedding in the same manner as the chalk-flints. The sponge-spicules are much more distinctly shown in these blue nodules than in the white portion of the rock. A magnified section of one of these nodules is shown on Plate 40, fig. 2.

Reigate and Betchworth, Surrey.—A short distance to the west of Reigate and again about two miles further west, close to Betchworth station, quarries are opened to obtain the hearthstones and firestones. In the upper beds of the quarries the base of the chalk marl is shown, and beneath this a bed from 4 feet to 8 feet in thickness of a soft greensand in which glauconite grains are very abundant. This bed represents the chloritic marl. The hearthstones are here 10 feet in thickness (3 m.) and the firestone beds beneath are of an equal thickness. The character of these rocks closely resembles that of the beds at Merstham and Godstone.

Farnham, Surrey.—The upper greensand strata near this town were formerly largely worked for agricultural purposes and for building, but good sections are no longer exposed. According to Messrs. Way and Paine,* who have given a detailed description of the beds, they have a total thickness of over 60 feet, of which the upper 40 feet (12 m.) is a soft friable rock, known as malm or soft Surrey sandstone, and the lower 20 feet (6 m.) is of a harder character, and is called firestone.

The malm rock is of a creamy-white tint, and very soft and porous, and of relatively light gravity when dried. With the exception of a few mica-scales and particles of glauconite it appears to be entirely siliceous. Its friable character does not permit microscopic sections to be made from it, but reduced to powder and examined in Canada balsam, fragmentary spicules both of silica and of glauconite can be seen in it. The greater part of the silica in the rock is in the globular form, and the minute globules or discs occur detached in the powdered material (Plate 45, figs. 18–18*b*). This globular silica, as well as that of the spicules, is in the amorphous condition and

* 'Journal of the Royal Agricultural Society,' vol. xiv., 1853, p. 232, *et seq.*

it is entirely negative between crossed nicols. The empty spicular cavities, to which the porous nature of the rock is due, appear in fractured surfaces as very minute, short, straight tubes or open canals from ·15 to ·4 mm. in length by ·03 mm. in width. They are undoubtedly the casts of acerate and trifid siliceous spicules, and they are sufficiently numerous to show that the silica derived from the solution of the contained spicules can have produced the colloid material of which the rock principally consists. There can be no doubt that this malm rock is derived from the spicules of siliceous sponges.

Messrs. Way and Paine first called attention to the fact that the Farnham malm and firestone contain a large amount, varying from 25 to 75 per cent., of soluble silica, but they do not appear to have noticed the peculiar globular form in which the silica has been deposited, and* they deny its organic origin.

Selborne, Hampshire.—The malm rock is well shown in the quarries at this village, where a section of 15 feet (4·5 m.) is exposed; also in road-cuttings and on the surfaces of the adjoining fields. It is of a much harder and more compact character than that at Farnham, and contains a fair proportion of calcite, which renders it more suitable as building material. Spicules and spicular casts abound in it. Some of the casts of spicules in the more calcareous beds have been refilled with crystalline calcite, whilst in sections of the more siliceous beds the spicules are seen thickly grouped together, surrounded by globular silica and also by transparent chalcedonic silica. The spicular canals are infilled by glauconite, which remains intact after the spicular walls have been dissolved.

Gilbert White, in the 'Natural History† of Selborne,' referring to this rock, states that it is but little removed from chalk in appearance, but seems so far from being calcareous that it endures extreme heat. It was known as the white malm, in contradistinction to the chloritic marl, which was called the black malm. Its organic origin does not seem to have been suspected.

Wallingford, Berkshire.—The malm rock is not limited to the northern margin of the Weald, for at Wallingford, which is situated about thirty-two miles to the north-west, there are exposed beds of siliceo-calcareous rock of the same character as those already described round the Wealden area. The rock is hard, and calcite is only occasionally present in it, but there is a greater relative amount of quartz-sand and other mechanically derived constituents than in the malm of the Wealden district. The sponge-remains are either as empty casts or of amorphous silica, and the siliceous matrix is partly in the globular form.

* "On the Silica Strata of the Lower Chalk." 'Journal of the Royal Agricultural Society,' vol. xiv., p. 243. These authors state that it is "not infusorial, for with the exception of a few foraminifera no traces of animal life can be observed in the rock by microscopical examination." They attribute its formation to "aqueous decomposition resulting from the contact of silicate of lime in solution, derived from the older rocks, with carbonic acid, produced either by vegetable and animal decay or by volcanic agency, so that at one and the same time carbonate of lime and gelatinous or soluble silica would be formed."

† Bell's edition, p. 3.

Eastbourne and Folkestone.—According to the Geological Survey Memoir,* the characteristic malm does not extend round the south-western margin of the Weald, but gradually disappears to the east of Steyning, in Sussex, and the upper greensand is represented by beds of sandy marl which extend as far as Eastbourne. In these beds I have not discovered any sponge-remains, but in some higher beds of dark grayish glauconitic sandstones, belonging to the horizon of the chloritic marl or the base of the chalk marl, there is a very characteristic sponge-fauna, which differs so greatly from that of the upper greensand proper that it merits notice, although beyond the limit assigned to my paper. These sponge-beds, or reefs, as they are termed by Mr. PRICE,† appear to consist exclusively of entire examples of hexactinellid sponges belonging to the genera *Plocoscyphia*, *Craticularia*, and *Stauronema*, growing together in such abundance that the exposed rock surface is covered with them. These and similar sponges are either wholly absent from the sponge-beds of the lower and upper greensand or represented by mere microscopic fragments of their skeletal mesh, and their presence in these deposits may be regarded as indicating the existence of different conditions, in which deeper water prevailed than when the greensand sponge-beds were formed.

Shanklin, Ventnor, and the *Undercliff, Isle of Wight.*—The sponge-beds in the upper greensand of these places principally occur in the upper division, characterised by bands of chert and siliceo-calcareous rock. The beds are well shown in the quarries near Shanklin and at Ventnor, and along the Undercliff. The striking parallel bands of cherty rock which project in relief in the upper portion of the cliff, and the huge masses of the same material scattered over the terraces beneath, consist of sponge-beds. At the Railway Station at Ventnor the following section is exposed; the bed marked 1 is immediately overlaid by the chloritic marl.

	feet.	inches.
(1.) Siliceo-calcareous rock, with bands of chert (sponge-bed)	21	0
(2.) Siliceo-calcareous rock	4	0
(3.) Building stone with nodular layer of bluish limestone	1	7
(4.) Freestone of quartz-sand, glauconite, and mica cemented by calcite	3	2
(5.) Freestone, similar to above, but with some silica in cementing material	4	10
(6.) Beds of siliceo-calcareous nodules with chert	1	4
(7.) Freestone of similar character to bed 5	4	9
(8.) Hard siliceo-calcareous rock, with chert	1	0
(9.) Freestone resembling bed 5	3	0
(10.) Freestone and chert to base of quarry.	3	0
	47	8 (=14·3 m.)

* "Geology of the Weald," p. 158. † Quart. Journ. Geol. Soc., vol. xxxiii., p. 435.

The upper layers (21 feet in thickness) of this section so abound with spicules that they may be considered as a continuous sponge-bed. The chert resembles that which has already been described from the lower greensand. It is usually of a light brown tint, and in thin sections under the microscope it is seen to be filled with spicules and spicular casts imbedded in a translucent matrix of chalcedonic silica. The spicules are likewise of chalcedony, and their canals are infilled with glauconite. Another variety of chert, also very abundant, is of a grayish or greenish-white tint; it differs from the former in that the matrix is of amorphous silica, whilst the inclosed spicules are of chalcedony. The chert bands here, similar to those of the lower greensand, are enveloped in an outer crust, of varying thickness, of white or yellow siliceous porous rock, which is interpenetrated with the empty moulds of spicules.

In some of the thicker masses of chert there are cavities or pockets filled with spicules, loosely mingled in a grayish siliceo-calcareous powder, in which there are also numerous well-preserved foraminifera, chiefly of the genus *Textularia*. The spicules in these cavities have undergone a remarkable alteration in structure; they appear to have lost their original silica, which has been replaced by glauconite and some other silicate of a greenish-white aspect. The replacing material has only partially filled the form of the original spicules, and thus they look like mere shadowy casts of complete spicules. These in many cases are peculiarly distorted and contracted (Plate 45, figs. 15*a*—*e*).

Spicules in varying quantities are also present in the beds of freestone and other siliceo-calcareous layers, in all about 27 feet in thickness, which form the lower part of the section at the Ventnor Station quarry. These beds consist principally of fine grains of quartz and glauconite with mica and calcite, and in appearance they resemble the malm rock of the northern margin of the Weald, and, in fact, they have thus been named by Mr. PARKINSON.* They differ, however, from the true malm in the small proportion of silica which they contain, and I have not seen in them any silica of the globular form so common in the malm. The sponge-spicules in these beds are mostly represented by their hollow moulds, and these are so minute that only by careful scrutiny can they be detected. They are, as a rule, more readily recognisable in the small dark phosphatic nodules which frequently occur in the beds. The spicules are not sufficiently abundant, however, for these beds to be considered as definite sponge-beds.

In the lower series of the upper greensand strata, consisting of about 50 feet (10·5 m.) of yellowish-gray micaceous sands, usually soft and unconsolidated, but with occasional harder bands, I did not detect any well-defined sponge-beds, but spicules are fairly abundant in some of the layers, and it is not improbable that a careful search would show their presence throughout.

The sponge-beds of the Isle of Wight, like those of the lower greensand, exclusively consist of the detached spicules of siliceous sponges. A few entire lithistid sponges

* Quart. Journ. Geol. Soc., vol. xxxviii., p. 370.

of the genus *Jerea* are occasionally met with, but the exact horizon in which they occur is not known, though I should judge from the character of the rock containing them that they belong to the horizon of the freestone beds.*

Warminster, Wiltshire.—From the upper greensand strata of this locality entire forms of lithistid sponges have been met with in one or two limited areas, but they are of comparatively rare occurrence, and appear to be seldom found at the present time. On the other hand, beds of rock filled with the detached spicules of lithistid and tetractinellid sponges are exposed at Warminster itself, and in other places south of it, to beyond the village of Crockerton. The sponge-beds are largely worked for road material, and fragments of them are abundant on the surfaces of the fields, under which the beds crop out.

The following section is shown in a quarry on the outskirts of Warminster :—

	feet.	inches.	
(1.) Broken fragmentary chert in a marly matrix . . .	3	0	
(2.) Greensand of quartz and glauconite grains . . .	1	0	
(3.) Chert (sponge-bed)	1	0	
(4.) Greensand similar to bed 2	1	0	
(5.) Siliceo-calcareous rock and chert, partly decomposed to a reddish loam (sponge-bed)	2	3	
(6.) Greensand similar to bed 2	1	3	
(7.) Siliceous rock with chert (sponge-bed)	2	6	
(8.) Greensand	1	0	
(9.) Siliceous rock with chert (sponge-bed) to bottom of quarry	1	3	
	14	3	(=4·275 m.)

Near Chute Farm, about four miles south of Warminster, another section is exposed as follows :—

	feet.	feet.	inch.	
(1.) Broken chert and siliceous rock	1 to	2	0	
(2.) Siliceo-calcareous rock and chert in beds of varying hardness (sponge-bed)		10	0	
		12	0	(=3·6 m.)

* I noticed large masses of rock with some of these sponges weathered out on their surfaces on the beach about two miles west of Ventnor. From their appearance I could understand the origin of the remarkable figures of the tulip alcyonium given by Webster in 'Transactions of the Geological Society,' 1814, vol. ii., p. 377, pl. 28. One need hardly say that the enormously long stems which are there represented as portions of the sponge have no real connection with it. These supposititious stems have been based on long cylindrical tubes with fluted walls, which interpenetrate the rock. Whatever may be the true nature of these tubes it is certain that they are not the stems of sponges.

The siliceo-calcareous rock of which the sponge-beds are largely composed is of a whitish appearance, and consists of silica, both amorphous and as chalcedony, calcite, a small quantity of mica, and grains of glauconite and quartz. It is filled with spicules of relatively large size; they are now of a porcelanic white by reflected light, and transparent in Canada balsam by transmitted light; they are usually perfect, and their canals are well-preserved. These spicules are of amorphous silica and are entirely negative between crossed nicols, though their canals are usually infilled with chalcedonic silica or with glauconite, and the transparent siliceous matrix in which they are imbedded is also chalcedonic. The spicules are in such profusion in some of the beds that they form distinctly-marked thin white parallel layers with intermediate layers, which, from the larger proportion of glauconite grains in them, are of a greenish tint. These parallel layers must have been thus arranged at the time the beds were forming, and they indicate the disintegration of the sponges and the mingling of their spicules before they were buried under fresh deposits. The perfect condition of the spicules likewise indicates that they could not have been drifted to any extent over the sea-bottom (Plate 40, fig. 1).

The white siliceous rock filled with spicules occasionally passes into a brown chert; in this rock the spicules are to a large extent dissolved away, and only their infilled canals and shadowy casts remain. It is evident that the cherty portions were originally as filled with perfect spicules as the siliceous rock into which they gradually pass, and the difference results from the solution of the amorphous silica of the walls of the spicules, and its re-deposition in the chalcedonic condition to form the chert.

In some of the sponge-beds there are cavities or pockets in the chert filled with a loose, powdery, unconsolidated material, of the same character as that already mentioned in the chert at Ventnor. The spicules in this material are, however, for the most part, in a perfect condition, and can be washed perfectly free from the matrix; but others have undergone a similar alteration to that of the Ventnor spicules, and they have been partially replaced by a nearly transparent mineral, and the silica is in the globular form (Plate 45, figs. 15, 16). Much of the silica in the powder of these cavities is in the form of minute globules or discs, and the silica is amorphous (Plate 45, figs. 19–19*e*).

Penzlewood, Somerset.—On the plateau formed by the summit beds of the upper greensand at this place, and at the adjoining villages of Stourton and Zeals, in Wiltshire, there are exposed beds of chert and siliceous rock, which are estimated by Mr. Horace B. Woodward* to be from 20 to 30 feet in thickness. The beds are largely worked for road material, and they have been formerly extensively employed for building and also for the manufacture of whetstones† for sharpening

* "Geology of East Somerset." 'Memoirs of the Geological Survey,' p. 138.

† My attention was first directed to these beds, by my friend Mr. Alfred Gillett, of Street, who showed me one of the whetstones made at this place. A fractured surface at once showed that the hard, porous siliceous rock, of which it was made, was a portion of a sponge-bed, since it was filled throughout with the empty casts of siliceous spicules.

scythes. The chert in these beds is of the same character as that which has been already described from Ventnor and elsewhere. Spicules are abundant in it, but their walls are frequently dissolved, and they are now largely represented by their infilled canals (Plate 40, fig. 3). This chert is accompanied, and frequently inclosed, by the hard, harsh, porous siliceous rock filled with spicular cavities so generally present. In some places this description of rock has been naturally broken up, so as to form beds, five or six feet in thickness, of small irregular fragments having a pumice-like aspect.

In addition to the chert and porous siliceous rock, some of the sponge-beds in this district consist of a hard, whitish, massive rock, of a granular character, filled with spicules of a white porcelanic tint. The rock appears to be nearly entirely siliceous, the silica is partly chalcedonic and partly amorphous, and in the globular form. The spicules are of amorphous silica, similar to those at Warminster. Small cavities in this rock are filled with spicules partially cemented together.

The sponge-beds of this locality are distinguished by the absence of calcite and the small proportion of glauconite and mica present in them. Their organic character does not appear to have been hitherto noticed, although they have excited much attention, owing to the fact that over an extensive area numerous conical pits have been made in them to extract the harder portion of the beds for economical purposes, and these pits have been surmised to be the* remains of primitive dwellings.

Blackdown Hills, Devonshire.—The beds of upper greensand age exposed in the higher portion of the Blackdown Hills have formed the subject of numerous memoirs, in which some reference is made to the occurrence in them of sponge-remains. The detached spicules met with here and at Haldon were first described and figured by Mr. E. Parfitt,† and later in a more complete manner by Mr. H. J. Carter, F.R.S.,‡ but these authors do not describe in detail the beds in which they occur. The beds have formerly been extensively worked for whetstones and for building, and Dr. Fitton§ gives full details of the beds employed, and the mode of manufacture of the whetstones, without, however, noticing their organic character.

The basal beds of the greensand consist of a fine-grained yellowish sand-rock, which, according to Mr. Downes,‖ is 30 feet (9 m.) in thickness. This is overlaid by a series of beds of sand and siliceous rock varying from 12 to 20 feet (3·6 to 6 m.) in thickness, which contain the so-called concretions used for whetstones. These are of a greenish tint when moist, and gray when dry. The rock is porous, very harsh to the feel,

* 'Proceedings of the Somerset Archæological and Natural History Society,' vol. xv.; also "Remarks on the Pen Pits and other supposed Early British Dwellings." By H. B. Woodward, in the 'Midland Naturalist,' 1883.

† 'Transactions of the Devonshire Association for the Advancement of Science, 1870.'

‡ Ann. Mag. Nat. Hist., s. 4, vol. vii., 1871, p. 112.

§ "Strata below the Chalk," p. 236.

‖ Quart. Journ. Geol. Soc., vol. xxxviii., p. 80.

partly hard, and partly soft and friable. It is filled with sponge-spicules and their empty casts, cemented together by chalcedonic silica. Quartz-sand and glauconite grains are also present, but no calcite, and only a small amount of mica. In the softer portions the spicules are so lightly cemented with the other materials that by gently breaking up the accretionary masses they may be obtained in great numbers and with their forms perfect. The spicules are chiefly of chalcedonic silica; some appear to be partly of crystalline silica; they are nearly transparent in Canada balsam, and require to be mounted for the microscope in glycerine. Spicules are abundant also in the unconsolidated sand of the whetstone series of beds, and they may be regarded as filled with sponge-remains throughout, and forming a continuous sponge-bed. The cementing silica, which renders this material suitable for whetstones, is derived from the solution of the spicules, and the chalcedonic silica, which has replaced the calcite of the molluscan shells in the same beds, may be attributed to the same source.

Above the whetstone series of sponge-beds, there are beds of sand and sandstone (beds 10, 11, of Mr. Downes), and these are again succeeded by sands with siliceous rock and chert, which are also composed of sponge-remains. On the summit of Blackdown, near Cullompton, there are beds of cherty débris 5 to 10 feet in thickness, imbedded in a clayey matrix. Though much weathered, spicules can yet be recognised in some of the masses of chert, and it seems to me highly probable that these beds of débris represent the chert series or sponge-beds, which at Warminster, Penzlewood, and at Ventnor, form the summit of the upper greensand.

Though detached spicules are so abundant, entire sponges are now comparatively rare in the Blackdown beds. The well-known *Siphonia pyriformis* (*S. tulipa*, Zittel), figured by Sowerby* in Dr. Fitton's memoir, is stated by Mr. Downes† to be limited to a single layer, and the spicules of this species very seldom occur in the sponge-beds, which are principally composed of the acerate and trifid forms belonging to tetractinellid sponges and of the large spicules of megamorine lithistids.

Haldon Hills, Devonshire.—The upper portion of these hills, like those at Blackdown, about twenty miles to the north-east, consists of beds of upper greensand age. A section of about 30 feet (8·7 m.) of strata is exposed by the side of the high road between Exeter and Dawlish. The beds are principally of quartzitic and glauconitic sands, with nodules and bands of chert. The sponge-remains principally occur in a layer of chert from 8 inches to 1 foot in thickness, and in a bed of reddish-brown quartz-sand, 10 feet in thickness, at the base of the section. The sand is unconsolidated, and it is in places filled with detached sponge-spicules, which can be obtained quite free from the matrix. The spicules are for the most part similar in form to those at Blackdown, but they are not so well preserved. The silica of these spicules is partly crystalline and partly chalcedonic.

Axmouth, Beer Head, Devonshire.—I have not had an opportunity of examining

* "Strata below the Chalk," 'Geological Transactions,' 2 ser., vol. iv., p. 340, pl. 15a.

† Quart. Journ. Geol. Soc., vol. xxxviii., p. 81.

the sections of the upper greensand at these places, but according to Mr. C. J. A. Meyer,* there is one bed (No. 6) of a light-coloured sand, with chert in nodules or layers, varying from 7 to 25 feet (2·1 to 7·5 m.) in thickness, which is crowded with sponge-spicules. Dr. Barrois† places this bed on the same geological horizon as the Blackdown deposits, but Mr. Meyer believes it to be above these latter.

Sponge-beds in deposits of corresponding age in Germany, France, and Belgium.

The general distribution of beds of sponge-remains in the lower and upper greensands of the south of England would of itself furnish a strong probability that similar beds might also occur in the same formations on the Continent. This has already been shown to be the case with respect to the Hils-sandstein in Westphalia, and I am enabled to bring forward some evidence of the presence of sponge-beds in strata of cenomanian age in France and Belgium.

Hils-sandstein, Westphalia.—This formation, which has a thickness of 150 m. (492 feet), was, until 1879, believed to be an ordinary sandstone composed of quartz-sand and mica cemented by silica. It was then stated by Herr Wœckener ‡ to be largely composed of huge sponges of irregular form, but Professor Zittel § showed that the supposed sponges were in reality nodular masses of rock filled with sponge-spicules or their casts, and that this rock was of organic origin and filled with the remains of sponges which lived on a sandy sea-bottom. The character of the rock very closely resembles the sponge-beds of Haslemere, and the formation is on the same geological horizon as the lower greensand.

France and Belgium.—The description given by Dr. Charles Barrois‖ of the petrological characters of the so-called gaize de l'Argonne corresponded so exactly with the characters of the sponge-beds in the upper greensand that, in default of an opportunity of seeing this rock *in situ*, I applied to Dr. Barrois to furnish me with hand-specimens of it. My request was most kindly complied with, and the specimens which Dr. Barrois supplied to me from different localities fully confirmed the anticipations which I had formed that the gaize was of organic origin, and largely composed of the remains of sponges. The specimens were obtained from the following localities, and I append short descriptions of their characters :—

(1.) *Grand Pré, Ardennes.*—The gaize is a gray or grayish-yellow rock with bluish spots, soft to the feel, friable, and very porous. It appears to be principally of amor-

* Quart. Journ. Geol. Soc., vol. xxx., 1874, p. 373.

† 'Recherches sur le terrain crétacé de l'Angleterre,' p. 70.

‡ 'Zeitschrift der deutschen geologischen Gesellschaft,' vol. xxxi., 1879, p. 663.

§ *Id.*, p. 786.

‖ 'Annales de la Société Géologique du Nord,' tome v., 1877–78, p. 153. The author says, "Cette gaize est une roche tendre, légère, argileuse et silicieuse, très poreuse; elle se délite à l'air avec la plus grande facilité; sa couleur est grise; elle contient cinquante pour cent de silice gélatineuse."

phous silica in minute granules; there are also grains of quartz-sand, minute scales of mica, and some glauconite. The rock is interpenetrated throughout with the empty casts of spicules, which give to it its porosity and lightness. A few of the spicules themselves also remain, and these are of amorphous silica, like the matrix. The specimen is a typical example of a fossil sponge-bed. Its characters correspond closely with the upper greensand malm of Merstham, but the gaize contains no calcite, and the silica, though amorphous, is not redeposited in the globular form. How far the specimen sent me may represent the general characters of the gaize de l'Argonne, which is stated to be 100 mètres (328 feet) in thickness, I am unable to determine, but if it is a fair sample of this deposit, it forms an immense bed of sponge-remains, which far exceeds in thickness those present in the greensands of this country.

(2.) *Montblainville, Meuse.*—The specimens of gaize are soft, minutely porous, of a bluish-gray tint, with yellowish-gray bands, which appear to be the remains of entire sponges, now decomposed beyond recognition. The rock is largely of amorphous silica, entirely negative to polarised light, and it is partly present in the globular form. The globules or discs are of the same transparent hyaline character as those from Farnham, but the majority of them are smaller; they are from ·006 to ·02 mm. in diameter. In addition to the remains of entire sponges, the matrix is filled with the minute casts of detached spicules as well as some of the spicules themselves. The silica of these is amorphous, and appears to be assuming the globular form, like that of the matrix. A further peculiarity of these spicules is that the concentric layers of silica composing their walls can be detected under the microscope. The specimens are from a true sponge-bed.

(3.) *Bracquegnies.*—Meule (gaize) zone *à Ammonites inflatus.*—The specimens of this rock are extremely light, porous, and friable. The rock is rough to the feel, of a greenish-white tint, with numerous green specks in it; it has generally the aspect of pumice. Its structure resembles that of the gaize from Grand Pré. The silica is largely amorphous, occasionally in the globular form, but usually in minute irregular particles. It is throughout filled with spicules and empty spicular casts. According to* M. M. Briart and Cornet, this meule de Bracquegnies contains large quantities of soluble silica, as well as silica in the form of chalcedony, and judging from the specimen forwarded to me by Dr. Barrois, this silica has been derived from the solution of the siliceous sponge-spicules with which the rock was filled. The able authors above mentioned have very fully described the larger fossils in the meule, which appear to be on the same geological horizon as the Blackdown sponge-beds, but they do not seem to have noticed the sponge-remains to which the deposit owes its principal mineralogical characters.

(4.) *Rumigny, Ardennes.*—Gaize (albien).—Thisrock is hard, of a greenish-gray tint, from the coarse glauconite grains with which it is filled. The silica which

* 'L'Académie Royale des Sciences, &c., de Belgique,' tome xxxiv., 1867—70.

cements the particles is partly amorphous and partly chalcedonic. It contains spicules and fragments of lithistid and hexactinellid sponges, but not sufficiently numerous to be deemed a sponge-bed.

(5.) *Chaumont de Porcion, Ardennes.*—Marne de Givron.—This rock, which occurs at a slightly higher horizon than the gaize, is of a grayish tint and soft structure. It consists largely of calcite, with glauconite, and a small quantity of colloid silica in the globular form. There are fragments of entire sponges in it, as well as traces of detached spicules, but these do not bear sufficient proportion to the other materials to constitute it a sponge-bed. In another specimen of the same rock, from Givron itself, the sponges have been replaced by iron peroxide, like those of the lower and upper chalk of this country.

II. Mineral Conditions of the Sponge-Remains and of the Beds derived from them.

With the single exception of the sponges in the lower greensand strata at Faringdon, which are calcisponges, the spicular remains forming the sponge-beds above referred to, both from the lower and upper greensand, are exclusively those of siliceous sponges. These spicular remains resemble so closely those of existing siliceous sponges, that the silica of which they are formed may be assumed to have been originally in the same colloid, hydrated condition in which it is present in the skeleton of recent forms. But no fossil sponge is yet known in which the spicular skeleton retains that transparent hyaline condition which is so striking a feature of recent siliceous sponges; the silica, in all, has undergone various molecular alterations, and now presents numerous gradations between the amorphous or colloid and the crystalline state. Further, the changes in the fossil sponges have not been limited to alterations in the condition of the silica merely, but the silica itself has frequently been partially or entirely dissolved and replaced by calcite, glauconite, and other minerals, or removed, leaving the empty cast of the spicule. These modifications of the fossil sponge skeleton were first made known by Zittel and Sollas, and though at first regarded with doubt, on account of the supposed stability of silica, are now generally recognised. The investigations of *Mr. Hannay, and those of M. Thoulet,† have proved that the silica of recent sponges is readily susceptible to the influences of the chemical ingredients of sea-water, and has a strong tendency to dissolution after the death of the animal, so that the changes in the silica of the fossil spicules are only such as may have been anticipated.

To a great extent the accumulated masses of detached spicules in the sponge-beds present the same modifications arising from fossilisation as those of entire sponges, which have hitherto been mostly studied, but in the sponge-beds we have also to take

* 'Memoirs of the Literary and Philosophical Society of Manchester,' vol. vi., 3 s., p. 234.

† 'Bulletin de la Société minéralogique de France,' tome vii., 1884, p. 147.

into consideration the mineral characters of the deposits in which the sponge-remains are imbedded, which in many cases are derived from the spicular remains themselves.

The detached spicules of the sponge-beds are now in the following mineral conditions:—

I. Silica; either in the amorphous or colloid state; or crypto-crystalline like chalcedony; or crystalline.

II. Glauconite and other silicates.

III. Crystalline calcite.

The beds or matrix inclosing the spicules, in addition to such mechanically derived constituents as quartz-sand, mica, and grains of glauconite, also contain silica in the same forms as in the spicules, and calcite, either granular or crystalline. The calcite is probably of organic origin, and derived from the remains of foraminifera and other organisms.

IV. The spicules also occur in the negative form, in which the silica has been entirely removed, and the casts are either empty, or merely contain the solidified canal of the spicule.

As the mineral characters of the spicular remains, and of the beds in which they are inclosed are, to a great extent, interdependent, it is more convenient to consider them in connection with each other.

I. *Spicules and matrix of colloid silica.*—The spicules which yet retain the silica in the amorphous condition are, as a rule, more perfectly preserved than those in which the silica is partially crystalline; their surfaces are smooth and even, and their canals are distinctly shown. They are of a milky-white or opal tint by reflected light, and, when mounted in Canada balsam are nearly transparent. Between crossed nicols, the spicular walls are entirely neutral, though the canals, which are frequently infilled with chalcedonic silica, are doubly refracting. Viewed by transmitted light the spicular walls can be seen to be traversed in all directions by very minute curved lines (Plate 40, fig. 8), which, under high powers, are resolved into incomplete elliptical rings, with smooth even rims (Plate 40, fig. 9). Besides this tendency to circular aggregation, the silica of the spicules frequently exists as very minute granules of irregular forms, showing by transmitted light a decided pinkish tint. In no instance does the spicular canal exist as a hollow tube, but it has been filled either with silica, glauconite, or other silicate, or, rarely, with a ferrous compound. The infilling material is generally of a more resistant nature than the walls of the spicules, so that the walls are frequently entirely dissolved, whilst the materials which have infilled the canal remain intact and retain the form of the spicule, and might readily be mistaken for the entire spicule (Plate 45, figs. 14, 14*a*).

By treating the spicules of colloid silica with heated caustic potash the greater portion of their substance is dissolved, but their infilled canals remain, and not infrequently the form of the spicule is retained by a greenish-white mineral, which,

when examined in Canada balsam, appears to be filled with exceedingly minute granules in a transparent ground-mass. These residuary spicules give a very slight double refraction between crossed nicols. In some instances one portion of a residuary spicule is of glauconite, or at least has the green appearance of this mineral, whilst the other portion is of this transparent mineral; and there is a gradual transition from one to the other. As nitric acid has no more influence on this mineral than it has on the glauconite, it is probably some silicate allied to glauconite.

These residuary spicules of glauconite and other silicates, which remain after dissolving away the silica by caustic potash are also produced under natural conditions, and they are very common in the loose material of the cavities in the chert of the upper greensand at Warminster in association with unaltered colloid spicules, and in similar positions at Ventnor, where they have been nearly wholly reduced to this condition (Plate 45, figs. 15–15*e* and 16).

Fossil siliceous spicules, with the silica in an amorphous condition, are of rare occurrence in comparison with the numerous instances in which the silica is crypto-crystalline or even crystalline, and they have not been previously noticed in this country. They are, however, abundant in the sponge-beds of the northern margin of the Weald, also at Wallingford, Warminster, and Penzlewood. Both spicules and entire sponges of amorphous silica are also common in glauconitic marls of senonian age in Westphalia and Hanover, from whence they have been described by Professor Zittel.*

The beds in which the spicules of amorphous silica are inclosed, also frequently contain an important quantity of silica in the colloid state, amounting in the case of the malm at Farnham, recorded by Messrs. Way and Paine,† to as much as 75 per cent. In some instances, however, whilst the spicules are of colloid silica, the inclosing matrix is of chalcedonic and even partially of crystalline silica, and calcite is also present.

The presence in the same beds of silica in an amorphous condition, with the siliceous spicules in a similar state, taken in connection with the fact that the beds are filled with empty spicular casts from which the spicules have been dissolved, and that many of the spicules are residuary forms which have lost all their soluble silica, points to the conclusion that the colloidal silica in the beds has been directly derived from the breaking up and dissolution of the sponge-remains.

The colloidal silica of the sponge-beds occurs in the form of minute granules, similar to those in the spicules, and also in a globular form, that is to say, in very minute bodies with circular outlines, though not strictly of a spherical form. The siliceous bodies can be examined in thin microscopic sections from the harder portions of the malm of Godstone and other places; but their characters are still better shown in the

* "Ueber Cœloptychium." 'Abhandlung der königl. Akad. der Wiss. zu München.' XII. Band, iii. Ab. Also, "Studien über fossile Spongien." *Id.*, XIII. Bd., i. Ab.

† 'Journal of the Royal Agricultural Society,' vol. xiv., p. 243.

fine powder from the cavities in the chert of Warminster, and in the powder which may be obtained with a needle from fractured surfaces of the friable portions of the malm of Merstham or Farnham. When this powder is mounted in water or in Canada balsam, it is seen under the microscope largely to consist of the globular bodies, either single and quite free (Plate 45, figs. 18, 19, 19*d*), or in groups of two, or three, or several individuals united together (Plate 45, figs. 18*a*, *b*, 19*a*, *b*, *c*). The single globules present well-defined circular outlines; the individuals forming the groups are also circular, except where in contact they partially coalesce, and their margins become abruptly truncated. In some instances the globules are ranged in a linear series, in which the individual components are only indicated by slight lateral indentations. Frequently these bodies are clustered round the residuary spicules and attached to the solidified canals of spicules, whose walls have been dissolved away (Plate 45, fig. 16). In the malm of Farnham the globules in some instances are aggregated into larger spherical bodies.

These globules, when magnified about 500 diameters, present well-marked variations. In the commonest forms there is a marginal ring, about one-sixth the diameter of the globule, which seems to be faintly striate, whilst the central portion has a granular appearance (Plate 45, fig. 18). In another form the surface is covered with faint striæ which radiate from an indefinite central granule (Plate 45, fig. 19*d*). In another form, which is nearly entirely confined to the fine material from the Warminster chert, there is a central well-marked circular area, from one-third to one-fifth the diameter of the globule (Plate 45, figs. 19, *a*, *b*, *c*). This central area is usually furnished with an ill-defined nuclear spot, and its margins are lighter than the outer portion of the globule, which is radiately striated. The striæ or rays are, in all cases, of a very faint character, and distinguishable only in the best preserved examples. The very smallest and simplest globules consist of perfectly clear silica without striæ or granules.

It is somewhat difficult to ascertain with precision the true form of these minute bodies. Some are certainly plano-convex discs, others are biconvex, and the smaller bodies may approach a spherical form.

The globules are very variable in size; the smallest forms are only ·0014 mm. in diameter, and from this there are all gradations of size to bodies of ·045 mm. in width, or more than thirty times the diameter of the smaller. The majority of them vary between ·013 and ·02 mm. in width.

By transmitted light the globules are translucent, and present the same pinkish tint as the colloidal silica of the spicules; by reflected light they have also the same bluish-white or opal aspect as the spicules. They are negative between crossed nicols, when freed from particles of calcite, which are often present, in the same deposits. They also readily dissolve when treated with caustic potash.

In the soft, friable beds of malm, the siliceous globules, which frequently constitute a large proportion of its mass, are mostly free, or but lightly consolidated together by

pressure, but in the harder nodular portions they are inclosed by transparent chalcedonic silica, and seem to be gradually passing into the crypto-crystalline condition. They also appear at times to be invested with a surface layer of minute crystals of chalcedony.

In some cases the silica seems to have assumed the globular form within the sponge-spicules, but, as a general rule, the silica of the spicules must have been dissolved and then re-deposited as minute globules. I am not prepared to explain the causes which have produced this singular method of deposition; the chemical and physical forces which have brought it about have not been affected by extraordinary influences of heat or pressure, since the rocks are normal sedimentary deposits, and show no traces of alteration from these causes. There is no apparent reason why, under similar conditions of deposition and fossilisation, the sponge-remains of the siliceous and siliceo-calcareous deposits of the malm and firestone should have retained the original colloid condition of the silica, and the silica of the spicules when dissolved, should have assumed the globular form without passing from its hydrated condition, whilst in similar remains in most of the other greensand sponge-beds, the silica of the spicules has become altered to the more stable condition of chalcedony, and even to crystalline quartz, and, when the spicules have been dissolved, the silica has been deposited in layers and nodules of chalcedonic chert. The change of the originally colloid silica of the sponge-remains into the crypto-crystalline and crystalline condition is, indeed, that which nominally takes place in the course of fossilisation, and has been noticed in connexion with fossil sponges from silurian to upper cretaceous strata, and it is only under the particular circumstances, of which we are at present ignorant, that the colloidal state of the silica has been retained.

Sedimentary deposits consisting in part of colloidal silica in a globular form seem to be of rare occurrence; at all events, I cannot find any* previous notice of this character. But colloidal silica in a globular form has been recorded in rocks of volcanic origin. Thus Vogelsang† describes and figures, from a quartz-trachyte from near Schemnitz, and also from Borsva, spherulitic bodies, the so-called *Globuliten*, of isotropic silica, and of approximately similar dimensions to the greensand globules. Michel Lévy‡ has also discovered silica in a globular form and isotropic condition in eurites from Les Settons. It is worthy of notice, also, that there is a remarkable similarity, both of form and size, between the colloid siliceous globules of the greensand and the calcareous bodies artificially produced by the reaction of ammonia carbonate

* Professor Zittel mentions the fact that after treating with acid the now calcified, or partially calcified, siliceous sponges from jurassic strata the residue contains numerous rounded, rough, deeply indented siliceous discs. "Studien über fossile Spongien," I. Hexactinellidæ. 'Abhandl. der königl. bay. Akad. der Wiss.,' II. Cl., XIII. Bd., I. Ab., p. 14. I have not seen these bodies, but I should judge from the description of them that they are much larger, and that they further differ from those under consideration in being either crystalline or crypto-crystalline.

† 'Die Krystalliten,' pp. 139, 142, taf. xv., figs. 1, 2.

‡ 'Bulletin de la Société Géologique de France,' 3me ser., t. v., 1877, p. 140.

and calcium chloride, which have been figured by VOGELSANG.* Similar calcareous globules or discs of natural origin have been discovered by BARROIS† filling the cavities of shells and corals in the devonian limestones of Spain, which, though inorganic, he believes to be of the same nature as fossil and recent coccoliths.‡

There is at first sight a great similarity between the colloidal globules from the greensand and recent coccoliths from the Atlantic deep-sea mud, but the globules are round and not elliptical in figure, they do not occur in the sleeve-link form, and they coalesce together in a manner which has never been observed in coccoliths. There is therefore no ground for the supposition that they are silicified coccoliths. For the sake of comparison, I have represented on Plate 45, fig. 20, a coccolith of average size from the Atlantic ooze on the same scale as the globules.

The sponge-remains which are now of chalcedonic and crystalline silica have a very different aspect to those in which the silica is colloidal. By reflected light they have somewhat the appearance of ground glass, and when mounted in Canada balsam they become nearly invisible; in glycerine, however, their forms are very distinctly shown. The surface of the spicules in this condition is usually rough and eroded, so that it sometimes has a reticulated appearance; at other times it seems covered with minute circular spots of about ·006 mm. in diameter. These spots sometimes appear to be minute excavations in the surface of the spicule, in other cases they look like minute mammiform or botryoidal elevations. This surface character is very conspicuous in the spicules from Haldon and Blackdown, and has already been noticed by Mr. CARTER.§ The surface of spicules which have been changed to crystalline silica is less eroded than the exterior of those in which the silica is in the condition of chalcedony. These crystalline spicules frequently exhibit a finely radiate structure (Plate 45, fig. 17).

The canals of spicules in which the silica is either chalcedonic or crystalline are less regularly preserved than those of spicules of colloidal silica. In the majority of spicules no canal at all is visible, but this probably arises from the fact that the canal having been infilled with chalcedonic silica of the same character as that of the walls, cannot, either by ordinary or by polarised light, be distinguished therefrom. In other instances the canals are infilled with glauconite or with a light-brown or opaque mineral, not improbably iron peroxide. Not infrequently the solidified canal is discontinuous and broken up, and no longer occupies the normal position in the axis of the spicule, but is deflected to near its outer surface (Plate 42, figs. 3*b*, 6*a*). In many instances, even in these spicules of chalcedonic silica, the silica which has infilled the

* 'Die Krystalliten,' p. 87, taf. xi., fig. 1.

† 'Recherches sur les Terrains Anciens des Asturies et de la Galice,' p. 45, pl. xx., fig. 4.

‡ Dr. GÜMBEL also regards these minute, round, calcareous bodies, which he has discovered in limestones and marls of various geological epochs, as coccoliths, but their identity with the genuine coccoliths of the Atlantic, described by HUXLEY and by WALLICH, is open to doubt.

§ "Fossil Sponge-spicules of the Greensand compared with those of existing Species." Annals and Mag. Nat. Hist., s. 4, vol. vii., p. 114.

canal resists dissolution more than the spicular walls, and the solidified canal remains either wholly or partially free after the enclosing wall has been removed (Plate 45, fig. 13).

II. *Spicules replaced by glauconite and other silicates.*—Not only do these minerals infill the canals of the spicules, but they also replace the spicular walls as well. The replacement seems to have been gradually effected from within outwardly; the dissolution of the silica of the spicule taking place by the enlargement of the axial canal, and as this progresses the glauconite and allied minerals occupy the place of the silica until they constitute the entire spicule. These glauconitic spicules are very common in the malm and firestone of Merstham and Godstone, and also in the cavities of the chert at Ventnor. In this latter they are very strangely contracted and distorted, and assume figures which might have been produced by the gradual desiccation of a gelatinous body. The normally straight shafts and rays of the spicules are bent and curved in various directions, and even occasionally become spiral (Plate 45, figs. 15*a*, *b*, *c*, *d*).

III. *Spicules replaced by crystalline calcite.*—In every case in which this replacement occurs the matrix enclosing the spicules is of calcite, generally in a finely granular condition. The change seems to have taken place in the detached spicules in the same manner in which it has long been known to occur in entire fossil sponges. The original silica of the spicules has been dissolved subsequent to the enclosure of the spicules in the calcitic matrix, and the crystalline calcite has then filled up the moulds (Plate 40, fig. 4). In some cases the spicular canals have been infilled with glauconite before the calcitic replacement has been effected. The change to calcite is not complete in all the detached spicules in the same bed, since some occur in which the siliceous structure remains unaltered. Sponge-beds, with the spicules replaced by calcite, occur in the lower greensand at Maidstone, Tilburstow Hill, and Hythe, and in the upper greensand malm at Selborne.

IV. *Spicules represented by empty moulds.*—One of the commonest features of the sponge-beds is the empty moulds or casts of spicules with which the beds are filled, which have been produced by the complete dissolution of the siliceous spicules. In some cases the solidified canal has resisted the solvent influence which has removed the spicular wall, and now remains as a slender, smooth, delicate rod in the centre of the mould. These empty casts are present alike in the soft, friable, siliceous, and siliceo-calcareous sponge-beds of the malm and firestone, in the fine sedimentary deposits of the lower division of the upper greensand in the Isle of Wight, in the porous siliceous rock which generally accompanies chert, occasionally also in chert, and in a matrix entirely of calcite. It is to the dissolution and removal of the siliceous spicules that the sponge-beds owe their porous character and low specific gravity. The silica derived from the solution of the spicules appears usually to have been redeposited in the small interspaces between them, or to have accumulated to form nodular masses and layers of chert. Where the silica has been redeposited in

the colloid condition the sponge-beds are usually friable and incoherent, but where it has been redeposited as chalcedony, the beds are of a hard, resistant character, and serve, as already mentioned, for whetstones and other economical purposes.

Beds of chert.—The chert associated with the sponge-beds of the lower and upper greensand varies from a light pellucid to a dark brown tint; it is readily translucent in thin sections. It is highly brittle, and sufficiently hard to scratch glass. It shows no reaction in nitric acid, and may thus be assumed to be free from calcite. Thin microscopic sections exhibit a nearly transparent ground-mass of chalcedonic—sometimes partially also of crystalline—silica, in which are numerous sponge-spicules, and also occasionally entire and fragmentary foraminifera and fragments of echinoderm structure. In some cases also grains of quartz-sand and glauconite are present. The chalcedonic silica forming the ground-mass can be seen in many cases to have been deposited in concentric layers over the enclosed siliceous and calcareous organic remains; it also, when partially crystalline, exhibits a radiate fibrous structure (Plate 40, fig. 3).

Throughout the chert, sponge-spicules are present, but owing to the fact that the spicules are usually of chalcedony, like the ground-mass of the chert in which they are enclosed, their outlines are very indistinct. But even when the spicules themselves cannot for this reason be recognised, their former presence is indicated by the solidified canals, which being of glauconite, or of silica of a different tint to that of the matrix, are distinctly shown (Plate 40, fig. 3.). In some cases the spicules have been dissolved after the chert has formed round them, leaving empty moulds.

Intimately allied to the chert is the yellowish-gray porous rock which frequently forms an exterior crust to the chert, but sometimes also forms independent beds. The rock is principally of chalcedonic silica, with which very frequently calcite is intermingled. The main difference between this rock and chert consists in its porous nature, arising from the fact that the spicules in it are but as empty casts, and the silica of these spicules has probably been deposited in the adjoining bands of chert.

There can scarcely be room for doubting that the beds and irregular masses of chert, which are found nearly everywhere in the strata of the lower and upper greensand in connexion with the detached spicules of sponges, have been derived from the silica of these sponge-remains; and from the same source has also originated the silica which, in many of the deposits, more particularly in the Blackdown Hills, has replaced the shells and tests of the mollusca and other calcareous organisms. The theory has, however, been advocated that the silica of chert has been derived rather as a direct deposit of this mineral from solution in sea-water, than as the product of the decomposition of the siliceous structure of sponges. Thus Dr. Bowerbank* held that the sponges imbedded in the chert of the greensand possessed horny and not siliceous skeletons, and that the silica of the chert in which they were imbedded was attracted from the exterior medium by the animal matter, and not secreted therefrom by the

* Trans. Geol. Soc., ser. 2, vol. vi., p. 181.

living sponge. Professor T. Rupert Jones * maintains the view that the silica of chert generally is derived directly from sea-water, and similar opinions as to the origin of the chert bands in the upper carboniferous limestones of Ireland have been put forward by Messrs. Hull and Hardman † and by M. Renard ‡ with respect to the phthanites in rocks of the same age in Belgium.

It is a significant fact, however, in connexion with the chert-beds of the Irish upper carboniferous strata that some have been discovered filled with sponge-spicules like the chert of the English greensand, and this indicates a similar origin for the silica, and negatives the supposition of Professor Hull § that it was deposited "from warm shallow water charged with silica in solution, in which chemical reactions would be at once set up, favoured and promoted by tidal and other currents."

III.—The Nature of the Sponges present in the Sponge-Beds.

As the skeletons of entire sponges, or even fragments of them, are nearly wholly absent in the sponge-beds, a determination of the character of the sponges of which they are formed must be based upon a study of the detached microscopic spicules which are indiscriminately mingled in the deposits. In the majority of cases these spicules are combined with the mechanically-derived materials of the beds, and with the silica produced from their own dissolution, into hard rock-masses, in which the form and proportion of the individual spicules are but very indistinctly and imperfectly shown on fractured or weathered surfaces or in thin microscopic sections. Under these conditions the knowledge to be obtained of the character of the spicules is very limited. In a few cases, however, the microscopic spicules occur loosely mingled together in incoherent beds of sand, or filling pockets or cavities in beds of chert, and it is then possible to obtain them perfectly free from the matrix, and to examine their characters under the microscope, in precisely the same manner as those of recent siliceous sponges. In one respect, indeed, these detached spicules sometimes present greater advantages for studying their individual forms than those of recent sponges, since, through gentle natural influences, they have become separated

* "Quartz and other Forms of Silica." 'Proceedings of the Geologists Association,' vol. iv., p. 447.

† "On the Nature and Origin of the Beds of Chert in the upper Carboniferous Limestone of Ireland." By Professor E. Hull, and "On the chemical Composition of Chert and the Chemistry of the Process by which it is formed." By E. T. Hardman. 'Scientific Transactions of the Royal Dublin Society,' vol. i., new series, 1878, p. 71.

‡ "Recherches lithologiques sur les Phthanites du Calcaire carbonifère de Belgique." 'Bulletin de l'Académie Royale de Belgique,' 2me s., t. xlvi., p. 471. The author expressly states that there is nothing to show that the infiltrated silica of these deposits has been derived from the decomposition of sponge-spicules or of diatoms. There are shown, however, in one of the figures (fig. 2) accompanying the paper, circular sections which more nearly resemble those of sponge-spicules than of crinoid-stems, to which they are assigned.

§ *Op. cit.*, p. 83.

from each other in a more perfect manner than it is always possible to obtain those of recent sponges by artificial means.

Spicules thus loosely imbedded have been met with in the lower greensand strata of Haslemere, Sevenoaks, and Tilburstow Hill, and from beds of upper greensand age in the vicinity of Warminster, Haldon, Blackdown, and Merstham. Figures are given in the accompanying plates of all the forms discovered; some of these are of general distribution, whilst others are restricted to the deposits of a single locality. With the exception of the spicules from Haldon and Blackdown, first partly described by Mr. Parfitt,* and subsequently and more completely by Mr. Carter,† this is the first time that those from the lower greensand and from the other localities above mentioned in the upper greensand strata have been figured in detail.

Mention has already been made of the fact that the spicules of the sponge-beds referred to in this paper (the Faringdon sponge-gravel not being included) are exclusively those of sponges with siliceous skeletons. Representatives of each of the four orders of siliceous sponges occur, but in varying proportions. Monactinellid spicules of acerate, acuate, and cylindrical forms are very abundant, but, judging from their relative dimensions, the majority of these spicules belong to the tetractinellidæ, where they are associated with trifid spicules, rather than to those monactinellids whose skeletons are exclusively composed of uniaxial spicules. It is quite possible, however, that monactinellid sponges may have been also numerous in these sponge-beds, though we have but few spicules which can be definitely recognised as belonging to this order, for the spicules of most of the recent monactinellids are so minute and delicate that they would most likely be destroyed in fossilisation. Tetractinellid sponges are strongly represented, and the majority of the spicules in the sponge-beds are included in this order. There is a very great variety of the "zone" spicules of these sponges, each of which probably indicates a distinct species. The so-called "anchor" spicules, the globates and stello-globates, which form a dermal layer in some of these sponges, are also present. The lithistid sponges are principally represented by spicules belonging to the megamorina family. These are very numerous and very generally distributed. The skeletal spicules of the other families of lithistids are less common, and their generally minute dimensions may partly account for this fact. There is, however, a great number of the dermal spicules of lithistid sponges, which most probably belong to the tetracladina family. These dermal spicules are but seldom preserved on the surface of the entire lithistid sponges which have been discovered in the greensand, but they are present, quite detached from each other, in the sponge-beds, in a great variety of forms. The remains of hexactinellid sponges are comparatively rare, and they are limited to small fragments of the mesh and a few detached spicules probably belonging to the surface layer of the sponge.

* 'Transactions of the Devonshire Association for the Advancement of Science,' 1870.

† Ann. Mag. Nat. Hist., 1871, ser. 4, vol. vii., p. 113, pl. 7–10.

In recent tetractinellid sponges, the spicular elements of the skeleton are merely held in position by the sarcode, and the decomposition of this substance after the death of the animal would at once set the individual spicules free from each other. This structural character readily explains the complete manner in which these sponges have been broken up into their component spicules, for, though these are so abundant in the sponge-beds, I have not met with a single fragment of the skeleton with the spicules in their proper relative positions. Only under exceptionally favourable conditions of preservation could the entire body of the sponge be preserved in the fossil state. A few specimens have, however, been met with in the upper chalk of Yorkshire and Westphalia, but none, that I am aware of, in the greensand.

The skeletal spicules of the lithistid sponges are more intimately intertwined and interlocked together than is the case with tetractinellid sponges, and it is therefore a matter for surprise that in these sponge-beds no fragments of the sponges themselves should be found. The spicules are thoroughly detached from each other, and I have not met with a single instance of even two spicules naturally united together. Entire lithistid sponges are, however, occasionally very abundant, in the upper greensand more particularly; but they appear in only a few limited localities, and their preservation seems to be owing to the infilling of the skeleton by chalcedonic silica before it was exposed to those influences which in the sponge-beds destroyed the union of the spicules with each other.

It is a difficult matter to determine the extent to which the varied forms of these detached spicules may represent distinct species. There can be no doubt that numerous species are present in these sponge-beds; but many of the characters, which in recent sponges determine the species and genus, are based on the arrangement of the spicules in the skeleton, the combination of various forms of spicules in the dermal layer, the characters of the canal system, and many other features, which, of course, are entirely unrecognisable in these detached spicules. The only classification which can be attempted in the circumstances must rest on the features of the isolated spicules, and where the form and proportions of these are sufficiently distinct from those of known recent and fossil sponges, they may reasonably be regarded as belonging to new species. These spicules, it is true, furnish us with only one character of the species, that is, the form of the skeletal elements; but this, after all, is the most important feature. If the entire sponges should at any future time be discovered, of which, judging from present experience, there is but little probability, the provisional characters based on the spicules can then be supplemented by those derived from the entire skeleton.

It is not my purpose in the present paper to discuss in detail the characters and relation of the spicular contents of these greensand sponge-beds, beyond what may be necessary to indicate the groups to which they may belong, and to furnish grounds for comparison with those of similar deposits elsewhere. In those cases in which the spicules seem to me to indicate new species I have assigned a provisional name to

them, following in this respect the example of Mr. CARTER and Prof. SOLLAS. Though these names may subsequently be displaced by the discovery of complete sponges, yet they are almost a necessity for purposes of reference.

The figures in the accompanying plates have all been drawn under the microscope by means of BECK's camera lucida, and their outlines may therefore be regarded as correct. It has not been practicable to figure all the forms on the same scale; the majority of them are enlarged either 40 or 60 diameters, a few are shown on the scale of 80, and in one instance the spicule has been enlarged 560 times. To avoid repetition, the titles of works in which similar detached spicules have been previously described, are given below under the names of the respective authors.*

Order Monactinellidæ ZITTEL.

1. *Reniera gracilis* (Plate 41, figs. 1–1*b*).—Cylindrical spicules, straight or slightly curved, smooth, evenly rounded at the ends. Average maximum length ·4 mm., width ·05 mm.

L. G. S.,† Haslemere, Surrey. U. G. S.,‡ Blackdown, Devonshire.

2. *Reniera obtusa* (Plate 41, figs. 2, 2*a*).—Smooth, slightly curved, cylindrical spicules, sometimes gradually tapering at one end. Length ·33 mm.; width ·058 mm.

U. G. S., Blackdown.

3. *Reniera cucumis* (Plate 41, figs. 3, 3*b*).—Slightly curved, smooth, either evenly cylindrical throughout, or slightly fusiform. Length ·466 mm., width ·066 mm.

U. G. S., near Warminster, Wilts; Haldon, Devonshire; Merstham, Surrey.

The cylindrical spicules above mentioned, though of similar form, are considerably

* CARTER, H. J.—"On Fossil Spicules of the Greensand compared with those of existing Species." Ann. Mag. Nat. Hist., ser. 4, vol. vii., pp. 112–141. 1871.

DUNIKOWSKI, E. V.—"Die Spongien, &c., der unterliassischen Schichten von Schafberg bei Salzburg." 'Denkschriften der kaiserlichen Akademie der Wissenschaften,' Wien. Bd. xlv., pp. 163–194. 1882.

HINDE, G. J.—I. 'Fossil Sponge-Spicules from the Upper Chalk.' 1880.

HINDE, G. J.—II. 'Catalogue of the Fossil Sponges in the British Museum.' 1883.

PARFITT, E.—"Fossil Sponge-Spicules in the Greensand of Haldon and Blackdown." 'Transactions of the Devonshire Association for the Advancement of Science.' 1870.

POČTA, P.—I. "Ueber isolirte Kieselspongiennadeln aus der böhm. Kreide-Formation." 'Sitzungsberichte der k. böhm. Gesellschaft der Wiss.' Prag, 1883.

POČTA, P.—II. "Ueber Spongiennadeln des Brüsauer Hornsteines." *Id.* 1884.

RUTOT, A.—"Note sur la Découverte de deux Spongiaires de l'Étage bruxellien." 'Annales de la Société Malacologique de Belgique,' tom. ix., 1874.

SOLLAS, W. J.—"On the Flint-nodules of the Trimmingham Chalk." Ann. Mag. Nat. Hist., ser. 5, vol. vi., p. 384. 1880.

WRIGHT, J.—"A List of the Cretaceous Microzoa of the North of Ireland." Belfast Nat. Field Club. 1875.

ZITTEL, R. A.—"Ueber Cœlophtychium." 'Abhandl. der könig. bay. Akad. der Wiss.,' II. Cl., XII. Bd. 1876.

† Lower greensand. ‡ Upper greensand.

larger than those of any recent species of *Reniera*. The form is very common in fossil sponge-beds, and cylindrical spicules of different dimensions have been described from the carboniferous (Carter); Lias (Dunikowski); upper chalk of Norfolk (Hinde, Sollas); Westphalia (Zittel); Bohemia (Počta).

4. *Reniera Zitteli*, Počta, (Plate 41, figs. 4–4*e*; *op. cit.*, p. 6, taf. I., figs. 10–14). Acerate spicules, smooth, fusiform, acutely pointed. Length ·3 to ·45 mm., width ·02 to ·05 mm.

L. G. S., Haslemere. U. G. S., Blackdown.

5. *Reniera truncata* (Plate 41, figs. 5–5*f*).—Slender acerate fusiform spicules, very gradually tapering to the extremities, which are frequently truncate. Canals prominent, enlarging at the spicular ends. Length ·33 to 1 mm., by ·04 to ·058 mm. in width.

With these also may be associated small fusiform spicules in which the canals are open throughout their entire length. These forms are regarded by Zittel as immature spicules (Plate 41, figs. 5*g*, 5*h*).

U. G. S., Warminster, Merstham.

6. *Axinella gracilis* (Plate 41, figs. 7, 7*a*).—Smooth, slightly curved acuate spicules, with rounded but not inflated summits; tapering from one-third to one-half their length to an acutely pointed extremity. Length from ·55 to ·66 mm., width ·033 to ·05 mm.

L. G. S., Haslemere. U. G. S., Blackdown, Haldon, Warminster, Merstham.

7. *Axinella dispersa* (Plate 41, figs. 6–6*e*).—Smooth, straight, or curved acuate spicules; summits rounded but not inflated. Some examples taper gradually from the summit, whilst others are of an uniform width to the centre or even below, and then commence to taper. Ends acute or slightly truncate. Length from ·51 to 1 mm., by ·05 to ·075 mm. in width.

L. G. S., Haslemere. U. G. S., Warminster, Blackdown, Merstham.

8. *Axinella stylus* (Plate 41, figs. 8–8*d*).—Relatively large, smooth, straight or slightly curved acuate spicules. Summits truncate, rounded, and occasionally with a slight inflation. Rarely the spicules taper from the summit, but more generally they are of the same width to near the extremity, when they rapidly taper to an acute point. Length from 1·5 to 3·1 mm., width from ·085 to 1 mm.

L. G. S., Haslemere. U. G. S., Blackdown, Haldon, Ventnor.

9. *Spirastrella neocomiensis* (Plate 41, figs. 9, 9*a*).—Smooth, fusiform, straight or slightly curved spinulate spicules. Length 95 mm., width ·05 to ·075 mm.

L. G. S., Haslemere.

10. *Monilites Haldonensis*, Carter (Plate 41, figs. 11–11*b*; *op. cit.*, p. 132, Plate 9, figs. 46, 47).—Cylindrical, slightly curved, moniliform spicules, with from five to eight rings. In some examples the inflations become depressed, giving an altogether different aspect to the spicule (fig. 11*b*). Length ·3 to ·53 mm., by ·09 mm. in thickness.

L. G. S., Haslemere. U. G. S., Haldon, Blackdown, Warminster.

11. *Esperites Haldonensis*, Carter (Plate 41, fig. 12; *op. cit.*, p. 131, Pl. 9, fig. 43).—

Slender bihamate spicules with short obliquely incurved extremities. Length ·533 by ·041 mm. in width. It may be doubted whether these forms are really analogous to the bihamate spicules of the existing genus *Esperia*, since, as Mr. CARTER has already remarked, they are five times the size of the recent spicules.

U. G. S., Blackdown, Merstham.

12. *Dirrhopalum neocomiensis* (Plate 41, fig. 14).—Robust, smooth, straight, conical spicules, with an inflated, evenly-rounded summit, a slight constriction immediately beneath; from this, a gradual tapering to a sharply-pointed extremity. Length 1·22 mm., greatest width ·275 mm. The affinities of this form are very doubtful; it may have some relationship to the smooth conical spicules in the upper chalk, which I have named *Dirrhopalum planum*, and I therefore include it provisionally in the genus.

L. G. S., Haslemere.

13. *Hamate spicule* (Plate 41, fig. 18).—A smooth, cylindrical, hamate spicule, apparently perfect. Length ·366, by ·06 mm. in width. I do not know of any fossil or recent spicule at all similar to this.

L. G. S., Haslemere.

14. *Anomalous acerate spicule* (Plate 41, fig. 12).—Minute, smooth, acerate spicule, evenly cylindrical for the greater portion of its length, then tapering. One end is acute, the other obtuse. This exterior acerate form appears to inclose a spinulate form with a pear-shaped head and straight shaft, and within this is a central axial rod which seems to be the infilled canal. Length ·12 mm., width ·018 mm. The affinities of this peculiar spicule are doubtful. It is the smallest form which I have discovered in the sponge-beds, and only one example of it has been met with.

U. G. S., near Warminster.

Order Tetractinellidæ MARSHALL.

The distinguishing characters of the recent genera, *Geodia*, *Stelletta*, *Erylus*, and other allied forms, are based on the structural features of the dermal layer. In the greensand sponge-beds there are numerous examples of zone and anchor spicules, together with body acerates and the globates and globo-stellates of the dermal layer, but in their detached condition it is impossible now to determine the particular zone-spicules which may have been associated with the dermal spicules, and thus there can be no certainty in assigning the various forms of the zone-spicules to one or other of these genera, and it may therefore be desirable to adopt CARTER'S general term *Geodites*. In recent sponges of this group, the species are characterised by a distinctive zone-spicule, and in like manner the different forms of these fossil zone-spicules may be regarded as indicating distinct species. The differences in the acerate spicules of distinct species are not recognisable in the fossil detached spicules, and they will have to be considered in a single group.

15. *Body-acerates of Geodites* (Plate 41, figs. 10–10*o*).—Straight, curved, or geniculate fusiform spicules, either acutely pointed or obtuse. The canals are only preserved as a rule in the spicules from Warminster and Merstham, in which the colloidal silica remains unaltered. The spicules vary from 1·1 to 3·12 mm. in length, and from ·04 to ·125 mm. in width. These spicules are very generally distributed in all the sponge-beds alike of the lower and upper greensand. The larger forms occur in the lower greensand of Haslemere, Sevenoaks, and Folkestone, and in the upper greensand of Blackdown and Haldon; whilst those from Warminster and the malm at Merstham are distinctly smaller forms.

16. *Pachymatisma virga* (Plate 41, figs. 15, 15*a*, 16).—Straight, slender, cylindrical spicules, terminating obtusely, occasionally also slightly inflated. Length 1·6 to 2 mm., by ·025 to ·041 mm. in width. With these there also occurs an inequally biclavate spicule (fig. 16), 1·16 mm. in length, by ·066 mm. in width. Dr. Bowerbank has figured spicules of similar form, but much slenderer proportions, in *Pachymatisma Johnstonia* ('Brit. Sponges,' vol. i., Plate 1, figs. 12, 20; vol. iii., Plate 8, figs. 2, 3), and I place these provisionally in the same genus.

U. G. S., Blackdown, Haldon.

17. *Geodites Carteri* (Plate 41, figs. 17–17*k*).—Zone spicules, with straight or slightly curved shafts, gradually tapering from the head, terminating obtusely. The rays straight or slightly curved, simple or furcate, acutely pointed, projecting forwards. In some forms one of the rays is suppressed and the spicule resembles a two-pronged fork (17*b*, *k*). The rays are frequently inequal in length, and bifurcate irregularly. Total length 1·4 to 1·75 mm., by ·1 to ·15 mm. in width; the rays ·375 to ·45 mm. in length, by ·1 mm. wide. Abundant.

L. G. S., Haslemere, Tilburstow Hill, Sevenoaks (Folkestone division). U. G. S., Blackdown, Haldon.

18. *Geodites prælongus* (Plate 42, figs. 1–1c).—Zone spicules, with elongated, nearly straight shafts, of nearly the same width throughout, or very gradually tapering. The head-rays are relatively short, simple, or bifurcate, usually slightly curved. They project obliquely forwards. Total length from 1·9 to 3·42 mm.; width, from ·113 to ·15 mm. The rays are from ·175 to ·425 mm. in length, by ·075 mm. in width.

L. G. S., Haslemere, Tilburstow Hill, Folkestone, Sevenoaks. U. G. S., Blackdown.

19. *Geodites robustus* (Plate 42, figs. 42, 2*a*).—Zone spicules robust, straight or slightly curved, tapering gradually, or of the same thickness for the greater part of their length. Rays also stout, simple, or bifurcate, projecting obliquely. Length 2·8 to 3·3 mm., width, ·17 to ·3 mm. The rays are from ·25 to ·45 mm. in length, by ·11 to ·15 mm. in thickness. Differs from the preceding form in being more robust throughout, and in the greater development of the rays.

L. G. S., Haslemere, Tilburstow Hill, Sevenoaks, Folkestone.

20. *Geodites audax* (Plate 42, figs. 3–3c).—Zone spicules robust, the shaft usually with a slight constriction just beneath the head, then gradually tapering to an acute

point, the head-rays all bifurcate, and in perfect specimens acutely pointed. Length 1·1 to 1·6 mm., width ·13 to ·2 mm.; rays ·33 mm. long, by ·116 mm. in thickness. Fig. 3*c* is probably the head of one of these spicules seen from beneath. Fig. 3*b* shows the peculiar manner in which the infilled canal has been distorted and broken up.

L. G. S., Haslemere.

21. *Geodites obtusus* (Plate 42, figs. 4, 4*a*).—Zone spicules robust, with straight shafts, usually slightly constricted near the head; below this they are tumid and then gradually taper. The rays are short, unusually thick, bifurcating near the summits, and forming short obtuse cones. Total length unknown; width ·35 mm.; rays ·35 mm. in length, by ·175 mm. in width.

L. G. S., Sevenoaks (Folkestone Division).

22. *Geodites politus* (Plate 42, figs. 5–5*c*).—Zone spicules, with straight, elongate, very gradually tapering shafts, and head-rays usually bifurcate, the terminations evenly rounded. The shafts incomplete in length, from ·17 to ·22 mm. in thickness. The rays ·25 by ·125 mm. Fig. 5*b* is probably the head of one of these spicules seen from below.

In the same beds an "anchor" spicule occurs (fig. 5*c*), with rounded head and three short, simple, recurved rays, which probably belongs to the same species as the zone spicules.

L. G. S., Folkestone, Tilburstow Hill. U. G. S., Merstham.

23. *Geodites pusillus* (Plate 42, figs. 6-6*b*).—Zone spicules, with straight or slightly curved shafts, usually constricted just below the rays; below this slightly tumid, and then gradually tapering. The rays are relatively short, simple, conical, and acutely pointed. Length 1·25 to 2 mm.; width ·125 to ·2 mm. Rays ·15 by ·075 mm.

L. G. S., Haslemere, Tilburstow Hill.

24. *Geodites Haldonensis*, Carter (Plate 42, figs. 7–7*b*, 16, 16*a*; *op. cit.*, p. 129, Plate 10, figs. 58–67).—Zone spicules, with straight or curved, elongated, tapering shafts and straight or curved head-rays, either simple or bifurcate. Length 1·5 to 2·5 mm.; width ·075 to 11 mm. The rays ·175 by ·075 mm. Mr. Carter has called attention to the different aspect of some of these spicules with curved shafts and simple rays, (figs. 7*b*, *e*), and suggests that they might belong to a different species. There are, however, intermediate forms between these and the spicules with bifurcate rays which indicate that they may all pertain to a single species. The "anchor" spicule (fig. 16) from the same beds was probably associated with these zone spicules. In comparing Mr. Carter's figures with those which I have given, allowance must be made for the different scale on which they are drawn, Mr. Carter's being on the scale of $\frac{1}{1}\frac{5}{5}$, whilst mine are only $\frac{4}{1}\frac{0}{0}$.

L. G. S., Sevenoaks. U. G. S., Haldon, Blackdown.

25. *Geodites divergens* (Plate 42, figs. 8, 8*a*, 11-11*b*).—Zone spicules, with short, stout shafts, straight or curved, gradually tapering, and terminating obtusely. The canals are prominent, and usually dilated at the extremity of the shaft. The rays are

strongly bifurcate, and the subdivisions usually diverge widely from each other. Total length from 1·07 to 1·77 mm.; width ·112 to ·16 mm. Rays ·17 to ·5 mm. in length by ·1 mm. in thickness.

L. G. S., Haslemere. U. G. S., near Warminster, Merstham.

26. *Geodites deflexus* (Plate 42, fig. 10).—Zone spicule, with straight, very gradually tapering shaft; the rays simple, extending for two-thirds of their length at approximately a right angle to the shaft, and then abruptly recurved. Total length unknown, the shaft is ·1 mm. in width. Rays ·375 mm. long by ·087 mm. wide.

U. G. S., Blackdown.

27. *Geodites Wrightii* (Plate 42, fig. 9; Plate 41, fig. 13).—Zone spicules with straight tapering shaft and stout rays, projecting obliquely forwards. The rays are incipiently bifurcate at their extremities. The rays and the greater portion of the shaft are moniliform. Length 1·375 by ·2 mm. in width. Rays ·3 by ·125 mm. Fragmentary specimens indicate much larger forms than the example figured. In the upper chalk examples of this form the rays are uniformly simple and more slender than in the greensand specimens.

Acuate moniliform spicules (Plate 41, fig. 13), rarely occur in the same beds with the trifid forms, and, though much larger in size, they may have belonged to the same species with the trifid spicules.

L. G. S., Haslemere, Godalming. U. G. S., Warminster.

28. *Geodites planus* (Plate 42, figs. 13–13*c*).—Zone spicules with slender, elongate, gradually tapering shafts, and simple or bifurcate head-rays, which extend nearly at right angles to the shaft. In some examples only two of the three head-rays are developed. Length ·875 to 1·72 mm.; width ·062 to ·125 mm. Rays ·175 to ·3 mm., by ·05 to ·09 mm. Very abundant.

L. G. S., Haslemere. U. G. S., Blackdown, Haldon, Warminster, Merstham.

29. *Geodites gracilis* (Plate 42, figs. 12–12*e*).—Slender spicules, with straight tapering shafts, and short simple or bifurcate head-rays, which extend obliquely forwards. Length 1 to 1·7 mm. by ·07 to ·1 mm. Rays ·225 by ·075 mm.

U. G. S., Blackdown, Haldon, Warminster, Penzlewood, Merstham.

30. *Globate and globostellate spicules of the dermal layer of Geodites* (Plate 43, figs. 2–2*f*).—The spicules forming the dermal crust of this group of sponges are abundantly present. There are two varieties of globates; one is nearly spherical in form (fig. 2*c*), the surface smooth and apparently destitute of hilum. The average diameter is ·233 mm. This form only occurs in the lower greensand at Haslemere. The other variety is elliptical or reniform (figs. 2*a*, 2*b*,), surface smooth, with a well-marked depression or hilum. The average size is ·137 by ·09 mm. It occurs in lower greensand at Haslemere, Sevenoaks, Tilburstow Hill, and in upper greensand at Warminster, Blackdown, Merstham.

One variety of globostellate is nearly spherical, ·137 mm. in diameter, its surface is thickly covered with minute blunted elevations (fig. 2*d*). It occurs in the upper

greensand at Warminster and Merstham, and it has been figured from the upper chalk of Westphalia (Zittel).

Another globostellate (figs. 2*e*, *f*) has a spherical body with stout, conical spines projecting from its surface. Entire diameter ·2 mm.; length of spines ·075 mm. Lower greensand, Haslemere.

31. *Stellettites?* sp. (Plate 42, figs. 14–14*e*).—Zone spicules, in which the shafts are greatly reduced, whilst the head-rays are, as a rule, widely extended in a nearly horizontal direction and symmetrically bifurcate. The great variety of size of these spicules indicates that they may belong to more than a single species. It is possible that some may belong to the dermal layer of lithistid sponges. It is a very common form, and occurs also in the lower lias of Schafburg (Dunikowski), in the upper chalk of Norfolk and Westphalia (Zittel), and in the eocene of Brussels (Rutot).

L. G. S., Folkestone. U. G. S., Blackdown, near Warminster, Penzlewood, Merstham.

32. *Stellettites callodiscus*, Carter sp. (Plate 43, fig. 3; *op. cit.*, p. 123; Plate 9, fig. 40).—Delicate flattened discs, circular or slightly elliptical in outline, with a well-defined smooth margin. Radially disposed at equal intervals near the margins are from twelve to fourteen flask-shaped apertures; between each of these there are two and occasionally three smaller sub-oval apertures. A series of canals radiating from the common centre of the disc extends to near its periphery. One of these canals is present between each of the larger flask-shaped apertures. A fairly large spicule is ·313 by ·263 mm. In spicules which are in process of breaking up, the smooth margin disappears, the flask-shaped apertures open on the outer border, and the canals are greatly widened, so that the spicule has an altogether different appearance.

U. G. S., Blackdown, Warminster. This form is also present in the upper chalk of Norfolk (Hinde) and Westphalia (Zittel).

33. *Tethyopsis Haldonensis*, Carter, pars. (Plate 44, figs. 15, 15*a*; Plate 43, figs 1–1*d*; *op. cit.*, Plate 10, fig. 70, non fig. 69).—Trifid spicules, with straight, elongate shafts and rays, usually inequal in length, and simple, which extend at a wide angle, and generally exhibit a slight recurvature. Frequently only two rays are developed. Length of shaft (incomplete) 1·9 mm.; thickness ·075 to ·13 mm. Rays ·3 to ·62 mm. in length, by ·075 to ·125 mm. in thickness.

L. G. S., Haslemere, Folkestone. U. G. S., Blackdown, Haldon.

34. *Small trifid spicules of different species* (Plate 42, figs. 17–17*k*).—The affinities of these small spicules are doubtful; figs. 17, 17*a*, *b* may be either young spicules of *Geodites*, or the dermal spicules of a lithistid sponge; the clavate spicules, 17*c*, *d*, *e*, approximate to the four-rayed spicules of *Pachastrella*; the obtuse spicules, 17*g*, *h*, *i*, are now in the condition of glauconite, and, though apparently perfect, may vary considerably from their original forms in silica; and the small spicule with much branched rays (17*k*) is very probably the dermal spicule of a lithistid sponge.

U. G. S., Blackdown, near Warminster, Merstham, Ventnor.

35. *Pachastrella* (*Dercitus*) *Haldonensis*, Carter (Plate 43, figs. 4–4 *i*; *op. cit.*, p. 130;

Plate 10, fig. 71).—Four-rayed spicules; the rays straight, simple, sub-equal, tapering from the central junction to an acute point, rarely rounded at the ends (fig. 4*d*). Very variable in size; the rays are from ·35 to ·825 mm. in length, and from ·075 to ·125 mm. in width.

With the above are sometimes associated smaller spicules (figs. 5, 5*a*, *b*) with relatively shorter and plumper rays, which may probably belong to a distinct species. They are similar to those from the upper chalk which I have named *P. Carteri* (*op. cit.*, i., p. 46; Plate 3, figs. 29, 31). The rays are ·225 to ·35 mm. in length, by ·08 mm. in width.

These spicules are very generally distributed throughout the sponge-beds of the lower and upper greensand. Similar forms are present in the carboniferous (Hinde), lias (Dunikowski), chalk of England (Sollas, Hinde), Ireland (Wright), Westphalia (Zittel), and in the eocene of Brussels (Rutot).

36. *Pachastrella* (*Monilites*) *quadriradiata*, Carter (Plate 43, figs. 6, 6*a*,; *op. cit.*, p. 132, Plate 9, figs. 44, 45).—Spicules of the same general form as the preceding, but, as a rule, much smaller, and the rays are throughout moniliform. Length of rays ·25 mm., thickness ·05 mm.

L. G. S., Haslemere. U. G. S., Blackdown, Haldon, Warminster.

Order: Lithistidæ, O. Schmidt.

Family: Megamorina, Zittel.

37. *Doryderma*, sp. (Plate 43, figs. 7–7*z*).—Detached spicules of sponges of the Megamorina family, of which the genus *Doryderma* may be taken as the type, are extremely abundant. They are of the most irregular and varied forms, generally elongate branching spicules with arms of unequal length, which terminate either in an obtuse point or in a slightly concave expansion. The branches in some of the larger spicules (figs. 7*q*, *r*, *s*, *u*) all terminate obtusely, and these appear to belong to the stem of the sponge, whilst the spicules whose rays terminate in an expansion belong to the body portion of the sponge. The larger spicules measure 2·5 mm. by ·175 mm., whilst a small form is only ·4 mm. by ·062 mm. These spicules probably belong to several different species.

L. G. S., Haslemere, Godalming, Tilburstow Hill, Sevenoaks. U. G. S., Blackdown, Haldon, Warminster, Penzlewood, Merstham, Ventnor.

38. *Carterella*, sp. (Plate 45, figs, 1–1*c*).—Elongate, curved spicules with blunted extremities and irregularly notched surfaces. Length ·95 to 1·4 mm., width 0·75 mm.

U. G. S., Warminster.

Family Anomocladina, Zittel.

39. *Mastosia neocomiensis*, Hinde (Plate 45, figs. 2–2*k*; *op. cit.*, ii., p. 57; Plate 10, fig. 4).—Spicules with spherical or sub-spherical centres, from which short arms or rays, from three to five in number, extend in various directions. Rays usually simple, though occasionally bifurcate near their ends. They terminate in slightly concave expansions. The nodes are about ·22 mm. in thickness, and the rays from ·12 to ·27 mm. in length, by ·62 mm. in thickness.

L. G. S, Haslemere, Tilburstow Hill, Petworth. U. G. S., Haldon, Warminster.

Family Rhizomorina, Zittel.

40. *Chenendopora*, sp. (Plate 45, figs. 3, 4–4*d*).—Branching spicules of irregular form, their surfaces covered with blunted spines or tubercles. Length ·53 mm. by ·085 to ·15 mm. in width.

L. G. S., Haslemere, Tilburstow Hill. U. G. S., Blackdown, Warminster, Penzlewood.

Family Tetracladina, Zittel.

41. *Ragadinia*, sp. (Plate 45, figs. 5, 5*a*, *b*).—Small, four-rayed spicules of the skeleton; one of the rays is reduced to a prominent tubercle, whilst the others are irregularly annulated.

L. G. S., Haslemere, Tilburstow Hill. U. G. S., Warminster.

42. *Siphonia*, sp. (Plate 45, figs. 6, 6*a*–6*f*).—Four-rayed spicules of the skeleton; the rays usually smooth, sub-equal, and branching at their extremities.

L. G. S., Tilburstow Hill, Sevenoaks. U. G. S., Blackdown, Haldon, Warminster, Penzlewood, Merstham.

Spicules of the dermal layer of Tetracladine Lithistids (Plate 44, figs. 1–15).

These spicules occur in an almost unlimited variety of form, but as suggested by Oscar Schmidt,* they all appear to originate from modifications of the summit-rays of the trifid spicule. The simplest form of dermal spicule in these sponge-beds consists of three horizontally extended simple rays with irregular lateral lobate extensions, and a minute ray or shaft at right angles to the others (figs. 4, 4*a*). This simple type is, however, of rare occurrence; the large majority of the dermal spicules are derived from forms in which the three primary rays are symmetrically bifurcate, and each of the subdivisions is variously lobed and branched in a horizontal plane. Thus in one of the less modified examples (figs. 1, 1*a*) the rays of the typical spicule are merely expanded laterally and sinuated; in another the lateral expansion is increased, and

* 'Die Spongien des Meerbusen von Mexico,' 1ster Heft, 1879, p. 22.

the margins are smooth (fig. 2), whilst in others the margins are sinuous and notched) (figs. 3, 3*a*, *b*). In the more complex forms the rays are divided and branched in an extraordinary variety of ways; in some cases the subdivisions are narrow (figs. 7, 7*a*), whilst in others they are mere lobate extensions (figs. 12, 12*b*). The greatest modification occurs in the spicules, which are simple horizontal discs with lobate (fig. 13*b*), jagged (13*c*), or smooth margins (14, 14*a*). In these all traces of the typical trifid spicule have disappeared in the external form, but the minute three-branched canal present in some of these discoid spicules (fig. 14) indicates very clearly that they have originated from the same fundamental type as the branched dermal spicules. As a general rule the canals in these dermal spicules extend only a short distance from their centres (figs. 10*a*, 11), and do not reach the extremities of the subdivided rays. In but slightly modified examples, however, the canals normally extend to the extremities of the rays (figs. 2, 3, 3*a*).

These dermal spicules vary in diameter from ·316 to ·816 mm., and the rays are from ·024 to ·1 mm. in width. They are as a rule much larger than the average dermal spicules of recent lithistids, as may be seen from those of *Discodermia sinuosa*, Carter (figs. 15, 15*a*), which have been figured on the same scale as the fossil forms for comparison.

These dermal spicules belong in all probability to many distinct species of lithistid sponges, but they are so rarely found in their natural position on the surface of these sponges in the fossil state, that very little is known at present of the particular forms belonging to each species. The only lithistid sponge from the English greensand in which the dermal spicules have been found in position, is *Hallirhoa agariciformis*, Bennett, sp.,* and they are of the lobate character shown in figs. 12, 12*b*, 12*d*. As numerous examples of the tetracladina genera *Siphonia*, *Hallirhoa*, *Ragadinia*, *Kalpinella*, and *Rhopalospongia* are known from the greensand, which, there can be no doubt, possessed a definite spicular dermal layer, we may conclude that these dermal spicules belong to these genera.

The dermal spicules are very generally distributed throughout the greensand, but they are more particularly abundant in the lower greensand at Haslemere, and in the upper greensand at Blackdown, Haldon, and Warminster.

Order Haxactinellidæ, O. Schmidt.

In contrast with the abundance of the detached spicules of tetractinellid and lithistid sponges in the sponge-beds, those of the above order are few and of rare occurrence. There are fragments of the skeletal mesh of two species, in which the nodes are compact; in one (Plate 45, figs. 7, 7*a*, *b*) the rays are robust and the distance from node to node is ·3 mm.; in the other (fig. 8) the mesh is smaller and the nodes are

* See Catalogue Fossil Sponges in the British Museum, pl. 15, fig. 1*b*.

only ·17 mm. apart. Both these forms are present in the lower greensand at Haslemere, and the smaller also occurs at Sevenoaks and Tilburstow Hill.

Portions of the mesh of another species in which the nodes are octahedral occur in the malm at Merstham (fig. 9).

43. *Stauractinella*, sp. (Plate 45, figs. 10, 10*a-c*).—Free hexactinellid spicules with straight or curved rays; the rays are nearly of an even thickness throughout and truncated at their ends. The nodes are compact, in some only five rays are developed. The rays are from ·2 to ·53 mm. in length, and ·04 mm. in thickness.

L. G. S., Haslemere, U. G. S., Blackdown, Merstham.

44. *Gomphites Parfitti*, Carter (Plate 45, figs. 11, 11*a* ; *op. cit.*, p. 127. Plate 9, figs. 38, 39).—Free spicules, in which only five rays are developed, the rays short, tapering, and either straight or curved. The rays are ·15 to ·416 mm. in length, by ·066 mm. in thickness.

U. G. S., Blackdown, Haldon.

Whilst some of the above-mentioned spicules are widely distributed, and range from the lower greensand to the upper chalk, and even to the eocene, there are others which, so far as our present knowledge extends, are restricted to a single geological horizon. Thus, for example, the distinctive zone spicules of *Geodites*, which I have named *Geodites robustus*, *G. pusillus*, *G. audax*, and *G. obtusus*, have as yet only been found in the lower greensand ; *Geodites deflexus*, *G. Haldonensis* and *Tethyopsis Haldonensis*, are apparently restricted to the lower or Blackdown division of the upper greensand ; *Geodites Carteri* and *G. prælongus* occur in both divisions ; and *G. Wrightii* ranges from the lower greensand to the upper chalk.

The character of the sponges in these lower and upper greensand sponge-beds bears a very general resemblance to those which have been deposited, under similar detached conditions, in the upper chalk of Norfolk, the North of Ireland, Westphalia, and Bohemia. In all these localities the large majority of the detached spicules belong to tetractinellid sponges of the same genera as those in the greensand, and with these are mingled, in varying proportion, those of the other orders of siliceous sponges. As a rule entire sponges do not occur in connexion with these detached spicules, but in Westphalia hexactinellid sponges are present in the same beds.

Recent deposits consisting largely of the detached spicules of disintegrated siliceous sponges, similar to those of the greensand, do not appear to have been met with in the dredging expeditions of these last few years, or at all events there is no record of them, but on the other hand *entire* siliceous sponges are stated by the late Sir Wyville Thompson* to be very abundant in some areas of the Atlantic.

* "Depths of the Sea," pp. 416, 430.

Summary.

I have pointed out in this paper the occurrence in the lower and upper greensand strata of the Wealden area, the Isle of Wight, and the South-Western counties, of beds of rock, formed to a large extent of the detached spicular remains of siliceous sponges, and thus distinctly of organic origin. Their true characters have not been generally recognised, and they have usually been described as deposits of sandstone, chert, malm, hearthstone, firestone, &c. In the lower greensand these beds are mainly developed in the lower or Hythe division, and they are exposed at Haslemere, Midhurst, Petworth, Godalming, Tilburstow Hill, near Godstone, Sevenoaks, Maidstone, and at Hythe. The sponge-beds vary from three-quarters of an inch to three feet in thickness; between them, as a rule, there are intervening beds of sand or sandstone. The greatest total thickness of the sponge-beds exposed in one section is 11 feet. Sponge-beds are less common in the higher or Folkestone division of the lower greensand, but they are numerous at Folkestone itself, and reach a total thickness of more than eight feet, and there is also a thin bed in this division at Sevenoaks. The lower greensand strata at Faringdon, in Berkshire, is of an altogether different character to those of the same formation, in the area treated of in this paper, and the sponges which abound therein are likewise entirely different, being calcisponges and retaining their entire forms.

The sponge-beds in the upper greensand are of two distinct types, one of which is shown on the northern and western margin of the Weald, and the other in the Isle of Wight, and further westwards in the counties of Wilts, Somerset, Dorset, and Devon. In the first-named district the sponge-beds are of a soft, grayish-white, siliceous or siliceo-calcareous rock, known under the names of malm, hearth, or firestone. In this the sponge-spicules principally occur in the negative form of minute empty casts, the presence of which renders the rock extremely light and porous. The beds can be traced nearly continuously along the northern and western margin of the Wealden area, and they are well shown at Godstone, Merstham, near Reigate, Betchworth, Farnham, and Selborne. Further northwards they are present at Wallingford, in Berkshire. The beds vary in thickness from 15 feet at Merstham to 60 feet at Farnham.

In the more typical upper greensand of the Isle of Wight and the south-western counties, the sponge-beds consist of thick layers of chert and porous siliceous rock at the summit of the series, immediately beneath the so-called chlorite marl; whilst in the lower division the sponge remains principally occur in loose quartzitic sands with siliceous accretions. The chert or sponge-beds at the top of the upper greensand are best exposed at Shanklin, Ventnor, and the Undercliff, in the Isle of Wight, at Warminster in Wiltshire, and Penzlewood in Somersetshire. They vary from 10 to 25 feet in thickness. The sponge-beds of the lower division are scarcely recognisable in the Isle of Wight, but they attain a thickness of 10 to 20 feet

on the summit of the Blackdown and Haldon Hills in Devonshire, and at Axmouth in Dorsetshire. The chert here is only present in beds of subordinate importance.

Sponge-beds of similar characters to those of the greensand have been described from the Hils-sandstein, in Westphalia, which is of neocomian age, and, judging from specimens which I have examined, the "gaize de l'Argonne," which is largely developed in the Ardennes, and the "meule de Bracquegnies," in Belgium, are sponge-beds, filled with spicules and spicular casts like those of the greensand.

The sponge-remains in the various beds are exclusively those of siliceous sponges. In some the silica of the spicules yet retains its original colloidal condition, in which it is negative to polarised light, and readily soluble in caustic potash. The matrix of the sponge-beds of the malm and firestone is also to a large extent of colloidal or amorphous silica, and this material has been deposited in the form of minute globules or discs, and it seems to have been derived from the sponge-spicules, with the empty casts of which the beds are throughout filled.

More generally the original amorphous silica of the sponge-remains has been altered to chalcedony, and the chert and porous siliceous rock accompanying it, which is filled with traces of the spicules, are likewise of chalcedony; occasionally the chalcedony gives place to crystalline silica.

Glauconite very commonly fills the canals of the spicules, and remains after the spicular walls have been removed; it also replaces the spicular walls.

In some sponge-beds the spicules have been nearly entirely replaced by crystalline calcite; they are imbedded in a matrix of granular limestone.

As a general rule the sponge-spicules are inclosed in a compact matrix, in which their forms can only be partially studied, but under certain conditions they are loosely distributed in sand or in fine powder in cavities in chert, from whence they can be obtained quite free from matrix. The sponge-beds appear to be wholly composed of detached, free spicules; entire sponges are absent. These spicules belong to numerous species. All four orders of siliceous sponges are represented, but those of monactinellid and hexactinellid sponges form but a small proportion, whilst those of tetractinellid and lithistid sponges, more particularly of the megamorina family, are extremely abundant.

Explanation of Plates.

PLATE 40.

Fig. 1. Microscopic section of a sponge-bed in the upper greensand at Warminster, showing entire and fragmentary spicules of colloid silica, inclosed in a matrix of transparent chalcedonic silica. Colloid silica in the globular form is also present. ×40 diameters.

Fig. 2. Microscopic section of a nodule from the upper greensand firestone at Godstone, filled with spicules of colloid silica in a siliceo-calcareous matrix. × 40 diameters.

Fig. 3. Microscopic section of chert from the upper greensand at Penzlewood, showing spicules and spicular canals of chalcedonic and crystalline silica, in a matrix of the same material. × 40 diameters.

Fig. 4. Microscopic section of a sponge-bed in the lower greensand at Tilburstow Hill, showing spicules replaced by crystalline calcite, inclosed in a matrix of granular calcite. × 40 diameters.

Fig. 5. Microscopic section of the malm rock from Merstham, showing a portion of a sponge-spicule in which the silica is in the globular form. The matrix largely consists also of colloid globular silica, with mica scales, grains of glauconite, and a small proportion of chalcedonic silica. × 150 diameters.

Fig. 6. Microscopic section of a sponge-bed in the upper greensand at Warminster, showing a portion of a sponge-spicule, the central part of which consists of colloid silica in a globular form, whilst the exterior is of chalcedonic silica. × 150 diameters.

Fig. 7. A fractured surface of the upper greensand "malm" at Merstham, showing the empty casts of spicules in a matrix of globular silica. × 40 diameters.

Fig. 8. A portion of a sponge-spicule from Warminster, in which the colloid silica is traversed by curved lines. × 150 diameters.

Fig. 9. A microscopic section of a portion of another spicule, still further magnified, in which the lines appear as incomplete elliptical rings with smooth rims. × 560 diameters.

PLATE 41.

Figs. 1, 1*a*, *b*. *Reniera gracilis*.—Haslemere, Blackdown. × 60 diameters.

Figs. 2, 2*a*. *Reniera obtusa*.—Blackdown. × 60 diameters.

Figs. 3, 3*a*, *b*. *Reniera cucumis*.—Near Warminster. × 60 diameters.

Figs. 4, 4*a–e*. *Reniera Zitteli*, Počta.—Haslemere, Blackdown. × 60 diameters.

Figs. 5, 5*a–f*. *Reniera truncata*.—Near Warminster. × 60 diameters.

Figs. 5*g–h*. *Reniera truncata*? Immature spicules in which the canal remains open throughout its length.—Near Warminster. × 60 diameters.

Figs. 6, 6*a–e*. *Axinella dispersa*.—Blackdown; near Warminster, Merstham. × 60 diameters.

Figs. 7, 7*a*. *Axinella gracilis*.—Haslemere, Blackdown. × 60 diameters.

Figs. 8, 8*a–d*. *Axinella stylus*.—Haslemere, Blackdown. × 40 diameters.

Figs. 9, 9*a*. *Spirastrella neocomiensis*.—Haslemere. × 60 diameters.

Figs. 10, 10*a–o*. *Geodites*. Acuate spicules of different forms and dimensions belonging to various species.—Haslemere, Blackdown, Haldon; near Warminster, Merstham. × 40 diameters.

Figs. 11, 11*a–b*. *Monilites Haldonensis*, Carter.—Blackdown; near Warminster ×80 diameters.

Fig. 12. *Esperites Haldonensis*, Carter.—Blackdown. ×60 diameters.

Fig. 13. *Geodites Wrightii*. Acuate spicule, fragmentary. ×40 diameters.

Fig. 14. *Dirrhopalum neocomiensis*.—Haslemere. ×40 diameters.

Figs. 15, 15*a*, 16. *Pachymatisma? virga*.—Blackdown, Haldon. 15*a*, ×40 diameters, figs. 15, 16, ×60 diameters.

Figs. 17, 17*a–k*. *Geodites Carteri*. Zone spicules, showing various modifications of the head rays. In figs. 17*b*, *k*, only two of the rays are developed.—Haslemere, Blackdown. ×40 diameters.

Fig. 18. *Hamate spicule*.—Haslemere. ×60 diameters.

PLATE 42.

Figs. 1, 1*a–c*. *Geodites prælongus*. Zone spicules with simple and bifurcate head-rays.—Haslemere, Sevenoaks. ×40 diameters.

Figs. 2, 2*a*. *Geodites robustus*. Zone spicules.—Haslemere. ×40 diameters.

Figs. 3, 3*a–c*. *Geodites audax*. Zone spicules. Fig. 3*b* shows the peculiar manner in which the infilled axial-canal has been contorted and displaced. Fig. 3 is the head of a spicule seen from beneath.—Haslemere. ×40 diameters.

Figs. 4, 4*a*. *Geodites obtusus*. Zone spicules, the shafts incomplete.—Sevenoaks. ×40 diameters.

Figs. 5, 5*a–c*. *Geodites politus*. Zone spicules and anchor spicule (5*c*.)—Merstham, Folkestone. ×40 diameters.

Figs. 6, 6*a*, *b*. *Geodites pusillus*. Zone spicules.—Haslemere. ×40 diameters.

Figs. 7, 7*a–g*. *Geodites Haldonensis*, Carter. Zone spicules with variously modified head-rays.—Blackdown. ×40 diameters.

Figs. 8, 8*a*. *Geodites divergens*. Zone spicules with very prominent canals.—Near Warminster. ×40 diameters.

Fig. 9. *Geodites Wrightii*. Zone spicule, moniliform.—Haslemere. ×40 diameters.

Fig. 10. *Geodites deflexus*. Zone spicule, shaft incomplete.—Blackdown. ×40 diameters.

Figs. 11, 11*a*, *b*. *Geodites divergens*. Zone spicules.—Near Warminster. ×40 diameters.

Figs. 12, 12*a–e*. *Geodites gracilis*. Slender zone spicules.—Blackdown; near Warminster, Merstham.

Figs. 13, 13*a–c*. *Geodites planus*. Zone spicules with horizontally extended head-rays. Haldon; near Warminster. ×40 diameters.

Figs. 14, 14*a–e*. *Stellettites?* sp. Zone spicules, with reduced shaft and horizontally extended head-rays.—Blackdown; near Warminster, Merstham. ×60 diameters.

Figs. 15, 15*a*. *Tethyopsis Haldonensis*, CARTER. Zone spicules.—Blackdown, Haldon. ×40 diameters.

Figs. 16, 16*a*. *Geodites Haldonenis*, CARTER. "Anchor" spicules, shafts incomplete. —Haldon, Penzlewood. ×40 diameters.

Figs. 17, 17*a–k*. Small trifid spicules of different species. Figs. 17, 17*a*, *b*, young spicules of *Geodites*. ×60 diameters. Figs. 17*c*, *d*, *e*, allied to *Pachastrella*. Figs. 17*g*, *h*, *i*, spicules replaced by glauconite. Fig. 17*k*, dermal spicule of lithistid. ×60 diameters.—Blackdown, Warminster, Merstham, Ventnor.

PLATE 43.

Figs. 1, 1*a–d*. *Tethyopsis Haldonensis*, CARTER.—Blackdown. ×40 diameters.

Figs. 2, 2*a–f*. *Geodites*. Globate and globostellate spicules of the dermal layer of various species.—Haslemere, Blackdown, near Warminster. ×80 diameters.

Fig. 3. *Stellettites callodiscus*, CARTER. Spicule of the dermal layer.—Near Warminster. ×80 diameters.

Figs. 4, 4*a–i*. *Pachastrella Haldonensis*, CARTER. Four-rayed spicules of the skeleton. Blackdown; near Warminster, Merstham. Fig. 4*a*. ×60 diameters; the others ×40 diameters.

Figs. 5, 5*a*, *b*. *Pachastrella Carteri*.—Haslemere, Blackdown, Merstham. ×40 diameters.

Figs. 6, 6*a*. *Pachastrella quadriradiata*, CARTER.—Blackdown, near Warminster. ×80 diameters.

Figs. 7, 7*a–z*. *Doryderma*. Spicules of various forms belonging to the body and stem of different species of this and allied genera of the megamorina family of lithistidæ.—Haslemere, Blackdown, Haldon, near Warminster, Merstham. ×40 diameters.

PLATE 44.

Figs. 1–14. Various forms of spicules of the dermal layer, probably belonging to different species of tetracladine lithistidæ, of the genera *Siphonia*, *Hallirhoa*, *Ragadinia*, &c.—From Haslemere, Blackdown, Haldon, near Warminster, Merstham. All drawn by means of the camera lucida to the same scale of 60 diameters.

Figs. 15, 15*a*. *Discodermia sinuosa*, Carter. Spicules of the dermal layer of this existing species, for comparison with those of the fossil lithistids. From the Gulf of Manaar. Drawn from examples of the typical species, kindly presented to the Author by Mr. H. J. Carter, F.R.S. ×60 diameters.

PLATE 45.

Figs. 1–1*c*. *Carterella*, sp. Skeletal spicules.—From near Warminster. ×40 diameters.

Figs. 2–2*k*. *Mastosia neocomiensis*. Various forms of skeletal spicules.—From Haslemere, Haldon; near Warminster. ×40 diameters.

Figs. 3, 4–4*d*. *Chenendopora*, sp. Skeletal spicules probably of more than one species.—Blackdown, Tilburstow Hill, near Warminster. ×60 diameters.

Figs. 5, 5*a*, *b*. *Rayadinia*, sp. Skeletal spicules.—Haslemere, near Warminster. ×60 diameters.

Figs. 6, 6*a*–*f*. *Siphonia*, sp. Skeletal spicules probably of different species.—Blackdown, near Warminster. ×60 diameters.

Figs. 7, 7*a*, *b*. Fragments of the skeletal mesh of a species of Hexactinellid.—Haslemere. ×60 diameters.

Fig. 8. Fragment of the skeletal mesh of another species.—Sevenoaks. ×60 diameters.

Fig. 9. Fragment of the skeletal mesh of another species in which the spicular nodes are octahedral.—Merstham. ×60 diameters.

Figs. 10, 10*c*. *Stauractinella*, sp.—Blackdown, Merstham. ×60 diameters.

Figs. 11, 11*a*. *Gomphites Parfitti*, Carter.—Blackdown, Haldon. ×60 diameters.

Fig. 12. *Acerate spicule*.—From near Warminster. ×560 diameters.

Fig. 13. An imperfect spicule, the walls of which have been partially dissolved away, whilst the infilled interior canal has remained intact.—From Blackdown. ×40 diameters.

Figs. 14, 14*a*. The infilled canals of a spicule of *Geodites* and one of *Pachastrella*, which have remained after the walls of the spicule have been entirely dissolved away.—From Chert, Penzlewood. ×40 diameters.

Figs. 15, 15*e*. Residuary spicules, in which the colloidal silica has been removed, and the form of the spicule is retained in glauconite or allied silicate. These spicules are frequently contracted and distorted in a peculiar manner.—From Ventnor, and near Warminster.

Fig. 16. A spicule of *Geodites*, in which the outer walls have been removed, and the infilled canal is surrounded by discs or globules of colloid silica.—From siliceous material in cavities in chert, near Warminster. ×150 diameters.

Fig. 17. A transverse microscopic section of a spicule in which the silica is crystalline and exhibits a radiated fibrous structure.—From chert at Petworth. ×150 diameters.

Figs. 18, 18*b*. Free discs or globules of colloidal silica.—From soft malm rock at Farnham. ×560 diameters.

Figs. 19, 19*e*. Discs or globules of colloidal silica, showing an indefinite nucleus, and faintly-marked radiating striæ. In 19*d*, *e*, the radiations start from a central point.—From loose powdery material in cavity in chert, near Warminster. ×560 diameters.

Fig. 20. A single coccolith from the Atlantic ooze for comparison with the siliceous globules. ×560 diameters.

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VIII. *Magnetisation of Iron.*

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[PLATES 46–52.]

Preliminary.

THE experimental determination of the relation between magnetisation and magnetising force, would be a simple matter if the expression of such relation were not complicated by the fact that the magnetisation depends not alone on the magnetising force at the instant, but also upon previous magnetising forces; in fact, if it were not complicated by the phenomena of residual magnetism. In the absence of any satisfactory theory, we can only experimentally attack particular cases, and the results obtained have only a limited application; for example, we may secure that the sample examined has never been submitted to greater magnetising force than that then operating, and we may determine a curve showing the relation of magnetisation to magnetising force when the latter is always increasing; we may also determine the residual magnetism when after each experiment the magnetising force has been removed. Such curves have been determined by ROWLAND (Phil. Mag., Aug., 1873) and others. For many purposes a more useful curve is one expressing the relation of the magnetising force and magnetisation when the former is first raised to a maximum and then let down to a defined point; such curves have been called descending curves. One or two descending curves are given in a paper by Mr. SHIDA (Proc. R. S., 1883, p. 404). It has been shown by Sir W. THOMSON and others that the magnetisation of iron depends greatly upon the mechanical force to which the iron is at the time submitted. In the following experiments the samples were not intentionally submitted to any externally applied force. CLERK MAXWELL gives in his 'Electricity and Magnetism,' chap. 6, vol. ii., a modification of WEBER'S theory of induced magnetism, and from this he deduces, amongst other things, what had been already observed, that iron may be strongly magnetised and then completely demagnetised by a reversed force, but that it will not even then be in the condition of iron which has never been magnetised, but will be more easily affected by forces in one direction than in the other. This I have verified in several cases. The ordinary determinations of residual magnetisation are not applicable to determine the permanent magnetism which a piece of the

material of suitable given form will retain after removal of external magnetising force, but, as will be shown, the descending curves which express the relation of magnetisation and force, where these are diminishing, can be at once used for this purpose. Such curves can also be used, as has been shown by WARBURG and by EWING (Report Brit. Assn., 1883), to determine the energy dissipated when the magnetisation of iron is reversed between given limits. That such dissipation must occur is clear, but some knowledge of its amount is important for some of the recent practical applications of electromagnetism. Probably Professor EWING has made a more complete experimental study of magnetisation of iron than any one else. The researches of Professor HUGHES should be mentioned here, as, although his results are not given in any absolute measure, his method of experiment is remarkable beyond all others for its beautiful simplicity. I have had great doubts whether it was desirable that I should publish my own experiments at all. My reason for deciding to offer them to the Royal Society is that a considerable variety of samples have been examined, that in nearly all cases I am able to give the composition of the samples, that the samples are substantial rods forged or cast and not drawn into wire, and that determinations of specific electric resistance have been made on these rods which have some interest from a practical point of view.

Method of experiment.

Let $\mathfrak{H}$ be the magnetic force at any point, $\mathfrak{B}$ the magnetic induction, and $\mathfrak{I}$ the magnetisation (*vide* THOMSON, reprint, MAXWELL, vol. ii., 'Electricity and Magnetism'), then $\mathfrak{B}=\mathfrak{H}+4\pi\mathfrak{I}$. We may therefore express any results obtained as a relation between any two of these three vectors; the most natural to select are the induction and the magnetic force, as it is these which are directly observed. $\mathfrak{B}$ is subject to the solenoidal condition, and consequently it is often possible to infer approximately its value at all points, from a knowledge of its value at one, by guessing the form of the tubes of induction. $\mathfrak{H}$ is a force having a potential, and its line integral around any closed curve must be zero if no electric currents pass through such closed curve, but is equal to $4\pi c$ if c be the total current passing through the closed curve. In arranging the apparatus for my experiments, I had other objects in view than attaining to a very small probable error in individual results. I wished to apply with ordinary means very considerable magnetising forces; also to use samples in a form easily obtained; but above all to be able to measure not only changes of induction but the actual induction at any time. The general arrangement of the experiments is shown in fig. 1, and the apparatus in which the samples are placed in fig. 2. In the latter, fig. AA is a block of annealed wrought iron 457 millims. long, 165 wide, and 51 deep. A rectangular space is cut out for the magnetising coils BB. The test samples consist of two bars CC′, 12·65 millims. in diameter; these are carefully turned, and slide in holes bored in the block, an accurate but loose fit; the ends which come in contact

are faced true and square; a space is left between the magnetising coils BB for the exploring coil D, which is wound upon an ivory bobbin, through the eye of which one of the rods to be tested passes. The coil D is connected to the ballistic galvanometer, and is pulled upwards by an india-rubber spring, so that when the rod C is suddenly pulled back it leaps entirely out of the field. Each of the magnetising coils B is wound with twelve layers of wire, 1·13 mm. diameter, the first four layers being separate from the outer eight, the two outer sets of eight layers are coupled parallel, and the two inner sets of four layers are in series with these and with each other. The magnetising current therefore divides between the outer and less efficient convolutions, but joins again to pass through the convolutions of smaller diameter. The effective number of convolutions in the two spools together is 2008. Referring to fig. 1, the magnetising current is generated by a battery of eight GROVE cells E, its value is adjusted by a liquid rheostat F, it then passes through a reverser G, and through a contact breaker H, where the circuit can be broken either before or at the same instant as the bar C is withdrawn; from H the current passes round the magnetising coils, and thence back through the reverser to the galvanometer K. The galvanometer K was one of those supplied by Sir W. THOMSON for electric light work, and known as the graded galvanometer, but it was fitted with a special coil to suit the work in hand. The exploring coil D was connected through a suitable key with the ballistic galvanometer L. Additional resistances M could be introduced into the circuit at pleasure, and also a shunt resistance N. With this arrangement it was possible to submit the sample to any series of magnetising forces, and at the end of the series to measure its magnetic state; for example, the current could be passed in the positive direction in the coils B, and gradually increased to a known maximum; it could then be gradually diminished by the rheostat F to a known positive value, or it could be reduced to zero; or, further, it could be reduced to zero, reversed by the reverser G, and then increased to any known negative value. At the end of the series of changes of magnetising current, the circuit is broken at H (unless the current was zero at the end of the series), and the bar C is simultaneously pulled outwards. Three successive elongations of the galvanometer L are observed. From the readings of the galvanometer K, the known number of convolutions of the coils B, and an assumed length for the sample bars, the intensity of the magnetising force $\mathfrak{H}$ is calculated. The exploring coil D had 350 convolutions. From its resistance, together with that of the galvanometer with shunts, the sensibility of the galvanometer, its time of oscillation, and its logarithmic decrement, a constant is calculated which gives the intensity of induction in the iron from the mean observed elongation of the galvanometer. The resistances have been corrected in the calculation for the error of the B.A. unit, and both galvanometers were standardised on the assumption that a certain CLARK's cell had an electromotive force of $1{\cdot}434 \times 10^8$ C.G.S. units. This CLARK's cell had been compared to and found identical with those tested by Lord RAYLEIGH.

Let the mean length of the lines of induction in the sample be l, and σ the section of the sample; let l' be the length of lines of induction in the block, and σ' their section, $\mathfrak{B}$ the intensity of induction in the sample, $\mathfrak{B}$ in the block, then $\sigma\mathfrak{B}=\sigma'\mathfrak{B}'=\mathrm{I}$; let

$$\mathfrak{B}=\mu\mathfrak{H},$$

and

$$\mathfrak{B}'=\mu'\mathfrak{H};$$

then

$$\int\left(\frac{\mathfrak{B}}{\mu}+\frac{\mathfrak{B}'}{\mu'}\right)ds=\int\mathfrak{H}ds$$

or

$$\frac{Il}{\mu\sigma}=4\pi nc-\frac{Il'}{\mu'\sigma'}$$

where n is the number of convolutions of the magnetising coils. Now in the instrument used σ' is large, and μ' is as large as can be obtained, hence the term $\frac{Il'}{\mu'\sigma'}$ is small comparatively. My first intention was to correct the magnetising force by deducting this small correction, but finally I did not do so, because in the more interesting results the magnetism of the block is dependent in part upon previous magnetising forces, the effect of which cannot be allowed for with certainty. We know then that in all the curves the magnetising force indicated is actually too great by a small but sensible amount, which does not affect the general character of the results or their application to any practical purpose. The magnetising force then at any point of the sample is $\frac{\mathrm{I}}{\sigma\mu}=\frac{4\pi nc}{l}$—a small correction which we deliberately neglect. There is another source of uncertainty in the magnetising force: the length l is certainly greater than the space within the wrought iron block, but it is not possible to say precisely how much greater. If the sample bars and the block were a single piece, the results of Lord RAYLEIGH for the resistance of a wire soldered into a block would be fairly applicable; but it is essential that there should be sufficient freedom for the bar to slide in the hole; the minute difference between the diameters of the sample and the hole will increase the value which should be assigned to l. Throughout, l is assumed to be 32 centims., and it is not likely that this value is incorrect so much as half the radius of the bar, or 1 per cent. The magnetising forces ranged up to 240 C.G.S. units when both bars were of the same material. In some cases a single bar only was available for experiment; the plan then was to use it as the bar which enters into the exploring coil, and for the other to use a known bar of soft iron. We have then to deduct from $4\pi nc$ the magnetising force required to magnetise the bar of soft iron to the state observed, and to distribute the remainder over the shorter length of sample examined. The results obtained in this way are subject to a greater error, because some lines of induction undoubtedly make their way across from the end of

the soft iron bar to the body of the block. A small correction is required, important in the case of bodies but slightly magnetic, for the fact that the area of the exploring coil is greater than the area of the bars tested. Thus the induction measured by the exploring coil is not only that in the sample, but something also in the air around the sample. The amount of this was tested by substituting for a sample of iron or steel a bar of copper, and afterwards a rod of wood, and it was found in both cases that the induction $\mathfrak{B}$ was 370 when the force $\mathfrak{H}$ was 230. The correction is in all cases small, but it has been applied in the column giving the maximum induction, as it materially affects the result when the sample contains much manganese, and is consequently very little magnetic.

The resistances were determined by the aid of a differential galvanometer, the connections being made as shown in the accompanying diagram, fig. 3. The additional resistance R was adjusted until the balance was obtained. The resistances actually measured are, some of them, as low as $\frac{1}{6000}$ of an ohm, they must not therefore be regarded as so accurate as determinations made upon samples of a more favourable form; they, however, do show the remarkable effect of several impurities in iron, though it is possible that some of the results may be in error nearly 1 per cent.

Results obtained.

In all, thirty-five distinct samples were tested, of twenty compositions. The first three were supplied to me by Messrs. MATHER and PLATT, and of these I have no analyses. All the rest were analysed for me in the laboratory of Sir JOSEPH WHITWORTH and Co., and the samples of material were actually prepared there, excepting, HADFIELD'S steel, No. X.; BESSEMER iron made by the basic process for telegraph wire, No. IV., from the North-Eastern Steel Company; and two Tungsten steels, Nos. XXX. and XXXI., which are in general use for permanent magnets.

I would express here my great indebtedness to Mr. GLEDHILL, one of the managing directors of Sir J. WHITWORTH and Co., for preparing for me the samples I desired, and having them analysed. The fact is, indeed, that any value this paper may possess really lies in the variety of samples tried and in the accompanying chemical analysis, both due to Mr. GLEDHILL. Samples Nos. I.–X. and XXXII.–XXXV. were tested with a pair of bars, the rest with a single bar of the sample used, in combination with a bar of wrought iron. The particulars of the several samples are most conveniently given in a table which follows, and to which I shall presently refer. With many samples observations were made sufficient to plot the ascending and descending curves which express induction in terms of magnetising force, but as these can make no pretence, for reasons already stated, to such accuracy as would warrant their use in testing a theory as to the form of curves of magnetisation, a few only are given as examples, and in other cases results are given in the table sufficient to define

in absolute measure the primary magnetic properties of the materials and the very characteristic way in which they differ from each other.

The curves given include in each case an ascending curve, taken before the sample had been submitted to greater magnetising forces; a curve of residual magnetisation, that is, a curve in which the ordinate is the residual induction left after application and removal of the magnetising force represented by the abscissa, and two descending curves.

Fig. 4 gives the curves from wrought iron No. I.
Fig. 5 the same to an amplified scale of abscissæ.
Fig. 6 for steel with ·32 per cent. carbon, annealed No. VI.
Fig. 7 for steel with ·32 per cent. carbon, oil hardened No. VII.
Fig. 8 for steel with ·89 per cent. carbon, annealed No. VIII.
Fig. 9 for steel with ·89 per cent. carbon, oil hardened No. IX.
Fig. 10 for cast iron No. III.

The descending curves, which express the passage from extreme magnetisation in one direction to extreme magnetisation in the opposite direction, may be roughly defined by the maximum ordinate to which they rise, and by the points AB in which they cut the coordinate axes. The ordinate OB is what is generally meant by the residual induction after great magnetising force, or the "retentiveness." The word "Coercive Force" has been long used, but, so far as I know, in a rather vague way and without accurate definition.

I propose to call OA the "Coercive Force" of the material, and define it as that reversed magnetic force which just suffices to reduce the induction to nothing after the material has been submitted temporarily to a very great magnetising force. It is the figure which is of greatest importance in short permanent magnets. The manner in which the dimensions of the ascending curves and the curves of residual magnetisation vary with the descending curves is sufficiently obvious from inspection. The slowness with which iron or steel yields to small magnetising forces is evidently intimately connected with the coercive force. Another force is worth noting, viz., that demagnetising force which not merely reduces the induction to zero whilst applied, but just suffices to destroy the residual magnetism so that when removed no permanent magnetisation remains. The area enclosed by the two descending curves divided by 4π represents the energy dissipated when the unit volume is magnetised to saturation, its magnetism reversed, and again reversed, and so brought to its first value. This area differs a little from 4 × coercive force × maximum induction. In the cases for which curves are given the results are as follows:—

Sample.	Area from curve. $\frac{}{4\pi}$	4 coercive force × max. induction $\frac{}{4\pi}$
No. I.	17247	13356
„ III.	15139	13037
„ VI.	45903	40120
„ VII.	61898	65786
„ VIII.	50521	42366
„ IX.	74871	99401

In this we note that for soft iron the area is greater than the product, the reverse for hard steel; for any practical purpose we may assume that the greatest dissipation of energy which can be caused by a complete reversal to and fro of magnetisation is approximately measured by $\frac{\text{coercive force} \times \text{maximum induction}}{\pi}$. An interesting feature in the curves is the manner in which the residual magnetism rapidly attains to near its maximum value, and is then nearly constant, whilst the induction continues to increase. This is very marked in the case of cast iron.

The column of figures in the general table of results almost explain themselves.

In the case of the cast iron, the total and the graphitic carbon are given, the difference being the combined carbon. In the case of the manganese steel and iron, the induction is almost proportional to the magnetising force, hence permeability is really the magnetic property to be noted: this is given below in a separate table. The demagnetising force is that reverse force which, when applied after great magnetising force, just suffices to remove all permanent magnetisation. The energy dissipated is $\frac{\text{OA} \times \text{maximum induction}}{\pi}$, and is approximately the energy in ergs. converted into heat in a complete cycle of magnetisation from the limit in one direction to that in the opposite and back again.

In the general table of results one of the striking features is the high specific resistance of some samples of cast iron, ten times as great as wrought iron. This fact is not without practical importance in some forms of dynamo machines, for the energy wasted by local currents induced in the iron by given variations of the magnetic force will be but $\frac{1}{10}$th as great with cast iron as with wrought iron. The high resistance of cast iron may be due in large measure to its heterogeneity; grey cast iron may be regarded as a mechanical mixture of more or less pure iron with very small bits of graphite.

[Jan. 15, 1886.—I have recently determined the rate of variation with temperature of the electric resistance of a sample of cast iron for the purpose of ascertaining whether it approximated more nearly to a pure metal, to an alloy, or to bodies the resistance of which decreases with rise of temperature. The sample examined was a thin rod of grey iron 6·71 millims. diameter and 24·85 centims. long between the contacts. The range of temperature was 10° C. to 130° C., and through this range

the rate of increase of resistance was nearly uniform. The specific resistance at 0° C. was inferred to be 0·000102, and the rate of increase was 0·00083 per degree centigrade.—J. H.]

Another very striking feature is the way in which any substantial proportion of manganese annihilates the magnetic property of iron; the sample with 12 per cent. of manganese is practically non-magnetic. The induction noted in the table =310 corresponds to a magnetising force of 244. If all the substances in this sample other than the iron were mechanically mixed with the iron, and arranged in such wise as to have the greatest effect upon its magnetic property, no such annihilation of magnetic property would ensue. This question of mixture will be considered somewhat more closely below.

The permeability and susceptibility are given in the following table for some of the samples containing much manganese :—

No.	Permeability.	Susceptibility.
X.	1·27	·0215
XIV.	3·59	·206
XVI.	3·57	·2046
XXXV.	1·84	·0668

It is therefore clear that the small quantity of manganese present enters into that which must be regarded for magnetic purposes as the molecule of iron, and completely changes its properties. The fact is one which must have great significance in any theory as to what is the molecular nature of magnetisation. Another clearly marked fact is the exceptionally great effect which hardening has both upon the magnetic properties and the electrical resistance of chrome steel.

Note also that in those cases where the maximum induction is low, the residual magnetism is proportionately lower still, but that the coercive force is not uniformly lower. This is in accordance with the supposition that these samples are to be regarded as mechanical mixtures of a strongly magnetic substance, such as ordinary iron or steel, and a non-magnetic substance, such as manganese steel with 12 per cent. of manganese. A feature present in all the curves may some day have a bearing on the molecular theory of magnetism. It is this: the ascending curve twice crosses the continuation of the descending curve; in other words, the fact that a sample has been strongly magnetised in a reverse direction, renders it for small forces, or for large forces, more difficult to magnetise than a virgin sample, but distinctly easier for intermediate forces. This is best seen in the case of the hardest steel, No. IX., fig. 9, in which the two curves cut in the points marked M, N. A similar phenomenon has been observed and investigated by G. Wiedemann (*vide* 'Die Lehre vom Galvanismus,' first edition, vol. ii., p. 340, *et seq.*).

No.	Description.	Temper.	Chemical analysis.						Specific resistance.	Magnetic properties.*				Energy dissipated.
			Total carbon.	Manganese.	Sulphur.	Silicon.	Phosphorus.			Maximum induction.	OB, residual induction.	OA, coercive force.	Demagnetising force.	
I.	Wrought iron	Annealed	..	..	..	..	..	..	·00001378	18,251	7,248	2·30	..	13,356
II.	Malleable cast iron	,,	..	..	..	..	..	..	·00003254	12,403	7,479	3·80	..	34,742
III.	Grey cast iron	..	..	..	..	..	..	..	·00010560	10,783	2,928	3·80	..	13,037
IV.	Bessemer mild steel	..	·045	·200	·030	None	·040	..	·00001050	18,196	7,860	2·96	..	17,137
V.	Whitworth mild steel	Annealed	·090	·153	·016	,,	·042	..	·00001080	19,840	7,080	1·63	..	10,289
VI.	,, ,,	,,	·320	·438	·017	·042	·035	..	·00001446	18,736	9,840	6·73	..	40,120
VII.	,, ,,	Oil hardened	,,	,,	,,	,,	,,	..	·00001390	18,796	11,040	11·00	..	65,736
VIII.	,, ,,	Annealed	·890	·165	·005	·081	·019	..	·00001559	16,120	16,740	8·28	..	42,866
IX.	,, ,,	Oil hardened	,,	,,	,,	,,	,,	..	·00001695	16,120	8,736	19·28	..	99,401
X.	Hadfield's manganese steel	..	1·005	12·360	·038	·204	·070	..	·00006554	810	..	..	..	..
XI.	Manganese steel	As forged	·674	4·730	·023	·608	·078	..	·00005368	4,623	2,202	23·50	37·13	34,567
XII.	,, ,,	Annealed	,,	,,	,,	,,	,,	..	·00003928	10,578	5,848	33·86	46·10	113,963
XIII.	,, ,,	Oil hardened	,,	,,	,,	,,	,,	..	·00005556	4,769	2,158	27·64	40·29	41,941
XIV.	,, ,,	As forged	1·298	8·740	·024	·094	·072	..	·00006993	747	..	..	..	..
XV.	,, ,,	Annealed	,,	,,	,,	,,	,,	..	·00006316	1,985	540	24·50	50·39	15,474
XVI.	,, ,,	Oil hardened	,,	,,	,,	,,	,,	..	·00007066	733	..	..	..	..
XVII.	Silicon steel	As forged	·685	·694	,,	3·438	·133	..	·00006163	15,148	11,073	9·49	12·60	45,740
XVIII.	,, ,,	Annealed	,,	,,	,,	,,	,,	..	·00006185	14,701	8,149	7·80	10·74	36,485
XIX.	,, ,,	Oil hardened	,,	,,	,,	,,	,,	..	·00006195	14,696	8,084	12·75	17·14	59,619
								Chromium.						
XX.	Chrome steel	As forged	·532	·393	·020	·220	·041	·621	·00002016	15,778	9,318	12·24	13·87	61,439
XXI.	,, ,,	Annealed	,,	,,	,,	,,	,,	,,	·00001942	14,848	7,570	8·98	12·24	42,425
XXII.	,, ,,	Oil hardened	,,	,,	,,	,,	,,	,,	·00002708	13,960	8,595	38·15	48·45	169,455
XXIII.	,, ,,	As forged	·687	·028	,,	·134	·043	1·195	·00001791	14,680	7,568	18·40	22·03	85,944
XXIV.	,, ,,	Annealed	,,	,,	,,	,,	,,	,,	·00001849	13,233	6,489	15·40	19·79	64,842
XXV.	,, ,,	Oil hardened	,,	,,	,,	,,	,,	,,	·00003035	12,868	7,891	40·30	56·70	167,050
								Tungsten.						
XXVI.	Tungsten steel	As forged	1·357	·036	None	·043	·047	4·649	·00002249	15,718	10,144	15·71	17·75	78,568
XXVII.	,, ,,	Annealed	,,	,,	,,	,,	,,	,,	·00002250	16,498	11,008	15·30	16·93	80,315
XXVIII.	,, ,,	Hardened cold water	,,	,,	,,	,,	,,	,,	·00002274	..	..	..	..	..
XXIX.	,, ,,	Hardened tepid water	,,	,,	,,	,,	,,	,,	·00002249	15,610	9,482	30·10	34·70	149,500
XXX.	,, ,, (French)	Oil hardened	·511	·625	None	·021	·028	3·444	·00003604	14,480	8,643	47·07	64·46	216,864
XXXI.	,, ,,	Very hard	·855	·312	..	·151	·089	2·353	·00004427	12,133	6,818	51·20	70·69	197,660
								Graphitic carbon.						
XXXII.	Grey cast iron	..	3·455	·173	·042	2·044	·151	2·064	·00011400	9,148	3,161	13·67	17·03	39,789
XXXIII.	Mottled cast iron	..	2·581	·610	·105	1·476	·435	1·477	·00006286	10,546	5,108	12·24	..	41,072
XXXIV.	White cast iron	..	2·036	·356	·467	·764	·458	None	·00005661	9,342	5,554	12·24	20·40	36,383
XXXV.	Spiegeleisen	..	4·510	7·970	Trace	·502	·128	..	·00010520	385	77	..	..	..

* The maximum magnetisation is obtained from the maximum induction by subtracting 240 (the magnetising force used) and dividing by 4π, or in most cases sufficiently nearly by multiplying by 0·08. The residual magnetism is obtained by dividing the residual induction by 4π. The numbers in the table are the actual magnetic forces in the infinitesimal gap between the two bars.

Magnetisation of a mixture of magnetic and non-magnetic substance.

We suppose that the mixture is purely mechanical, and that the two substances each retain their magnetic properties.

We may regard as an element of the substances a portion great in comparison with the size of the pieces of the two substances constituting the mixture, or we may be more analytical and regard as an element a portion very small in comparison with such pieces.

Let the volume of magnetic substance be λ, of non-magnetic $1-\lambda$. The magnetic properties of the mixture will depend, not only upon λ, but upon the relative arrangement of the magnetic and non-magnetic parts.

Let α, a, A be the magnetic force, induction and magnetisation, regarding the sizes of the parts of the two substances as infinitely small; let α_0, a_0, A_0 be their values within a portion of magnetic substance. α, a, A are what we could actually observe. The relations of α_0, a_0, A, may be known from experiments on the magnetic substance when unmixed.

1. Suppose the magnetic substance to be arranged in the mixture in the form of filaments or laminæ parallel to the lines of magnetic force, then $\alpha=\alpha_0$, and $A=\lambda A_0$. Hence the effect of admixture in this case is to reduce the magnetisation for a given force in the ratio $1:\lambda$.

2. Let the non-magnetic substance be in thin laminæ lying perpendicular to the lines of force; we shall then have again $A=\lambda A_0$; but $a=a_0$ instead of $\alpha=\alpha_0$, whence

$$\begin{aligned} \alpha &= a-4\pi A \\ &= a_0-4\pi\lambda A_0 \\ &= (1-\lambda)a_0+\lambda\alpha_0 \end{aligned}$$

α_0 being supposed known in terms of a_0, this gives us the means of calculating the properties of the mixture.

These two are the extreme cases; all other arrangements of the two substances will have intermediate effects approximating to the one extreme or the other in a manner which we can judge in a rough way.

For example, if the magnetic substance be in separate portions bedded in the non-magnetic substance, the result will be somewhat analogous to the case of plates perpendicular to the lines of force; if on the other hand the non-magnetic substance be in separate portions bedded in the magnetic, the result will approximate rather to the case of filaments parallel to the lines of force.

Suppose that in the case of HADFIELD'S steel, No. X., the mixture be of pure iron in very small quantity in a non-magnetic matrix, how much pure iron is it necessary to suppose to be present, supposing the arrangement to be as unfavourable as possible? Here $a=a_0=310$, $\alpha=244$, $\alpha_0=$sensibly zero, whence $\lambda=\frac{66}{310}=0{\cdot}21$. Suppose, however, that the iron were arranged as small spheres

bedded in the non-magnetic substance, we have $\lambda = -\frac{1}{2} + \frac{3}{2}\frac{\mu}{2+\mu}$, μ being the observed value of $\frac{a}{\alpha}$, viz., 1·27; whence $\lambda = 0{\cdot}09$. We may say that of the 86 per cent. iron in this sample not more than 9 per cent. is magnetic.

If hard steel were bedded as small particles in a non-magnetic matrix, we should expect the mixture to have low retentiveness, but comparatively high coercive force, such as we see in the case of samples XI., XIII., and XV. If our apparatus had been sufficiently delicate to detect residual magnetism in samples X., XIV., and XVI., it is probable enough that we should have found the coercive force to be considerable.

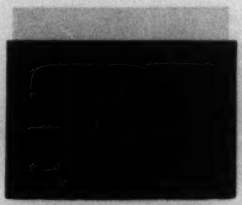
In the case of mixtures much will depend on the relative fusibility of the magnetic and non-magnetic substances. If the former were less fusible, it would probably occur as crystals separated from each other by a non-magnetic matrix; if on the other hand it were more fusible, it would remain continuous. It is easy to see the kind of difference in magnetic property which would result.

Determination of permanent magnetisation of an ellipsoid.

If an ellipsoid be placed in a uniform magnetic field, its magnetisation will be uniform.

If the externally applied magnetising force be zero, the force at any point within the ellipsoid will be AL, BM, CN, where A, B, C are the components of magnetisation of the ellipsoid, and $L = 4\pi abc\frac{d\phi_0}{da^2}$, &c., where $\phi_0 = \int_0^\infty \frac{d(\phi^2)}{\sqrt{(a^2+\phi^2)(b^2+\phi^2)(c^2+\phi^2)}}$, and a, b, c are the semi-axes of the ellipsoid. Suppose the forces have all been parallel to the axis a, we have then $\mathfrak{H} = L\mathfrak{J} = \frac{L\mathfrak{B}}{4\pi}$ very nearly.

Let the curve PQ be the descending curve of magnetisation (the ordinates being



induction), draw OR so that $\frac{RN}{ON} = \frac{4\pi}{L}$; then RN is clearly the induction in the ellipsoid when the external force is removed. In the case of a sphere $L = -\frac{4}{3}\pi$, therefore $RN = -3ON$. The greatest residual induction which a sphere of the materials can

retain is a very little less than three times the force required to reduce the magnetisation to zero.

In a similar way any spheroid could be readily dealt with, and the best material judged for a permanent magnet of given proportions. It should, however, be noted that any conclusions thus deduced might be practically vitiated by the effect of mechanical vibration in shaking out the magnetism from the magnet.

Dissipation of energy by residual magnetism.

Imagine a conducting circuit of resistance R, let x be the current in it at time t, E the electromotive force other than that due to the electro-magnetic field, and a the total magnetic induction through the circuit, then

$$Rx=E-\frac{da}{dt}.$$

The work done in time dt by the electromotive force is $xEdt=(Rx^2+x\frac{da}{dt})dt$; of this Rx^2dt goes to heat the wire, the remainder, or xda, goes into the electro-magnetic field. Imagine a surface of which the conducting circuit is a boundary, and on it take an elementary area; through this area draw a tube of induction returning into itself; the line integral of force along the closed tube is $4\pi x$. If therefore we assume that the work done in any elementary volume of the field is equal to that volume multiplied by the scalar of the product of the change of induction, and the magnetising force divided by 4π, the assumption will be consistent with the work we know is done by the electromotive force E. Now apply this to any curve connecting induction and magnetic force. Let PQ be two points in the curve, draw PM and QN parallel to the

axis of magnetic force OX; the work done on the field per cubic centimeter passing from P to Q is equal to $\frac{\text{area PMNQ}}{4\pi}$. Some of this is converted into heat in the case of iron, for we cannot pass back from Q to P by diminishing the magnetising force.

Let AKB be the curve connecting A and B when the magnetising force is reversed, BLA when it is again reversed in this cycle; the final magnetisation is the same as it

was initially; hence the balance of work done upon the field must be converted into heat; this heat will be represented by the area AKBLA$\div 4\pi$ in ergs. per cubic centimeter.

An approximation to the values of this dissipation is given in the table of results. It may be worth while to call attention to their practical application. Take the case of a dynamo-machine with an iron core, finely divided to avoid local electric currents. Note that we are going to assume—though whether true or false we do not know—that the dissipation is the same whether the magnetisation is reversed by diminishing and increasing the intensity of magnetisation without altering its direction, or whether it is reversed by turning round its direction without reducing its amount to zero.

A particular machine has in its core about 9000 cubic centims. of soft iron plates; the resistance of its armature is 0·01 ohm, of its shunt magnets 8·0 ohms, and when running 900 revolutions per minute, its E.M.F. at the brushes is 55 volts. When the current in the armature is 250 ampères we have

	Ergs. per second.
Total energy of current	$= 144 \times 10^9$.
Loss in armature resistance	$= 625 \times 10^7$.
Loss in magnet resistance	$= 378 \times 10^7$.
Loss in magnetising and demagnetising iron core of armature	= 9000 cubic centims. $\times$ 15 revolutions per second $\times$ 13,356 (from table of results) $= 18 \times 10^9$.

From this we see at once that the heat generated in the core of the armature by reversal of magnetisation is about one-half of that arising from the resistance of the copper wire of the electro-magnet. If a hard steel were used the loss from reversal might amount to 20 per cent. or more of the useful work done.

WEBER'S *Theory of Magnetism.*

In WEBER'S theory it is, in effect, assumed that the magnetic force tending to deflect a molecule is that which it would experience if it were placed in a long cylindrical cavity, the axis of the cylinder being in the direction of magnetisation. This seems a rather unnatural supposition. If instead of this we assume that the deflecting force is that which it would experience in a spherical cavity, and draw a curve connecting either the induction or magnetisation with the deflecting force on a molecule within a spherical cavity, we shall find that the curve differs very little from a straight line. In the curves already given we have taken $\mathfrak{B}$ and $\mathfrak{H}$ as the variables, where

$$\mathfrak{B}=\mathfrak{H}+4\pi\mathfrak{I}.$$

Suppose we take $\mathfrak{B}$ and $\mathfrak{K}$ where

$$\mathfrak{B}=\mathfrak{H}+4\pi\mathfrak{I}.$$

and

$$\mathfrak{K}=\mathfrak{H}+\frac{4\pi}{3}\mathfrak{I}.$$
$$=\tfrac{1}{3}\mathfrak{H}+\tfrac{1}{3}\mathfrak{B}.$$

The curves would then be hardly distinguishable from straight lines, the same scales being used for ordinates and abscissæ; it requires no great stretch of imagination to suppose that if this curve were continued far enough it would differ but little from that given by MAXWELL, vol. ii., p. 79.

Now, in dealing with WEBER'S theory it would seem more suitable to take $\mathfrak{K}$, the magnetic force in a spherical cavity, as the independent variable. If we assume WEBER'S theory with this modification we arrive at the following conclusions:—

1. All observations yet made upon the magnetisation of iron are upon the straight part of WEBER'S curve.

2. The particular features of curves of magnetisation as ordinarily observed arise from a slight irregularity in WEBER'S curve, magnified by the near approach of iron to a state in which a random distribution of the magnetic axes of the molecules is unstable.

I do not put these remarks forward as indicating more than the fact that we are a very long way from obtaining a range of facts sufficiently extended for testing a molecular theory of magnetism. The broad fact which strikes the mind most forcibly is the specific difference which exists between magnetic and non-magnetic bodies. Most bodies are either very slightly ferro-magnetic or very slightly diamagnetic. On the other hand iron, nickel, and cobalt are enormously magnetic.

Iron with 12 per cent. of manganese, and some small quantities of carbon and other substances, is so little magnetic that its magnetism would be accounted for by supposing that in its mass were distributed a few little bits of pure iron. There

seems to be a certain instability of something we know not what; bodies fall on one side practically non-magnetic, on the other enormously magnetic, but hardly any intermediate class exists.

The number of actual observations made on each of the samples named has been very considerable, though I have not thought it necessary to set them out at length, as I base no general conclusion upon them. The bulk of these observations were made by my assistant, Mr. E. Talbot, and my pupil, Mr. Paul Dimier, to whom my thanks are due for their patience and care.

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IX. *The Absorption Spectra of the Alkaloids.*

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[Plates 53–56.]

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Introduction.

Many of the poisonous alkaloids give no distinctive chemical reactions by which they can be identified, and in certain cases the means of recognising them are restricted to observations on their crystalline form and physiological action. Some of the alkaloids have never been crystallised, and even such as are usually obtained in crystals are not always recognisable, because their shape is not invariably a well-marked or characteristic feature; moreover the form and grouping of crystals is occasionally modified by such reactions or treatment as may be necessary in the extraction of an organic base. Small admixtures of foreign substances also modify crystalline character, and not infrequently cause a crystalline substance to become amorphous. No absolute reliance

* This communication was presented to the Society in two parts. The information concerning aconitine, pseudaconitine, japaconitine, morphine, narcotine, codeine, thebaine, papaverine, oxynarcotine, apomorphine hydrochloride, cotarnine hydrobromide, tetracetyl morphine, diacetyl codeine, quinine, quinine sulphate, cinchonine sulphate, quinidine sulphate, cinchonidine sulphate, veratrine, piperine, brucine, strychnine, narceine, aconitine (foreign), cevadine, atropine, solanine, hyoscyamine, digitaline, picrotoxine, nicotine, and caffeine formed Part I. (Received November 19, 1884. Abstract published in Proc. Roy. Soc., vol. xxviii., p. 1.) That concerning pyridine, piperidine, quinoline, tetrahydroquinoline, quinoline hydrochloride, and the specimens of aconitine received from Dr. Stevenson formed Part II. (Received March 5, 1885. Abstract published in Proc. Roy. Soc., vol. xxviii., p. 191.)

can be placed upon the mere appearance of crystals; they must be submitted to recrystallisation by sublimation or some other process. The physiological action of certain alkaloids of an extremely deadly character is remarkable enough to prove a means of identifying the substance when the effect on the human subject is under observation; but it is to some extent capable of being modified by the extent of the dose, the administration of other drugs, or the idiosyncracy of the patient. These facts are well known, and are generally made use of by the counsel for the defence in medico-legal cases. When the identification of an alkaloid is a necessary part of the evidence of the administration of poison, experiments of a physiological character have necessarily to be made on the lower animals. It has been objected that the action of drugs on creatures of a smaller size and different organisation may be, and, in certain cases, indeed is, unlike that produced in the human organism, and comparative experiments are necessary in which the suspected substance is compared with authentic specimens before its identity can be established. But in such comparative experiments an element of uncertainty is introduced, because various preparations supposed to be the same alkaloid may differ in physiological action to such an extent that reasonable doubts may be entertained as to whether they consist of the same substance or, indeed, of one substance only. The whole subject of the modification of alkaloids by the reagents used in their extraction, variations in their crystalline character and in their physiological action, can be well illustrated by reference to the researches of Dr. C. R. A. Wright on the various preparations known as aconitine.

The evidence given at the trial of George Henry Lamson, a surgeon, at the Central Criminal Court in 1882, for poisoning with aconitine, conferred greatly increased importance upon any method of absolute physical measurement which might be substituted for the ordinary tests in the identification of the dangerous alkaloids. Almost all active alkaloids have a complex chemical constitution, and every complex molecule in which carbon is in a certain state of condensation has a definite absorption spectrum in the ultra-violet if not in the visible region. In most cases the absorption curve is peculiar and often strikingly characteristic, and only minute quantities of material are actually required for obtaining measurements of spectra from which such curves may be plotted. The interest that attaches to an examination of the absorption spectra of the alkaloids is not alone the fact that a means of recognising, detecting, and estimating, such substances may be devised, but still more that we may learn something of their chemical constitution.

The experimental details.

The method of examination employed was that described in the Phil. Trans., Vol. 170, p. 257, 1879, but the spectroscope was the short focussed instrument employed for photographing the spectra published in the 'Journal of the Chemical Society,' vol. xli., p. 84.

Several modifications were introduced with the object of gaining accuracy and reducing the amount of labour entailed by a large series of observations.

1st. The solutions were as far as possible made up to the same strength, and cells of definite thickness were employed in series as 5 millims., 4 millims., 3 millims., 2 millims., and 1 millim. This obviated the necessity for repeatedly diluting the solutions, and so greatly facilitated the work; while it diminished the possibility of errors, especially in the descriptions of the spectra.

2nd. The measurements were all reduced to wave-lengths, and the absorption curves were plotted accordingly. As the curves have been made continuous the necessity for shading has been avoided.

3rd. While the absorption curves serve for the comparison of one alkaloid with another, as a ready means of distinguishing them and of estimating the quantity present in a solution, a very careful description of the spectra from which the curves were drawn has been furnished, so that no essential details have been omitted such as were found to be imperfectly represented by the drawings.

4th. The rays intended to give a suitable spectrum were obtained from metallic points, which, for the purpose, are superior to all others. In previous researches, electrodes of cadmium or of nickel alloyed with a small quantity of copper, in certain cases metallic indium and iron have been used. None of these are quite satisfactory. The cadmium and indium lines are not sufficiently close together or numerous, the wave-lengths of the iron and nickel lines had not been determined when these experiments were in progress; moreover, they were deemed unsuitable because there is too great a difference between the intensity of the lines in different parts of the spectrum, so that deceptive results are obtained when feeble absorptions occur.

The electrodes I prefer to all others are cadmium, tin, and lead; the lines of tin and lead are numerous and well distributed throughout the whole spectrum; they are distinctive in character, and therefore easily recognisable, which is not the case with the iron and nickel lines. The cadmium lines preponderate in intensity, but the emissive power of the cadmium is purposely reduced by diminishing the quantity of that metal in the spark.

Thus I take for one electrode an alloy of tin with 25 per cent. of cadmium, for the other an alloy of lead with the same proportion of cadmium. When the two are used as opposite electrodes a spark spectrum is obtained every line in which can be easily recognised; the tin lines are on one side, and stretch half-way across; the lead are on the other side; while the cadmium contained in both electrodes causes the lines of that metal to stretch across from point to point. A somewhat longer exposure than usual fills the intervals between the lines with a weaker continuous spectrum. The period was three minutes. The extent of the continuous rays is always noted in the descriptions of the absorption spectra. To secure well-defined spectra the photographs were taken with the solutions placed in front of the slit, and the rays were concentrated by a condensing lens of two inches diameter and three inches focal length.

Method of determining the wave-lengths.

It was usual to take a series of five or six photographs of each solution on one plate, and thus the spectra represent the rays transmitted by 1, 2, 3, 4, 5, or more millimetres of the absorbent liquid. The wave-lengths referred to are derived from those given in the Phil. Trans., Vol. 175, p. 63, 1884, for cadmium, tin, and lead The scale numbers in hundredths of an inch are arbitrary, and quite different from those given beside the wave-lengths in the paper quoted; they were actually read off from photographs of the electrodes by means of an ivory scale, the end of which coincided with the end of the glass plate. The scale originally used was a photograph of the spectrum of the electrodes with divisions etched on the glass, the gelatine film being scraped off just below the lines to admit of the glass being etched. The divisions were hundredths of an inch, and every fifth line was distinguished by the corresponding wave-length being written on the back of the plate opposite to the longer line. The figures, however, are too small, and, otherwise, from the transparency of the glass difficult to read when applied to the face of another photograph. It was found more satisfactory to apply an ivory scale with a bevelled edge, divided as aforesaid, so that the scale numbers arbitrarily fixed first for the cadmium lines are read off on all the other lines. The scale is held in position by a pair of wooden spring clips. Supposing an absorption band occurs near the line 120 ($\lambda=3245{\cdot}5$), then 120 on the scale is brought over this line on the photograph; if it be seen to extend to 240 or thereabouts ($\lambda=2593$), the reading on the scale is brought up to this line, and a more exact measurement is made. If the adjustment is not further from the correct number than $\frac{1}{100}$th of an inch, it is sometimes not worth while to alter it; but whenever it is necessary to make accurate measurements, the scale is capable of easy and perfect adjustment to any part of the spectrum. For the purpose of converting the scale measurements into wave-lengths and their reciprocals, two curves have been drawn on one sheet of paper. The scale numbers, wave-lengths, and oscillation frequencies of the fiducial lines employed for the interpolation curves are stated on p. 475. Fractions of a tenth-metre have been omitted as being unnecessary in dealing with absorption spectra.

On the diagrams the scale of wave-lengths is reduced to millionths of a millimetre.

LINES of cadmium, tin, and lead used for the interpolation curve.

Scale-numbers.		λ.	$\frac{1}{\lambda}$	Scale-numbers.		λ.	$\frac{1}{\lambda}$
Hundredths of an inch.				Hundredths of an inch.			
0	..	..	..	190·0	Sn	2812	*3556*
4·5	..	..	..	192·5	Pb	2801	*3571*
20·5	Cd (7)	4414	*2265*	197·0	Sn	2778	*3599*
22·5	Pb	4386	*2280*	203·5	Cd (17)	2747	*3640*
30·5	Pb	4245	*2355*	213·0	Sn	2705	*3696*
42·75	Pb	4061	*2462*	223·0	Pb	2662	*3756*
	Air	3994		235·0	Pb	2613	*3827*
62·0	Sn	3800	*2503*	240·0	Sn	2593	*3856*
67·3	Pb	3739	*2674*	244·0	Pb	2576	*3882*
72·0	Pb	3683	*2715*	245·0	Cd (18)	2572	*3888*
76·3	Pb	3639	*2748*	245·5	Sn	2570	*3891*
79·0	Cd (9)	3610	*2770*	247·0	Pb	2561	*3904*
82·7	Pb	3572	*2799*	252·5	Sn	2545	*3929*
93·7	Cd (10)	3465	*2886*	266·5	Sn	2495	*4008*
97·0	Air	3437	*2910*	270·0	Sn	2483	*4027*
101·0	Cd (11)	3403	*2938*	272·0	Pb	2475	*4040*
106·5	Sn	3352	*2983*	281·3	Pb	2445	*4090*
109·5	Sn	3330	*3003*	282·0	Pb	2442	*4095*
115·0	Sn	3283	*3045*	286·5	Sn	2429	*4116*
118·0	Sn	3262	*3065*	289·0	Sn	2422	*4128*
119·5	Cd (12)	3260	*3067*	295·0	Pb	2402	*4163*
129·5	Sn	3174	*3150*	298·0	Pb	2393	*4178*
135·0	Pb	3137	*3187*	306·0	Sn	2368	*4223*
151·0	Sn	3033	*3297*	311·0	Sn	2355	*4246*
155·0	Sn	3008	*3324*	318·0	Sn	2335	*4282*
159·5	Cd	2980	*3355*	320·0	Cd	2329	*4293*
165·0	Pb	2949	*3391*	322·7	Cd	2321	*4308*
171·0	Sn	2912	*3434*	325·7	Cd (23)	2313	*4323*
177·0	Cd	2880	*3472*	335·0	Cd & Sn	2288	*4370*
178·5	Pb	2872	*3481*	344·0	Cd (24)	2265	*4415*
180·0	Sn	2862	*3494*	351·0	Sn	2247	*4450*
182·5	Sn	2849	*3510*	353·5	Cd	2241	*4462*
185·0	Sn	2839	*3524*	368·5	Pb	2205	*4535*
186·0	Pb	2832	*3531*	372·5	Cd	2195	*4555*
188·0	Pb	2822	*3543*	395·0	Cd	2145	*4662*

In this research the wave-length numbers have alone been quoted. The mode of proceeding after a series of photographs had been taken was, after measuring, to record and reduce the measurements to wave-lengths. The numbers obtained were then laid down on suitably ruled paper for the absorption curve to be plotted. It was then generally found necessary to execute a further series of photographs, and sometimes two or three series of the absorptions caused by more dilute or stronger solutions, in order to obtain the maximum and minimum effect of the absorbent substance as well as the coefficients of absorption for all intermediate wave-lengths. The curves drawn are continuous, so that the necessity for shading is obviated; the unshaded diagrams are not such true representations of the photographs, but these less exact diagrams are supplemented by very careful and accurate descriptions. Authentic

specimens of alkaloids and other substances in a state of great purity were for the most part obtained from the chemists by whom they were prepared, namely, Messrs. T. and H. Smith and Son, of Edinburgh, Dr. C. R. A. Wright, and Mr. David Howard. I am particularly indebted to Dr. Stevenson and Dr. C. R. A. Wright for several aconitine specimens, also to Mr. David Howard for the cinchona alkaloids.

The substances examined were the following :—

Alkaloids of the Aconites.

Aconitine from *A. Napellus*	$C_{33}H_{49}NO_{12}$
Pseudaconitine from *A. Ferox*	$C_{36}H_{43}NO_{12}$
Japaconitine from a species of Japanese aconite	$C_{66}H_{88}N_2O_{21}$
Aconitine from a foreign source ; probably picraconitine.	

Alkaloids of the Cinchonas.

Cinchonine	$C_{20}H_{24}N_2O$
Cinchonidine sulphate	$(C_{20}H_{24}N_2O)_2.H_2SO_4.2Aq$
Quinine	$C_{20}H_{24}N_2O_2$
Quinine sulphate	$(C_{20}H_{24}N_2O_2)_2.H_2SO_4.2Aq$
Quinidine sulphate	$(C_{20}H_{24}N_2O_2)_2.H_2SO_4.2Aq$

Alkaloids of the Papaveraceæ.

Morphine	$C_{34}H_{38}N_2O_6$
Codeïne	$C_{36}H_{42}N_2O_6$
Narcotine	$C_{22}H_{23}NO_7$
Narceïne	$C_{23}H_{29}NO_9$
Papaverine	$C_{21}H_{21}NO_4$
Thebaïne	$C_{19}H_{21}NO_3$

Derivatives of the Opium bases.

Tetracetyl-morphine	$C_{34}H_{34}(C_2H_3O)_4N_2O_6$
Diacetyl-codeïne	$C_{36}H_{40}(C_2H_3O)_2N_2O_6$
Oxynarcotine	$C_{22}H_{23}NO_8$
Cotarnine hydrobromide	$C_{12}H_{13}NO_3.HBr$
Apomorphine hydrochloride	$C_{17}H_{17}NO_2.HCl$

Alkaloids of the Solanaceæ.

Nicotine	$C_{10}H_{14}N_2$
Solanine	$C_{42}H_{67}NO_{15}$
Atropine	$C_{17}H_{23}NO_3$

ALKALOIDS of Strychnos.

Brucine	$C_{23}H_{26}N_2O_4.4H_2O$
Strychnine	$C_{21}H_{22}N_2O_2$

ALKALOIDS of Veratrium.

Veratrine	$C_{37}H_{53}NO_{11}$ (COUERBE's Veratrine)
Cevadine	$C_{32}H_{49}NO_9$ (MERCK's Veratrine)

VARIOUS other Alkaloids.

Digitaline	$C_{54}H_{48}O_{30}$, a glucoside
Hyoscyamine	$C_{17}H_{23}NO_3$
Caffeïne	$C_8H_{10}N_4O_2.H_2O$
Piperine	$C_{17}H_{19}NO_3$
Picrotoxine; composition doubtful.	

Index.

The specimens of alkaloids and their derivatives, the strength of the solutions examined, the solvents employed, the sources whence the alkaloids were obtained, or by whom they were prepared, are given in the following tabular statement.

Double normal sulphuric acid signifies a gramme molecule of H_2SO_4 in a litre of liquid, or $\frac{H_2SO_4}{1000}$.

Name.	Proximate strength of solutions.	Solvent.	From whom obtained.	Page.
Aconitine, Plate 53, fig. 1	$\frac{1}{100}$, $\frac{1}{1000}$	Alcohol, sp. gr. 0·8	Dr. C. R. A. WRIGHT	479
Aconitine, Plate 53	$\frac{1}{100}$	Proof spirit	Dr. STEVENSON	479
Aconitine, Plate 53, fig. 2	$\frac{1}{100}$, $\frac{1}{1000}$	Alcohol, sp. gr. 0·8	T. and H. SMITH and Co.	480
Aconitine (foreign), Plate 53	$\frac{1}{100}$	,, ,,	,, ,,	481
Japaconitine, Plate 53, fig. 3	$\frac{1}{100}$, $\frac{1}{1000}$	,, ,,	Dr. C. R. A. WRIGHT	481
Pseudaconitine (4), Pl. 53, fig. 4	$\frac{1}{100}$, $\frac{1}{1000}$	,, ,,	,, ,,	482
Aconitine	$\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{10000}$	,, ,,	Dr. STEVENSON	483
MORSON's aconitine (5), Plate 53, fig. 5	$\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{10000}$	,, ,,	,,	484
Aconitine (6), Plate 53, fig. 6	$\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{10000}$	,, ,,	,,	485
Aconitine	$\frac{1}{100}$, $\frac{1}{1000}$	,, ,,	,,	486
Quinine, Plate 53, fig. 7	$\frac{1}{1000}$	,, ,,	Mr. DAVID HOWARD	487
Quinine sulphate, Plate 54	$\frac{1}{1000}$	Alcohol, sp. gr. 0·8, with $\frac{1}{4}$ volume of $\frac{H_2SO_4}{1000}$	,, ,,	487
Cinchonine, Plate 54	$\frac{1}{1000}$		,, ,,	488
Quinidine sulphate, Plate 54	$\frac{1}{1000}$		,, ,,	488
Cinchonidine sulphate, Pl. 54	$\frac{1}{1000}$		,, ,,	489

Name.	Proximate strength of solutions.	Solvent.	From whom obtained.	Page.
Morphine, Plate 54, fig. 5 . .	$\frac{1}{200}$, $\frac{1}{1000}$ . . .	Alcohol, sp. gr. 0·8	T. and H. Smith and Co.	489
Morphine, Plate 54, fig. 6 . .	$\frac{1}{200}$, $\frac{1}{1000}$ . . .	,, ,,	Macfarlan and Co.	490
Tetracetyl-morphine, Plate 55	$\frac{1}{200}$, $\frac{1}{1000}$ . . .	,, ,,	Dr. C. R. A. Wright	491
Codeïne, Plate 55, fig. 2 . .	$\frac{1}{200}$, $\frac{1}{1000}$ . . .	,, ,,	,, ,,	492
Codeïne, Plate 55, fig. 3 . .	$\frac{1}{200}$, $\frac{1}{1000}$ . . .	,, ,,	T. and H. Smith and Co.	492
Diacetyl-codeïne, Pl. 55, fig. 4	$\frac{1}{200}$, $\frac{1}{1000}$ . . .	,, ,,	Dr. C. R. A. Wright	493
Thebaïne, Plate 55, fig. 5 . .	$\frac{1}{200}$, $\frac{1}{100}$, $\frac{1}{1000}$, $\frac{1}{2000}$	,, ,,	T. and H. Smith and Co.	494
Papaverine	$\frac{1}{200}$, $\frac{1}{1000}$, $\frac{1}{2000}$.	,, ,,	,, ,,	495
Narcotine, Plate 55, fig. 6. .	$\frac{1}{200}$, $\frac{1}{1000}$, $\frac{1}{2000}$.	,, ,,	,, ,,	497
Narceïne	$\frac{1}{200}$, $\frac{1}{1000}$, $\frac{1}{10000}$	,, ,,	,, ,,	498
Apomorphine hydrochloride, Plate 56, fig. 1	$\frac{1}{200}$, $\frac{1}{1000}$, $\frac{1}{2000}$, $\frac{1}{20000}$	,, ,,	,, ,,	501
Cotarnine hydrobromide . .	$\frac{1}{200}$, $\frac{1}{1000}$, $\frac{1}{10000}$	,, ,,	Dr. C. R. A. Wright	499
Veratrine, Plate 56, fig. 4. .	$\frac{1}{200}$, $\frac{1}{1000}$ . . .	,, ,,	T. and H. Smith and Co.	505
Cevadine	$\frac{1}{200}$	,, ,,	Dr. C. R. A. Wright	506
Brucine, Plate 56, fig. 5 . .	$\frac{1}{200}$, $\frac{1}{1000}$, $\frac{1}{2000}$.	,, ,,	,, ,,	506
Strychnine, Plate 56, fig. 6 .	$\frac{1}{200}$, $\frac{1}{100}$, $\frac{1}{2000}$, $\frac{1}{20000}$, $\frac{1}{125000}$	,, ,,	,, ,,	507
Piperine, Plate 56, fig. 3 . .	$\frac{1}{200}$, $\frac{1}{2000}$	,, ,,	T. and H. Smith and Co.	509
Oxynarcotine, Plate 56, fig. 2	$\frac{1}{200}$, $\frac{1}{1000}$, $\frac{1}{2000}$ $\frac{1}{20000}$	Absolute alcohol, with $\frac{1}{4}$ vol. glacial acetic acid	Dr. C. R. A. Wright	502
Nicotine	$\frac{1}{100}$	Alcohol, sp. gr. 0·8	T. and H. Smith and Co.	513
Solanine	$\frac{1}{200}$	,, ,,	,, ,,	511
Atropine	$\frac{1}{200}$	,, ,,	,, ,,	510
Hyoscyamine	$\frac{1}{200}$	,, ,,	,, ,,	511
Digitaline	$\frac{1}{200}$	,, ,,	,, ,,	512
Picrotoxine	$\frac{1}{200}$	,, ,,	,, ,,	512
Caffeïne	$\frac{1}{200}$	,, ,,	,, ,,	513

In all cases where the strength of the solutions have been stated as $\frac{1}{200}$ or $\frac{1}{1000}$, it is to be understood as one part by weight of the substance dissolved in liquid equal in volume to 200 times the weight of distilled water; thus 0·2 grm. dissolved in 40 cub. centims. of alcohol is represented by $\frac{1}{200}$th; or 0·1 grm. dissolved in 20 cub. centims. of alcohol, to which 5 cub. centims. of acetic acid were added in order to prepare a perfectly clear solution, is indicated by $\frac{1}{250}$. In thus making the liquids up to proportionate volumes we avoid the complication of having to weigh the solvents, while in every case the cells of varying thicknesses represent proportional weights of the alkaloids.

The measurements and descriptions of spectra.

ACONITINE. From Dr. C. R. A. WRIGHT. Plate 53, fig. 1.

0·2 grm. in 40 cub. centims. of alcohol of 0·8 sp. gr.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	2860
4	Continuous to	2860
3	Continuous to	2770
	ABSORPTION BAND	**2799 to 2714**
	Rays transmitted	2614
	Spectrum ends at	2573
2	Continuous to	2747
	ABSORPTION BAND	**2747 to 2614**
	Rays transmitted very faintly till	2614
	Spectrum ends at	2573
1	Continuous	2747
	Very weak	2746 to 2614
	Weak to	2547

This solution, which had been prepared with dilute alcohol (proof spirit), was kept two years to ascertain whether it would undergo any change likely to make an alteration in its spectrum. About 20 cub. centims. were contained in a bottle holding 50 cub. centims., it was freely exposed to air in this manner, by occasional removals of the stopper, in order to subject it to the conditions which might affect an ordinary pharmaceutical preparation. It yielded the same spectrum before and after this period of keeping. In order to test whether violent chemical treatment might affect the spectrum, it was boiled with concentrated sulphuric acid for fifteen minutes, till all the alcohol was expelled; it was then diluted with a known volume of water and again examined, but it was found quite unaltered. It was not subjected to treatment with alkalies, though it is known that it may be thus converted into benzoic acid and a new base. If the treatment with sulphuric acid is sufficiently prolonged it equally effects this transformation, and there is less liability to darkening of the solution by the formation of resinous products than by saponification with alkaline solutions. The liquid should then show the absorption band characteristic of benzoic acid. (See Phil. Trans., Vol. 170, p. 257, 1879.)

ACONITINE. From Dr. STEVENSON.

0·2 gr. in 40 cub. centims. of alcohol of 0·8 sp. gr., or approximately 1 in 200.

Unfortunately, no exact measurements of this spectrum can be given, but the diagram which was made at the time when that of the previous sample of aconitine was drawn, was precisely the same, hence it may be inferred that the measurements were

the same. It is as well to mention here that the whole of this work was executed in 1882, some of it even at an earlier date, and curves were plotted according to the method described in the paper in the Phil. Trans., Vol. 170, p. 257, but two years subsequently the work was repeated, after the modifications in the method of working had been introduced.

ENGLISH ACONITINE. From Messrs. T. and H. SMITH and Co., of Edinburgh and London. Plate 53, fig. 2.

0·2 grm. in 40 cub. centims. of alcohol of 0·8 sp. gr.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong	3171
4	Continuous, strong	3171
	Faint	3138
3	Strong	3131
	Weak	3008
2	Continuous, strong	3128
	Weak	3008
	ABSORPTION BAND	**3008 to 2747**
	Rays transmitted, faint	..
	Spectrum ends at	2573
1	Continuous, strong	2978
	ABSORPTION BAND	**2978 to 2872**
	Rays transmitted, weak, to	2573
	Spectrum, faint, end at	2547

0·02 grm. in 20 cub. centims. of alcohol of 0·8 sp. gr.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	3020
	Feeble to	2995
	ABSORPTION BAND	**2995 to 2735**
	Spectrum ends at	2556
4	Continuous, strong, to	2965
	ABSORPTION BAND	**2965 to 2863**
	Spectrum ends at	2556
3	Spectrum continuous to	2556
	Very feeble to	2518
2	Spectrum ends at	2466
1	Fairly strong, continuous spectrum to	2466
	Spectrum ends at	2255

FOREIGN ACONITINE. From Messrs. T. and H. SMITH and Co.

0·2 grm. in 40 cub. centims. of alcohol of 0·8 sp. gr.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	2839
4	Continuous to	2839
3	Continuous to	2573
2	Continuous to	2547
1	The same spectrum, but stronger and extending to a faint ray	2486

JAPACONITINE. From Dr. C. R. A. WRIGHT. Plate 53, fig. 3.

0·2 grm. in 40 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	2861
4	Continuous to	2839
3	Continuous, strong	2861
	ABSORPTION BAND	**2799 to 2614**
	Spectrum ends at	2573
	From 2614 to 2573 the spectrum is only very faintly visible.	
2	Continuous	2747
	ABSORPTION BAND	**2747 to 2614**
	Spectrum ends at	2573
1	Rays transmitted to	2547
	Remains of the absorption band are seen in the very weak transmission of rays lying between	2747 and 2614
0·8	Rays transmitted to	2529
0·6	Rays transmitted to	2529
0·4	Rays transmitted to	2526
0·2	Continuous to	2429

This substance gave the same spectrum after a solution in weak alcohol (proof spirit) had been kept for two years.

PSEUDACONITINE. From Dr. C. R. A. WRIGHT. Plate 53, fig. 4.

0·2 grm. in 40 cub. centims. of alcohol of sp. gr. 0·8.

This solution was diluted to five times its original volume for these photographs.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	3127
4	Continuous, strong to	3084
3	Continuous, strong to	3028
	ABSORPTION BAND	**3028** to **2431**
	Rays faintly transmitted to where spectrum ends at	2316
2	Continuous to	2747
	ABSORPTION BAND	**2747** to **2504**
	Rays transmitted faintly to	2431
	Rays transmitted weakly to	2316
1	Continuous to	2316
	Spectrum ends at a line the position of which is just indicated	2292

This substance yielded the same spectrum after it had been kept for two years in solution in weak alcohol (proof spirit).

A series of aconitines from different sources of manufacture was kindly sent to me by Dr. STEVENSON, Lecturer on Chemistry and Medical Jurisprudence at Guy's Hospital, London.

The list of specimens, which was accompanied by remarks upon them, is the following :—

No. 1. "Exotic aconitine, probably German, rather inert."

No. 2. "A fine specimen of crystallised aconitine, special, prepared by T. MORSON and SON, 124, Southampton Row, London."

No. 3. "Aconitine, from BURGOYNE, BURBIDGES and Co., 16, Coleman Street, London."

No. 4. "Nitrate of Aconitine." (This specimen was accidentally destroyed.)

No. 5. "Aconitine of uncertain source."

No. 1. ACONITINE. Exotic aconitine, probably German; rather inert.

0·1 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
10	Continuous to	3245·5
9	Continuous to	3245·5
8	Continuous to	3245·5
7	Same as last	
6	Continuous to	3245·5
5	Continuous to	3130
	Spectrum extends feebly to	2863
4	Continuous to	2948
	Spectrum extends feebly, but not continuous to	2568
3	Continuous to	2948
	A feeble spectrum extends to	2568
2	Continuous strong to	2863
	A feeble spectrum extends to	2568
1	Continuous to	2863
	Spectrum extends weakly to	2568

0·1 in 20 dilutes 2 cub. centims. to 10 cub. centims., or 0·1 grm. in 100 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	2863
	A feeble spectrum extends to	2568
4	Continuous spectrum to	2542
3	Same as last	
2	Continuous spectrum to	2472·5
1	Continuous to	2472·5
	Spectrum extends feebly to	2387

0·1 grm. in 500 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	2387
	Spectrum extends faintly to	2259
4	Same as last up to	2259
	Spectrum extends feebly to	2144
3	Same as last.	
2	Same as last, but stronger.	
1	Same as last.	

No. 2. Aconitine. From F. Morson and Son. Plate 53, fig. 5.

0·1 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	3130
4	Same as last.	
3	Continuous to	3130
2	Same as last.	
1	Continuous, strong to	3130

0·1 grm. in 100 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	3085
4	Continuous to	3085
3	Continuous to	3033
	Absorption band from	3033 to 2836·5
	Rays transmitted, not continuous to	2740
	Absorption band from	2740 to 2424·3
	Rays transmitted very feebly to	2310
2	Continuous to	3002
	Absorption band from	3002 to 2863
	Rays transmitted weakly to	2740
	Absorption band from	2740 to 2424·3
	Rays transmitted weakly to	2310
1	Continuous to	2740
	Very feeble, but not continuous to	2440·5
	Very feebly continuous to	2286

0·1 grm. in 500 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	2740
	Very feeble, but not continuous to	2440·5
	Feebly continuous to	2310
4	Continuous to	2696·5
	Very feebly continuous 205	...
	Faint but not continuous to	2472·5
	Feebly continuous to	2310
3	Some at last.	
2	Continuous to	2259
1	Continuous to	2259

No. 3. ACONITINE. From BURGOYNE, BURBIDGES and Co. Plate 53, fig. 6.

0·1 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous	2976·5
4	Continuous to	2863
	ABSORPTION BAND from	**2863 to 2613**
	Rays transmitted at	2568
3	Continuous to	2863
	Weak, not continuous to	2740
	ABSORPTION BAND from	**2740 to 2658**
	Rays transmitted at	2568
2	Continuous to	2863
	Weak, but not continuous.	2568
1	Continuous to	2493·5

0·1 grm. in 100 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	2493·5
4	Continuous to	2476
3	Continuous strong to	2476
2	Continuous to	2472·5
	Spectrum extends feebly to	2418
1	Continuous to	2418

There is a *weakening of the spectrum* between 2402 and 2310, but it can scarcely be called an *absorption band*.

0·1 grm. in 500 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	2418
	Weakening of the spectrum from	2387 to 2310
	Rays transmitted feebly to	2259
4	Continuous to	2387
	Spectrum extends to	2145·9
3	Same as last.	
2	Same as last, but stronger.	
1	Continuous to	2190

No. 5. Aconitine. Aconitine of uncertain source.

0·1 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	2812
	Spectrum faint, but not continuous, extends to	2568
4	Continuous to	2812
	Spectrum extends to	2568
3	Continuous to	2568
2	Continuous to	2542
1	Continuous to	2472·5

0·1 grm. in 100 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	2472·5
4	Continuous to	2418
3	Same as last.	
2	Continuous to	2418
1	Continuous to	2387
	Spectrum extends to	2145·9

Of these specimens, the spectra of which have been described, only two exhibit absorption bands. Curves have been drawn of these.

Morson's preparation, the crystals of which were one and even two millimetres in length, was found to absorb the rays at two points, the two absorptions being equally strong. The measurements and descriptions of the transmitted spectra are here given, together with the curves.

It is noticeable that the most active aconitines are those with strongest absorption bands; and of the commercial samples scarcely two yield the same absorption spectra. The variations in the curves indicate that not only may there be considerable differences in their composition, but also in their chemical constitution.

QUININE. From Mr. DAVID HOWARD. Plate 53, fig. 7.

0·02 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	3465
4	Continuous, strong to	3465
3	Continuous, strong to	3403
	ABSORPTION BAND from	**3403 to 3033**
	Rays transmitted to	2995·5
2	Continuous, strong to	3403
	ABSORPTION BAND from	**3403 to 3033**
	Rays transmitted very weakly to	2980
	ABSORPTION BAND, but weaker than former, from	**2974 to 2854**
	Rays transmitted very weakly to	2572
1	Continuous, strong to	3393
	*Feeble, but not continuous to	3123
	Continuous, strong to	2936
	*Feeble to	2880
	Continuous, weak to	2469

QUININE SULPHATE. From Mr. DAVID HOWARD. Plate 54, fig. 1.

0·02 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8, and 5 cub. centims. of double normal sulphuric acid.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	3746
	ABSORPTION BAND from	**3746 to 2865**
	Rays transmitted to	2747
	Spectrum ends at	2660
4	Continuous to	3746
	ABSORPTION BAND from	**3746 to 2875**
	Spectrum ends at	2660
3	Continuous to	3691
	ABSORPTION BAND from	**3691 to 3028**
	Spectrum ends at	2644
2	Continuous, strong to	3606
	ABSORPTION BAND from	**3606 to 3466**
	Rays transmitted to	2632
1	Continuous, strong to	3606
	Weak, scarcely continuous to	3465
	This may be called a faint absorption band.	
	Continuous, strong to	2632
	Spectrum ends at	2515

* At these points there are traces of absorption bands. They may be said to be still there, but feeble.

CINCHONINE. From Mr. DAVID HOWARD. Plate 54, fig. 2.

0·02 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8, and 5 cub. centims. of double normal sulphuric acid.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	3465
	ABSORPTION BAND from	3465 to 2735·5
	Rays transmitted to	2569
4	Continuous to	3356
	ABSORPTION BAND from	3356 to 2834
	Rays transmitted to	2569
	Spectrum ends at	2544
3	Continuous to	3356
	ABSORPTION BAND from	3356 to 2834
	Spectrum ends at	2547
2	Continuous to	3260
	ABSORPTION BAND from	3260 to 2860
	Spectrum ends at	2467
1	Continuous, strong to	3260
	ABSORPTION BAND from	3260 to 3022
	Spectrum ends at	2467

QUINIDINE SULPHATE. From Mr. DAVID HOWARD. Plate 54, fig. 3.

0·02 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8, with 5 cub. centims. of double normal sulphuric acid added.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	3737
	ABSORPTION BAND from	3737 to 2867
	Spectrum ends at	2660
4	Continuous to	3690
	ABSORPTION BAND from	3690 to 2875
	Spectrum ends at	..
3	Continuous to	3691
	ABSORPTION BAND from	3691 to 2891
	Spectrum ends at	2644
2	Continuous to	3178
	ABSORPTION BAND from	3178 to 3022
	Spectrum ends at	2644
1	Continuous, strong to	3606
	Continuous, weak to	3461
	Continuous, strong to	2628
	Spectrum ends at	2610

CINCHONIDINE SULPHATE. From Mr. DAVID HOWARD. Plate 54, fig. 4.

0·02 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8, with 5 cub. centims. of double normal sulphuric acid added.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	3364
	ABSORPTION BAND from	**3364** to **2860**
	Spectrum ends at	2614
4	Continuous to	3364
	ABSORPTION BAND from	**3364** to **2860**
	Spectrum ends at	2614
3	Continuous to	3348
	ABSORPTION BAND from	**3348** to **2860**
	Spectrum ends at	2614
2	Continuous to	3258
	ABSORPTION BAND from	**3258** to **2864**
	Spectrum ends at	2569
1	Continuous, strong to	3258
	Absorption incomplete from	**3258** to **2907**
	Spectrum ends at	2572

MORPHIA. From Messrs. T. and H. SMITH and Co. Plate 54, fig. 5.

0·1 grm. in 25 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	3008
4	Strong, continuous to	3008
3	Continuous to	2980
2	Strong, continuous to	3008
	Rather weak to	2980
	ABSORPTION BAND	**2980** to **2660**
	Rays transmitted faintly to	2572
1	Continuous to	2957
	ABSORPTION BAND	**2956** to **2747**
	Rays transmitted to	2572

0·1 grm. in 10 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	2980
	Absorption band	**2980 to 2747**
	Rays transmitted weakly to end at	2572
4	Continuous, strong to	2949
	Absorption band	**2949 to 2838**
	Spectrum ends at	2572
3	Continuous to	2980
	Absorption band	**2949 to 2870**
	Spectrum ends at	2547
1	Continuous, but rather weak	2504
	Rays transmitted feebly	2316
	Spectrum ends at	2316

Morphine. From Macfarlan and Co., Edinburgh. Plate 54, fig. 6.

0·1 grm. in 25 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous	3008
4	Same as five.	
3	Continuous to	3008
	Absorption band	**3008 to 2613**
2	Continuous to	2980
	Absorption band	**2980 to 2613**
	Rays feebly transmitted to	2570
1	Continuous to	3008
	Absorption band from	**2980 to 2836**
	Rays feebly transmitted to	2570

Solution of 0·1 in 25 cub. centims. of alcohol of sp. gr. 0·8, diluted to four times its volume, equivalent to 1 in 1000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
4	Continuous to	2980
	Absorption band from	**2980 to 2836**
	Rays feebly transmitted to	2570
3	Continuous to	2980
	Absorption band	**2980 to 2857**
	Rays transmitted feebly at	2570
2	Continuous to	2980
	Absorption band	**2980 to 2880**
	Rays transmitted to	2542
1	Continuous to	2980
	Very feebly continuous to	2542

TETRACETYL-MORPHINE. From Dr. C. R. A. WRIGHT. Plate 55, fig. 1.

0·1 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	2949
4	Continuous to	2949
3	Continuous to	2926
	ABSORPTION BAND from	**2926 to 2572**
	Rays transmitted very faintly	2572
2	Continuous to	2902
	ABSORPTION BAND from	**2902 to 2611**
	Rays transmitted faintly to	2547
1	Continuous to	2864
	ABSORPTION BAND from	**2864 to 2745**
	Rays transmitted to	2544

Should there appear to be any discrepancy between the measurements of absorption bands seen in the spectra of strong and weak solutions of the same substance which might be expected to give the same readings on the scale, they may be accounted for by the fact that two series of photographs were taken by different persons and the measurements made and recorded by different observers, the later observations being made after the lapse of several months. In no case have the differences between any two sets of measurements been sufficiently great to affect the curves.

0·2 grm. in 200 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	2965
	ABSORPTION BAND	**2964 to 2601**
	Spectrum ends at	2556·5
4	Continuous to	2821·5
	ABSORPTION BAND	**2821 to 2735**
	Spectrum ends at	2535·5
3	Rays continuous to	2821·5
	Rays transmitted feebly to	2593
	A little stronger to	2556·5
	Very feebly transmitted to where spectrum ends at	2482
2	Spectrum ends at	2387
1	Spectrum ends at	2302·3

CODEINE. From Dr. C. R. A. WRIGHT. Plate 55, fig. 2.

0·1 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave lengths.
millims.		
5	Continuous to	3008
4	Continuous to	2980
3	Continuous to	2980
2	Continuous, strong to	2980
	ABSORPTION BAND from	**2977 to 2613**
	Rays transmitted at	2613
1	Continuous, strong to	2977
	ABSORPTION BAND from	**2977 to 2747**
	Rays transmitted to	2572

0·02 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	2965
4	Continuous to	2965
	ABSORPTION BAND	**2965 to 2735**
	Spectrum ends at	2556
3	Continuous to	2942
	ABSORPTION BAND	**2942 to 2798**
	Spectrum ends at	2556
2	Continuous to	2942
	ABSORPTION BAND	**2942 to 2863**
	Rays transmitted to	2556
1	Spectrum strong to	2942
	Rays transmitted are weak to end at	2556

CODEINE. From Messrs. T. and H. SMITH and Co. Plate 55, fig. 3.

0·2 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Strong, continuous to	3008
4	Strong, continuous to	3008
	Very faint to	2980
3	Strong, continuous to	3008
	Faint to	2980
2	Strong, continuous to	2980
	ABSORPTION BAND	**2964 to 2660**
	Rays transmitted faintly to	2613
1	Continuous, strong to	2980
	ABSORPTION BAND	**2747 to 2572**
		transmitted
	Continuous, strong to	2980
	ABSORPTION BAND	**2962 to 2747**
	Rays transmitted to	2572

The original solution diluted to 5 times its original volume=1 in 1000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	2976
	Absorption band	**2962 to 2747**
	Rays transmitted to	2613
4	Continuous to	2976
	Absorption band	**2976 to 2747**
	Rays transmitted to	2570
3	Continuous to	2976
	Absorption band	**2976 to 2747**
	Rays transmitted to	2570
2	Continuous to	2976
	Absorption band	**2976 to 2740**
	Rays faintly transmitted to	2740
	Very faint at	2568
1	Continuous to	2875
	Spectrum ends at	2542

Diacetyl-Codeïne. From Dr. C. R. A. Wright. Plate 55, fig. 4.

0·2 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5–4–3	Continuous, strong in all three cases to	2976
2	Continuous to	2947
	Absorption band	**2947 to 2613**
	Rays transmitted to	2572

The original solution was diluted to 5 times its original volume=1 in 1000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	2976
	Absorption band	**2976 to 2609**
	Rays transmitted to	2568
4	Continuous, strong to	2942
	Absorption band	**2942 to 2740**
	Rays transmitted to	2568
3	Continuous, strong to	2942
	Absorption band	**2942 to 2740**
	Rays transmitted to	2556
2	Continuous to	2556
	Rays transmitted at	2542
1	The same to	2542
	Rays transmitted feebly to	2210

THEBAINE. From T. and H. SMITH and Co. Plate 55, fig. 5.

0·1 grm. in 25 cub. centims. of alcohol of sp. gr. 0·8 or 1 in 250.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	3250
4	Continuous, strong to	3250
	Faint to	3230
	Very faint to	3174
3	Continuous, strong to	3250
	Faint to	3174
2	Continuous, strong to	3250
	Weak to	3174
1	Continuous, strong to	3250
	Rather weak to	3174
	Faint to	2265*

A diluted solution of this showed an abnormal absorption at one end of the spectrum, *i.e.*, 5 millims. dilute should be equivalent to 1 millim. undilute, dilution being five times; probably due to a change in the solution from being kept.

0·1 in 20 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	3238
	Feebly continuous to	3215·5
4	Continuous, strong to	3238
	Feebly continuous to	3215·5
3	Continuous to	3171
2	Continuous to	3171
1	Continuous to	3171

0·1 grm. 20 cub. centims. of alcohol of sp. gr. 0·8, diluted 2 cub. centims. to 10 cub. centims. =1 in 1000.

Thickness of layer of liquid.	Description of spectrum.	Wave lengths.
millims.		
5	Continuous, strong to	3238
	Weak, continuous to	3171
4	Continuous to	3140
3	Continuous to	3123·5
2	Continuous to	3123·5
	ABSORPTION BAND from	**3171 to 2568**
1	Continuous to	3171
	ABSORPTION BAND from	**3171 to 2568**
	Rays transmitted to	2568

* It is remarkable how the spectrum suddenly lengthens at this point.

0·1 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8, diluted 2 cub. centims. to 50 cub. centims. =1 in 5000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	3123·5
	Absorption band from	3171 to 2568
	Rays transmitted faintly to	2490·5
4	Continuous to	2976·5
	Absorption band from	2976·5 to 2740
	Rays transmitted to	2469·5
3	Continuous to	2976·5
	Absorption band	2976·5 to 2863
	Rays transmitted to	2418
2	Continuous to	2863
	Fairly strong but *not continuous*	2863 to 2653
	Continuous to	2472·5
1	Continuous to	2466

Papaverine. From T. and H. Smith and Co.

0·1 grm. in 40 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	3403
4	Continuous, strong to	3403
3	Continuous, strong to	3403
	Continuous, weak to	3352
2	Continuous, strong to	3352
1	Continuous, strong to	3352
	Faint to	3330

0·1 grm. in 40 cub. centims. of alcohol of sp. gr. 0·8, diluted 2 cub. centims. to 10 cub. centims.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	3982
	Faint to	3468
4	Continuous, faint to	3352
3	Continuous to	3352
2	Continuous to	3330
1	Continuous to	2980

0·1 grm. in 40 cub. centims. of alcohol of sp. gr. 0·8.

This solution was kept for some time previous to being photographed.

Thickness of layer of liquid.	Description of Spectrum.	Wave-lengths.
mil'lms.		
5	Very faint to	4237
4	Faint to	4237
3	Weak, continuous to	4237
	Spectrum extends to	3982
2	Continuous to	3982
	Faint to	3600
1	Continuous to	3456

0·1 grm. in 40 cub. centims. of alcohol of sp. gr. 0·8, diluted 2 cub. centims. to 10 cub. centims. or 1 in 2000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	3456
4	Continuous to	3340
3	Continuous to	3324
2	Continuous to	3002
1	Continuous to	2740

0·1 grm. in 40 cub. centims. of alcohol of sp. gr. 0·8, diluted to 1 in 2000, again diluted 2 cub. centims. to 10 cub. centims. or 1 in 10,000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	2740
4	Weak, continuous to	2568
3	Same as last spectrum.	
2	Rays transmitted to	2568
1	Continuous to	2545
	Absorption band, from	**2545 to 2343**
	Rays transmitted faintly to	2265

0·1 grm. in 40 cub. centims. of alcohol of sp. gr. 0·8, third dilution or 1 in 50,000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to termination of spectrum	2572
4	Continuous to	2545
	Ill-defined ABSORPTION BAND from	2545 to 2313
	Rays transmitted feebly to	2265
3	Very strong, continuous to	2862
	Continuous, and fairly strong to	2572
	Ill-defined ABSORPTION BAND from	2545 to 2313
	Rays transmitted to	2265
2	The same description and measurement are applicable, the lines being a little stronger.	
1	Continuous to	2572
	ABSORPTION BAND visible	**2545 to 2313**
	But lines are transmitted very feebly at	2495, 2483, 2429, 2422, 2355
	Spectrum ends at	2255

NARCOTINE. From Messrs. T. and H. SMITH and Co. Plate 55, fig. 6.

0·2 grm. dissolved in 20 cub. centims. of alcohol of sp. gr. 0·8 = 1 in 200.

Thickness of layer of liquid.	Description of spectrum.	Wave lengths.
millims.		
5	Strong, continuous to	3464
4	Strong, continuous to	3403
3	Strong, continuous to	3403
2	Strong, continuous to	3352
1	Strong, continuous to	3352

0·2 dissolved in 200 cub. centims. of alcohol of sp. gr. 0·8 = 1 in 1000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Strong, continuous to	3352
	Somewhat doubtful trace.	
4	Strong, continuous to	3352
	ABSORPTION BAND from	**3330 to 2707**
	Rays transmitted to	2705
3	Strong, continuous to	3330
	ABSORPTION BAND from	**3262 to 2839**
	Rays weakly transmitted to	2572
2	Strong, continuous to	3330
	ABSORPTION BAND from	**3262 to 2839**
	Rays transmitted to	2572
1	Strong, continuous to	3260
	Spectrum continuously transmitted to	2545

0·2 dissolved in 200 cub. centims. of alcohol of sp. gr. 0·8, 2 cub. centims. diluted to 10 = 1 in 5000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Strong, continuous to	3245·5
	Weakening of the spectrum between	3245·5 and 2863
	Continuous to	2482·5
4	Continuous to	2418
3	Continuous to	2418
2	Continuous to	2310
1	Continuous to	2259

NARCEINE. From T. and H. SMITH and Co.

0·1 grm. in 25 cub. centims. or 20 cub. centims. of alcohol of sp. gr. 0·8, with 5 cub. centims. of glacial acetic acid added thereto.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	3572
	Weak to	3465
4	Continuous, strong to	3465
	Weak to	3330
3	Continuous to	3330
	Faint to	3276
2	Continuous to	3260
1	Continuous, strong to 119	3260

0·1 grm. in 25 cub. centims. of alcohol of sp. gr. 0·8. Solution kept some time.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Faint, continuous to	4237
	Feebly continuous to	4041
4	Weak, continuous to	4041
	Faint, continuous to	3982
3	Same as last.	
2	Fairly strong, continuous to	3982
	Feebly continuous to	3600
1	Fairly strong, continuous to	3982
	Faint to	3456

2 cub. centims. of 0·1 grm. in 25 cub. centims. of alcohol of sp. gr. 0·8, diluted to 8 cub. centims. = 1 in 1000

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
4	Continuous to	3245·5
3	Same as the last spectrum.	
2	Continuous to	3171
1	Continuous to	3008

2 cub. centims. of 0·1 grm. in 25 cub. centims. of alcohol of sp. gr. 0·8, diluted to 80 cub. centims. = 1 in 10,000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
10	Continuous to	2740
9	Same as last spectrum.	
8	Same as last spectrum.	
7	Same as last spectrum, but slightly stronger.	
6	Same as last spectrum.	
5	Same as last spectrum.	
4	Same as last, stronger.	
3	Continuous to	2418
2	Continuous to	2418
1	Continuous to	2310

COTARNINE HYDROBROMIDE. From Dr. C. R. A. WRIGHT.

The original solution $\frac{1}{100}$ diluted to five times its volume, or 1 in 1000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	4143
	ABSORPTION BAND	**4061** to **2747**
4	Continuous to	4061
	ABSORPTION BAND	**4061** to **2747**
3	The spectrum is substantially the same as that transmitted by 4 millims.	
2	Continuous to	3982
	ABSORPTION BAND	**3982** to **2747**
	Spectrum extends to	2696
1	Continuous as above to	3982
	ABSORPTION BAND	**3982** to **2872**
	Rays transmitted weakly to	2696

1 in 10,000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
10	Continuous to	3917
	ABSORPTION BAND	**3982 to 2747**
	The absorption is weakening and rays appear feebly, though in reality strong lines, as far as	3800 3739 3682
	Rays very feebly transmitted to	2863
	Continuous, but weak to	2747
9	Continuous to	3600
	ABSORPTION BAND	**3600 to 3033**
	Rays feebly continuous to	2653
8	Continuous to	3600
	ABSORPTION BAND	**3600 to 3033**
	Rays continuous but feeble to	2653
7	The description of the spectrum transmitted by 8 millims. is the same, with the exception of a trace of a line at	2310
6	Spectrum precisely the same as 7 millims.	
5	Spectrum precisely the same as the foregoing as far as	2653
	Apparently an ABSORPTION BAND	**2740 to 2310**
	Rays feebly transmitted to	2310
4	Precisely the same as above, but in addition to the rays transmitted weakly to	2310
	There are others extending to	2255
3	Continuous	3600
	ABSORPTION BAND	**3600 to 3033**
	This band is greatly weakened, so that rays are faintly transmitted to	3033
	Continuous to	2740
	ABSORPTION BAND	**2740 to 2310**
	Rays transmitted, the absorption being weakened between 2740 and	2568
	Also from 2430 to	2255
2	Continuous to	2863
	Continuous to	2740
	ABSORPTION BAND, diminished in intensity	**2740 to 2310**
	Rays transmitted weakly to	2568
	Very feebly to	2255

APOMORPHINE HYDROCHLORIDE. From T. and H. SMITH and Co. Plate 56, fig. 1.

0·1 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	3465
4	Continuous, strong to	3465
3	Continuous, strong to	3465
	Faint to	3434
2	Continuous, strong to	3465
	Faint to	3403
1	Continuous to	3330

0·1 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8 or 1 in 200. Fresh solution.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Weak, continuous to	3456
4	Weak, continuous to	3456
3	Weak, continuous to	3438
2	Weak, continuous to	3403
1	Fairly strong, continuous to	3982
	Weak, continuous to	3403

0·1 grm. in 20 cub. centims. 2 cub. centims. diluted to 10 cub. centims =1 in 1000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	3403
4	Continuous to	3403
3	Continuous to	3403
	Faint to	3331·5
2	Continuous to	3261
1	Continuous to	3261
	Feeble lines at	3072

2 cub. centims. of the last solution diluted to 10 cub. centims. =1 in 5000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	3261
4	Same as last, but a little stronger	
3	Continuous to	3245·5
2	Continuous to	3245·5
	ABSORPTION BAND from	**2976·5 to 2568**
	Trace of a line at	2479·5

Three solutions of oxynarcotine were examined, this solution, which was kept for some time after the notes written below had been taken, yielded a spectrum from which the curve was drawn. It was found however that the liquid had undergone great change, so a second solution was made which yielded the dotted curve when quite fresh.

OXYNARCOTINE. From Dr. C. R. A. WRIGHT. Plate 56, fig. 2.

0·1 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8 and 5 cub. centims. of acetic acid.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	3465
4	Continuous to	3348
3	Continuous to	3332
2	Continuous to	3268
1	Continuous to	3186

Another solution of the same specimen, examined immediately after its preparation.

0·1 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8 and 5 cub. centims. of acetic acid.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	3456
	Two weak lines at	
4	Continuous to	3324
3	Continuous	3245·5
2	Continuous to	3245·5
1	Continuous to	3245·5

2 cub. centims. diluted to 10 cub. centims. =1 in 1000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
4	Continuous, strong to	3245·5
3	Continuous, strong to	3245·5
2	Continuous to	3171
1	Continuous to	3123·5

2 cub. centims. of the last solution diluted to 10 cub. centims. =1 in 5000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	3033
	AN ABSORPTION BAND from	**3008 to 2568**
	Rays feebly transmitted at the following points	2976·5
		2857·5
		2836·5
		2545
4	Continuous to	3033
	ABSORPTION BAND from	**3008 to 2568**
	Rays transmitted at the following points (spectrum not continuous)	2976·5
		2857·5
		2836·5
		2744·5
		2705
		2597
		2545
		2497
		2482·5
		2430·7
		2421
3	Continuous to	3033
	Two strong lines at	
	Spectrum extends to	2421
2	Same as last spectrum. The lines being a little stronger.	
1	Continuous to	2863
	Spectrum extends to (not continuous)	2310

2 cub. centims. of the last solution diluted to 10 cub. centims. =1 in 25,000.

Thickness of layer of liquid.	Description of spectrum.	Wave lengths.
millims.		
5	Continuous	2482·5
	Spectrum extends to (not continuous)	2310
4	Continuous	2418
	Spectrum extends to (not continuous)	2259
3	Same as last spectrum.	
2	Continuous	2310
	Spectrum ends at	2259
1	Same as last.	

The selective absorption in this substance is very slight, beginning at a thickness of 5 millims. and ending at the next thickness of 4 millims. in a solution of 1 grm. in 5000 cub. centims. (alcohol).

This solution gives a blue fluorescence similar to that given by pseudaconitine and apomorphine-hydrochloride.

A solution which had been kept for about six months and had evidently become altered.

0·1 grm. in 25 cub. centims. of alcohol of sp. gr. 0·8, and 5 cub. centims. of acetic acid or 1 in 250.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Faint to	4376
4	Feebly continuous to	4340
3	Feebly continuous to	4340
2	Feebly continuous to	4340
1	Weak, continuous to	4255

This substance gave a blue fluorescence similar to pseudaconitine and apomorphine-hydrochloride.

2 cub. centims. of the last solution diluted to 8 cub. centims. =1 in 1000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
4	Weak, continuous to	3994
3	Continuous to	3994
2	Continuous to	3825
	To wave-length 3600 the spectrum is discontinuous.	
1·8	Continuous to	3825
	To wave-length 3600 the spectrum is feeble and discontinuous	
1·6	Continuous to	3825
1·4	Continuous to	3825
	Absorption band from	3456 to 2740
	Rays transmitted to	2740
1·2	Continuous to	3825
	Rays are transmitted as far as wave-length 3456, but the spectrum is not continuous.	
	Absorption band from	3456 to 2740
	Rays transmitted to	2740
1	Continuous to	3600
	Weak, but not continuous, to	2709·5

2 cub. centims. diluted to 20 cub. centims. =1 in 10,000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
10	Continuous to	3610
	Very feebly continuous from	3600 to 3456
	Weakening of the spectrum between	3171 and 2863
9	Same as last, but stronger.	
8	Same as last, with this exception.	
7	Same as last.	
6	Strong, continuous	3982
	Weak, continuous to	3600
	Rays transmitted to	2568
5	Strong, continuous to	3245·5
	Weak, continuous.	
	Faint, continuous to	2568
	Not continuous, but the lines are strongly marked to	2545
4	Continuous to	3245·5
	Spectrum extends to	2421
3	This spectrum is continuous to	2310
2	Continuous spectrum to	2259
1	Same as last spectrum, but slightly stronger all through.	

VERATRINE. From Messrs. T. and H. SMITH and Co. Plate 56, fig. 4.

0·1 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	3145
4	Continuous to	3132
3	Continuous to	3111
2	Continuous to	3034
	ABSORPTION BAND	**3034 to 2848**
	Rays weakly transmitted to	2747
1	Continuous to	3016
	ABSORPTION BAND	**3016 to 2860**
	Rays transmitted to	2747
	Rays weakly transmitted to end of spectrum at	2707

0·1 grm. in 100 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	3008
	Absorption band	3008 to 2847
	Rays transmitted faint to	2740
4	Continuous to	3033
	Absorption band	2976·5 to 2869
	Rays transmitted to	2705
3	Continuous to	3033
	Rays transmitted feebly and continuously to	2869
2	Continuous to	3171
	Weak to	2418
1	Continuous to	2307·5

Cevadine. From Dr. C. R. A. Wright (Merck's Veratrine).

0·1 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
	A thickness of liquid equal to 20 millims. transmits all rays strongly to	2735
5	Transmits all to	2556
1	Transmits all to	2518
	No absorption band visible.	

Brucine. Plate 56, fig. 5.

0·1 grm. in 25 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	3246
4	Continuous, strong to	3246
3	Continuous, strong to	3246
2	Continuous, strong to	3246
1	Continuous, strong to	3246

0·1 grm. in 25 cub. centims. 2 cub. centims. diluted to 8 cub. centims. =1 in 1000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	3208
4	Continuous to	3171
3	Continuous to	3171
2	Continuous to	3171
1	Continuous to	3171
	Absorption band from	**3171 to 2857**
	Rays transmitted to	2831·5
	Absorption band from	**2831·5 to 2427·5**
	Rays transmitted to	2418

2 cub. centims. of the last solution diluted to 10 cub. centims. =1 in 5000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	3171
	Absorption band from	**3171 to 2857**
	Rays transmitted to	2793
	Absorption band from	**2793 to 2424·3**
	Rays transmitted to	2418
4	Continuous, strong to	3171
	Feeble absorption band	**3171 to 2869**
	Rays are transmitted feebly all through this band, and extend stronger from 2869 to	2740
	Absorption band from	**2740 to 2568**
	Rays transmitted to	2310
3	Continuous to	2310
2	Continuous to	2836·5
	Weak to	2310
	Faint to	2259
1	Continuous to	2259

Strychnine. Plate 56, fig. 6.

0·1 grm. in 25 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	2976
4	Continuous, strong to	2976
3	Continuous, strong to	2976
2	Continuous, strong to	2976
	Very faint to	2946
1	Continuous, strong to	2976
	Weak to	2946

0·1 grm. in 50 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Strong, continuous to	2976·5
4	Strong, continuous to	2976·5
3	Strong, continuous to	2976·5
2	Strong, continuous to	2976·5
1	Strong, continuous to	2976·5
	Faint, continuous to	2948

2 cub. centims. of the last solution diluted to 10 cub. centims. =1 in 2500.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	2948
	Weak lines at	2863
	And	2836·5
	Trace of a line at	2740
4	Continuous to	2948
	Faint to	2740
3	Continuous to	2948
	Weak to	2740
	Faint line at	2701
	ABSORPTION BAND	**2740 to 2310**
	Rays transmitted at	2310
2	Continuous to	2740
	ABSORPTION BAND	**2740 to 2310**
	Rays transmitted from	2740 to 2701
	Also from (very faint)	**2352 to 2259**
1	Continuous to	2740
	ABSORPTION BAND	**2568 to 2427·5**
	Rays transmitted to	2259

2 cub. centims. of the last solution diluted to 10 cub. centims. =1 in 12,500.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous to	2740
	ABSORPTION BAND	**2568 to 2427·5**
	Rays transmitted to	2259
	Two very faint lines at	2493·5 2482·5
4	Continuous to	2259
3	Continuous to	2259
2	Continuous to	2259
1	Continuous to	2193

PIPERINE. From T. and H. SMITH and Co. Plate 56, fig. 3.

0·1 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8. (1 in 200 cub. centims.)

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, fairly strong to	4061
4	Continuous, fairly strong to	4061
	Continuous, fairly weak to	4028
3	Continuous, strong to	4028
	Continuous, faint to	4000
2	Continuous, strong to	4000
1	Continuous, strong to	4000
	Continuous, weak to	3952

This solution was kept twelve months before the next series of photographs was taken.

The following measurements are the more exact, though there is but little difference between them.

0·1 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	3982
4	Continuous, strong to	3982
3	Continuous, strong to	3982
	Faint, continuous to	3955
2	Continuous, strong to	3982
	Faint, continuous to	3955
1	Same as last.	

2 cub. centims. of the last solution diluted to 10 cub. centims. =1 in 1000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	3982
	Faint, continuous to	3917
4	Continuous, strong to	3891
	Faint, continuous to	3825
3	Continuous, strong to	3891
	Faint, continuous to	3825
	Faint line at	3788
2	Continuous, strong to	3891
	Weak, continuous to	3825
	Weak line at	3788
1	Continuous, strong to	3825
	Weak to	3742
	Faint line at	3673

2 cub. centims. of the last solution diluted to 10 cub. centims. =1 in 5000.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	3825
	Weak to	3742
	Weak line at	3673
4	Continuous, strong to	3825
	Weak, continuous to	3706
	Faint to	3600
3	Continuous to	3600
	Absorption band from	**3600 to 2740**
2	Continuous to	3600
	Absorption band from	**3600 to 2740**
	Rays transmitted from	3600 to 3456
	Also from	2813 to 2568
	Absorption band from	**2568 to 2310**
	With two lines faintly transmitted at	2427·5 2418
1	Continuous spectrum with indications of the absorption bands to	2201

Atropine. From T. and H. Smith and Co.

0·1 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
20	Continuous, strong to	2707
5	Continuous, strong to	2707
	Very faint as far as	2429
4	Continuous, strong to	2644
	Weak to	2573
	Very weak to	2429
3	Continuous, strong to	2573
	Rather weak to	2429
	Very faint extension to	2398
2	Continuous, strong to	2398
	Very faint to about	2355
1	Continuous, strong to	2398
	Weak to	2355
	Very faint to	2340

SOLANINE. From T. and H. SMITH and Co.

0·1 grm. in 25 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Strong, continuous to	2570
	Weak to	2265
4	Nothing characteristic	
3	Nothing characteristic	
2	Nothing characteristic	
1	Strong, continuous to	2265

A thickness of liquid as great as 20 millims. transmits all rays without selective absorption as far as 249, a faint appearance of a line at 256 and at 276 and at 293 are just sufficiently visible to be recorded.

HYOSCYAMINE. From T. and H. SMITH and Co.

0·1 grm. in 20 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	2747
	Continuous, faint to	2572
4	Continuous, strong to	2747
	Continuous, faint to	2572
	Continuous, very faint to	2436
3	Continuous, fairly strong to	2613
	Continuous, weak to	2572
	Continuous, faint to	2430
2	Continuous, strong to	2572
	Continuous, weak to	2430
	Continuous, faint to	2422

A thickness of 20 millims. absorbs no less refrangible ray than 206·6=2747. No band distinctly visible.

DIGITALINE. From T. and H. SMITH and Co.

0·1 grm. in 25 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	3610
	Very faint to	3465
	Exceedingly faint to.	3033
4	Continuous, strong to	3610
	Weak to.	3465
	Faint to.	2572
3	Continuous, strong to	3465
	Weak to.	2980
	Faint to.	2572
2	Continuous, fairly strong to	2837
	Fairly strong to	2572
	Faint to.	2422
1	Continuous, strong, but not of normal strength to	2545
	Weak to.	2313

Thicker layers of liquid, namely, 10 millims., 15 millims., and 20 millims., were examined, but the absorption was continuous and increased down to transmission of rays at 75.

PICROTOXINE. From T. and H. SMITH and Co.

0·1 in 20 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Continuous, strong to	2398
	Very weak to	2261
4	The same increasing in strength.	
3	The same.	
2	Continuous and fairly strong	2261
1	Continuous and strong	2261
20	Continuous, strong to	2521
10	Continuous, strong to	2500
	Weak to.	2418

This substance shows no absorption band.

NICOTINE. From T. and H. SMITH and Co.

0·367 in 36·7 cub. centims. of alcohol of sp. gr. 0·8.

This substance exhibits no absorption band.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
20	Continuous, strong to	
5	Continuous, strong to	3132
	Weak to	3008
4	Continuous, strong to	3033
	Continuous, weak to	2980
3	Continuous, strong to	3033
	Continuous, weak to	2977
2	Continuous, strong to	2977
	Continuous, weak to	2947
	Continuous, very faint to	2834
1	Continuous, strong to	2875
	Continuous, weak to	2801

CAFFÉINE. From T. and H. Smith and Co.

0·1 in 40 cub. centims. of alcohol of sp. gr. 0·8.

Thickness of layer of liquid.	Description of spectrum.	Wave-lengths.
millims.		
5	Transmits all rays to	2965
1	All to.	2942
	No absorption band visible.	

Having recently examined the spectra transmitted by pyridine, piperidine, quinoline, tetrahydroquinoline, and the hydrochlorides of these bases, it has been found that the absorption band of pyridine does not appear in the spectrum transmitted by piperidine, and that quinoline and tetra-hydroquinoline exhibit selective absorption very strongly, the curves differing from each other and from those of their hydrochlorides. Observations made on simple bases differ from those made on substitution products in this way that the bases are the more diactinic, while addition products are more diactinic than the bases.

SUMMARY.

Many alkaloids are capable of saponification, the products of the reaction are a new base and an organic acid which is frequently of the aromatic series.

The acid is not invariably of the aromatic series, otherwise its presence would

probably be evident in the absorption spectrum of the alkaloids. There are, however, two conditions under which an aromatic derivative would certainly not affect the spectrum in a characteristic manner, and these are, when the acid is a hydrogenised body like hydrophthalic acid, or when the basic portion of the molecule has so powerfully absorbed the rays that the band of selective absorption is obscured. The spectrum of narceïne is undoubtedly of this character, and so likewise is that of oxynarcotine. In order to comment upon the nature of the various absorption spectra described, it will be advantageous to refer briefly to what is at present known concerning the constitution of some of the alkaloids.

Alkaloids may be divided into two groups: (*a*) those which exhibit spectra with absorption bands; and (*b*) those which transmit continuous spectra.

ALKALOIDS and derivatives exhibiting spectra with absorption bands.

Aconitine	Papaverine	Cinchonine-sulphate
Pseudaconitine	Oxynarcotine	Quinidine-sulphate
Japaconitine	Apomorphine-hydrochloride	Cinchonidine-sulphate
Morphine	Tetra-acetyl-morphine	Veratrine
Narcotine	Di-acetyl-codeïne	Piperine
Codeïne	Quinine	Brucine
Thebaïne	Quinine-sulphate	Strychnine

ALKALOIDS yielding continuous spectra.

Narceïne	Solamine	Picrotoxine
Aconitine (foreign)	Hyoscyamine	Nicotine
Cevadine	Digitaline	Caffeïne
Atropine		

The properties of the aconitines.

By saponifying the aconitines with acids, alkalies, or even with water at a high temperature under pressure, aconitine, japaconitine, and picraconitine yield benzoic acid and a new base, while pseudaconitine furnishes di-methyl-protocatechuic acid $C_6H_3\begin{cases}(OCH_3)_2\\COOH\end{cases}$. It was thought probable that by the action of bromine on the alkaloids substitution would occur in the benzene nucleus, and a definite absorption of a pronounced character would result. This was however not the case, probably because the alkaloids were altogether altered, for after treatment they exhibited no absorption bands. In comparing the diagrams it is interesting to note the difference between pseudaconitine and the other aconitine bases, it evidently possesses a nucleus with a similar constitution, but the acid residue which is different modifies the absorption curve.

It has been shown by the researches of Dr. C. R. A. WRIGHT that the aconitines all yield apo-derivatives with such facility that to obtain the bases free therefrom is not an easy matter, and further they undergo saponification by acids and alkalies, very readily yielding an inert base in each case. Hence different preparations yield different absorption curves. The aconitine and japaconitine of Dr. WRIGHT have practically the same absorption spectrum and yield similar curves, but that of japaconitine is just what we might expect from a body with a nucleus of a similar constitution but twice the molecular weight of aconitine, namely, a much greater absorptive power. It has been shown that japaconitine has such a difference, for it is in fact a sesqui-apo-aconitine.*

The English aconitine of Messrs. T. and H. SMITH and Co. appears to be a modified form of japaconitine, and it is possible that the modification of the spectrum curve is in part due to an admixture of another body.

Foreign aconitine is doubtless an entirely different substance, since it yields no absorption bands, and it is known to be comparatively inactive physiologically.

On two separate occasions it was found necessary to estimate the strength of solutions of aconitine, and this was in each case successfully accomplished by taking a series of photographs and constructing a curve therefrom. That the results so obtained were correct was proved by a discovery of some missing notes giving particulars of the preparation of the solutions.

The constitution of the cinchona bases.

The examination of these bases was made by me five years ago, and although there were no means of obtaining accurate measurements of wave-lengths of the absorbed rays at that time, owing to the requisite data being wanting, yet some idea of the constitution of quinine and cinchonine was derived from a comparison of the absorption bands seen in the spectra of the substances themselves and of their derivatives and decomposition products.

The bands in the case of quinine and cinchonine extend into a region of less refrangibility than the absorption caused by pyridine, at the same time the position of the band agrees with that of quinoline.

In one of a series of papers contributed to the 'Transactions of the Chemical Society,' vol. xli., p. 45 ("Researches on the Relation of the Molecular Structure of Carbon Compounds to their Absorption Spectra."—Part VI.), I remarked that the quinine spectrum was probably due to the conjugation of four pyridine or two quinoline nuclei. Since then, several researches on the cinchona alkaloids have extended our knowledge, and formulæ have been proposed for quinine and for

* Vide the structural formulæ in the papers by WRIGHT and LUFF, and WRIGHT and MENKE, J. Chem. Soc., vol. xxxiii., p. 173; vol. xxxv., p. 404.

cinchonine by WISCHNEGRADSKY, WEIDEL, KŒNIGS, and SKRAUP, in all of which a more or less modified quinoline nucleus plays a part. Spectrum observations cannot at present decide upon the respective merits of their formulæ.

The constitution of the opium bases.

It is easy to see that the opium bases, morphine, codeïne, and their derivatives, apomorphine-hydrochloride, di-acetyl-codeïne, and tetra-acetyl-morphine, all have a similarly constituted nucleus. This nucleus is a benzene or a pyridene derivative, as shown by the absorption band, extending from about wave-length 3000 to 2600; the effect of alkyl and acetyl substitutions upon the curve of absorption is well exemplified by codeïne, and the acetylated derivatives of codeïne and morphine.

In apomorphine-hydrochloride the intensity of the general absorption is greatly increased, but the position of the band is but little altered, as it lies within wave-lengths above mentioned.

In the greatly increased length of the curve of thebaïne, we have evidence which is not in harmony with the comparatively simple formula $C_{19}H_{21}NO_3$. The substances narceïne, narcotine, and oxynarcotine are all considered to be derivatives of hemipinic acid (WRIGHT and BECKETT, J. Chem. Soc., vol. xxviii., p. 629, and vol. xxix., p. 461), the relationship between the substances being expressed by the following formulæ :—

Hemipinic acid.	Narceïne.	Narcotine.	Oxynarcotine.
$C_6H_2\begin{cases}CO.OH\\CO.OH\\O.CH_3\\O.CH_3\end{cases}$	$(C_{13}H_{20}NO_4)$	$(C_{11}H_{11}(CH_3)NO_3)$	$(C_{11}H_{11}(CH_3)NO_3)$
	CO	CO	CO
	$C_6H_2\begin{cases}CO.OH\\O.CH_3\\O.CH_3\end{cases}$	$C_6H_2\begin{cases}COH\\O.CH_3\\O.CH_3\end{cases}$	$C_6H_2\begin{cases}CO.OH\\O.CH_3\\O.CH_3\end{cases}$

From these expressions it may be seen that the same benzenoid grouping is common to all, and the narcotine and oxynarcotine stand to each other in the relationship of aldehyde and acid as far as this grouping is concerned.

In attempting to trace a connexion between the absorption spectra and the formulæ given above, one is met by a difficulty caused by the large number of oxygen atoms in the molecules, and particularly in those portions which have hitherto yielded no clue to their constitution. The difficulty is due to the fact that carboxyl and COH groups on side-chains, or as forming a portion of the substituted benzene nuclei exhibit great absorptive power. The occurrence of several oxidised radicals may cause the following variations in spectra : (*a*), the absorption bands become so widened as to extend into the region of rays affected by naphthalene, quinoline, and their derivatives ; (*b*) or the absorption is so powerful that it extends to rays less refrangible than

those in which the band is situated, and continues so far down the curve that the selective absorption is not made manifest. Narcëine appears to be a good example of this; its absorptive power is very great, extending into the region of such low refrangibility as wave-length 3000 when 1 millim. of liquid is examined containing only $\frac{1}{1000}$ of substance, so that no band is visible. In the case of narcotine and oxynarcotine it is difficult to arrive at any conclusion, but, from the extension of their absorption bands, it is probable that the portion of the molecule which has a constitution hitherto undetermined is the principal cause of the absorption. In the case of apomorphine the loss of water appears to cause two molecules of morphine to coalesce, the result being that the band does not suffer displacement to any great extent; it still absorbs rays of approximately the same wave-length, but the band appears far down the curve, or, in other words, the intensity of the absorption, is increased.

The remarks on narcëine are also applicable to papaverine in every particular.

Cevadine.

Cevadine yields on saponification a new base, provisionally called cevine and cevadic or methyl-crotonic acid; its formula therefore as shown by WRIGHT and LUFF (J. Chem. Soc., vol. xxxiii., p. 351) is the following:—

$$(C_{27}H_{41}NO_6)\begin{matrix}OH\\O.CO.C(CH_3)=C_2H_4\end{matrix}$$

Veratrine.

Veratrine $[(C_{28}H_{44}NO_7)-O.CO.C_6H_3(O.CH_3)_2]$ when saponified decomposes into a new base and di-methyl-protocatechuic acid. Its curve greatly resembles that of codëine and morphine with their derivatives (*ibid.*, p. 355), and is distinctly different in character to that of cevadine, as the latter shows no absorption band. At the present time, however, no inference is to be drawn from this, for we have no knowledge of the constitution of the basic portion of the molecule which is the major part in veratrine.

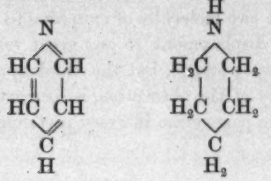
Brucine and Strychnine.

The absorption curves of brucine and strychnine are interesting, the former being essentially different from the latter, as it extends into a much less refrangible portion of the spectrum. It will be observed that the brucine shows two absorption bands, the second, however, is very like that of strychnine, so much so indeed as to cast suspicion upon the purity of the specimen of brucine examined. This, however, cannot be questioned. (See pp. 519–520.)

The constitution of piperine.

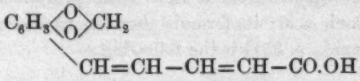
According to researches of A. W. HOFMANN (Berl. Berichte, vol. 12, p. 984) and of KŒNIGS (*idem*, p. 2341) piperine has a constitution somewhat similar to that of atropine, that is to say, it is a derivative of a hydrogenised pyridine and of piperic acid, which is a benzene derivative, thus:—

Pyridine. Piperidine.



Piperidine . . .	$C_5H_{10}.NH$
Piperic acid . .	$C_{12}H_{10}O_4$
Piperine . . .	$C_{17}H_{19}NO_3$

Piperic acid has the structure represented by FITTIG in the following manner:—



Unlike atropine, piperine shows a very powerful absorption and a persistent absorption band extending from wave-length 3600 to 2740.

The constitution of nicotine.

According to WISCHNEGRADSKY (Ber. 13, p. 2315, 1880), nicotine is an alkyl derivative of a di-hydro-pyridine in which the hydrogen united to the nitrogen is replaced by ethyl, and the second hydrogen is replaced by allyl, thus:—$C_5H_4\begin{cases}C_3H_5\\NC_2H_5\end{cases}$.

The spectrum is quite in accordance with such a constitution, as it yields no absorption bands.

The constitution of atropine.

The constitution of atropine is that of tropine and tropic acid, less a molecule of water, thus:—

$C_5H_7(C_2H_4.OH)N.CH_3$	Tropine
$C_6H_5.CH\begin{cases}CH_2.OH\\CO.OH\end{cases}$.	Tropic acid
$C_{17}H_{23}NO_3$	Atropine

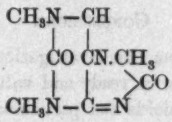
According to LADENBURG tropine is probably an ethylene derivative and a hydro-

genised addition-product of methyl-pyridine. A pyridine nucleus hydrogenised would yield no absorption band and would be very diactinic.

In tropic acid we have a substance capable of manifesting the benzene nucleus by giving a banded spectrum; if however it be modified by hydrogenising, this property disappears. As atropine not only exhibits no absorption band, and is, moreover, a remarkably diactinic substance, it is highly probable that both the pyridine and benzene residues are hydrogenised. (Compare hyoscyamine and piperine.)

Hyoscyamine is isomeric with atropine; on treatment with baryta water it yields tropine and tropic acid. Like atropine it is remarkably diactinic, layers of solutions 20 millims. in thickness, which contained one part of the substance in 200 of alcohol, transmitted all rays in the case of atropine to wave-length 2700, and of hyoscyamine to wave-length 2740.

Caffeine, or *methyltheobromine*, $C_8H_{10}N_4O_2+H_2O$, has been shown to have the constitution of a tri-methyl-xanthine. According to EMIL FISCHER its structure is represented thus :—



There are no reasons for believing that such a body would yield an absorption band, indeed there is much evidence to lead one to think that it would not. We may therefore say that its supposed constitution is in accordance with its absorption spectrum.

Pyridine, quinoline and derivatives.

The absorption band characteristic of pyridine is strongest between wave-lengths 2700 and 2300, with diminished intensity it extends from 2570 to 2400. It appears highly probable that this substance enters into the constitution of morphia and some other of the opium bases.

As was predicted piperidine shows no absorption band, but still, like the terpenes, it possesses considerable absorptive power, which is exerted on the more refrangible rays, extending continuously to wave-length 2420.

The absorption spectra of two specimens of quinoline yield curves which are precisely similar. The band of absorption is strongest between wave-lengths 3170 and 2600, continuing with diminished intensity it lies between 2980 and 2830. There is in addition a remarkable narrow band of absorption which is very well defined, it lies between 3085 and 3039, and continues with but slightly diminished intensity between 3078 and 3039.

Quinoline hydrochloride yields a spectrum differing from that of the base inasmuch

as it exhibits two absorption bands lying between (1) 3320 and 2740, (2) 2418 and 2199.

Tetra-hydro-quinoline is a more diactinic substance than quinoline, it is characterised by two absorption bands—(1) between 3180 and 2750, and continuing to 2870; (2) between 2700 and 2300, extending further to between 2650 and 2370.

Substances, such as any of the natural alkaloids, which may be derived from di-hydroquinoline or tetra-hydroquinoline, by replacement of the hydrogens by other elements or radicals in such a manner as to have the nucleus of the compound unchanged, must be expected to exhibit absorption bands.

Strychnine.—Strychnine appears to be a derivative of pyridine.

Brucine.—The substance brucine is most probably a derivative of tetra-hydroquinoline, or an addition product of quinoline of the same character, since there is a remarkable similarity between the curves of the two substances.

I cannot close this paper without acknowledging indebtedness to the great skill and care that my assistant Mr. W. R. Barnett has bestowed on these later observations.

Conclusions.

The conclusions to be drawn from this investigation are the following:—

1. The absorption spectra offer a ready and valuable means of ascertaining the purity of preparations of the alkaloids and particularly of establishing their identity. The quantity of some of the alkaloids present in a solution may be estimated by means of the absorption curves. The difference in character of the various specimens known as aconitines may be recognised; thus the comparatively harmless base known as foreign aconitine may be distinguished from those of great physiological activity by its transmission of a continuous spectrum, while the three active specimens of aconitine are distinguished from one another by their characteristic absorption curves.

That each of the three active aconitine bases are substances with a different constitution is a conclusion confirmed by optical examination.

The purity of quinine and absence of any admixture of cinchonine can be readily determined, by reason of the latter substance being much less diactinic than the former, but for the same reason quinine cannot be directly estimated in a mixture of the two bodies.

Drugs of such potency as aconitine, morphine, quinine, strychnine, &c., which ought to be prescribed only when of absolute purity, should be submitted to spectroscopic examination, so that their exact nature and degree of purity may be guaranteed.

2. In comparing the spectra of substances of similar constitution it is observed that such as are derived from bases by the substitution of an alkyl radical for hydrogen, or of an acid radical for hydroxyl, the curve is not altered in character, but may vary in length when equal weights are examined.

This is explained by the absorption bands being caused by the compactness of structure of the nucleus of the molecule, and that equal weights are not molecular weights, so that by substituting the hydrogens of the nucleus by radicals which exert no selective absorption, the result is a reduction in the absorptive power of a given weight of the substance. Examples are afforded by morphine and codëine (methyl-morphine), diacetyl-codëine and tetra-acetyl morphine.

3. Bases which contain oxydised radicals, as hydroxyl, methoxyl, and carboxyl, increase in absorptive power in proportion to the amount of oxygen they contain. This is exemplified by papaverine, narcëine, narcotine, and oxynarcotine. The apo-derivatives are less diactinic than the parent bases. Examples are apo-morphine and pseudaconitine.

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X. *Experimental Researches in Magnetism.*

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[Plates 57–68.]

§ 1. The experiments to be described in this paper deal with a number of parts of one general subject—the magnetisation of iron and steel. They relate to—

(1.) The magnetic susceptibility of iron and steel; the form of the curve of magnetisation and magnetising force; and the changes of magnetism caused by gradual reversal and other cyclic changes of magnetising force.

(2.) The influence of vibration on magnetic susceptibility and retentiveness.

(3.) The influence of permanent strain on magnetic susceptibility and retentiveness.

(4.) The energy expended in producing cyclic changes of magnetisation.

(5.) The ratio of residual to induced magnetism.

(6.) The changes of induced and residual magnetism caused by changes of stress.

(7.) The effects of constant stress on magnetic susceptibility and retentiveness.

(8.) The changes of magnetism caused by changes of temperature.

(9.) The effects of temperature on magnetic susceptibility.

In connexion with the above, experiments of a closely related character have been made on the following additional subjects, but an account of them will be reserved for another paper:—

(10.) The changes of thermoelectric quality caused by changes of stress.

(11.) The changes of thermoelectric quality caused by magnetisation.*

* The following previous publications may be referred to as preliminary notices of some of the results:—

(1.) "Effects of Stress on the Thermoelectric Quality of Metals" (Abstract), Proc. Roy. Soc., No. 214, 1881, p. 399.

(2.) "On Effects of Retentiveness in the Magnetisation of Iron and Steel," Proc. Roy. Soc., No. 220, 1882, p. 39.

(3.) "On the Magnetic Susceptibility and Retentiveness of Iron and Steel," Brit. Ass. Rep., 1883, and Phil. Mag., Nov., 1883. And an intimately related investigation is described in a paper "On the Production of Transient Electric Currents in Iron and Steel Conductors by Twisting them when Magnetised, or by Magnetising them when Twisted," Proc. Roy. Soc., No. 228, 1883, p. 117.

The effect of stress on thermoelectric quality was the first of these subjects to engage my attention.* In testing the changes of thermoelectric quality which a stretched iron wire underwent when successively loaded and unloaded so as to suffer alternate application and removal of tensile stress, I found that during increment and decrement of the load equal values of load were associated with widely different values of thermoelectric quality;† the difference being mainly of this character, that the changes of thermoelectric quality lagged behind the changes of stress. This lagging is, however, a *static* phenomenon, for it is sensibly unaffected by the speed at which the load is changed; and again, when any state of load is maintained constant the thermoelectric quality does not change with lapse of time. I then proceeded to search for other instances of the same kind of lagging, in other physical effects of stress, and also in effects caused by the change of other physical conditions besides stress. Magnetic phenomena present many instances of a similar action—some of which will be described below. Thus, when a magnetised piece of iron is alternately subjected to pull and relaxation of pull sufficiently often to make the magnetic changes cyclic, these lag behind the changes of stress in much the same way as the changes of thermoelectric quality do. And more generally, if a piece of iron, whether magnetised or not, be subjected to cyclic stress variations, it can be shown that certain of its physical qualities, while varying cyclically in consequence of the changes of stress, exhibit this same lagging. I found it convenient to have a name for this peculiar action, and accordingly called it Hysterēsis (from ὑστερέω, to lag behind).‡ Thus, when there are two qualities M and N such that cyclic variations of N cause cyclic variations of M, then if the changes of M lag behind those of N, we may say that there is hysteresis in the relation of M to N. The value of M at any point of the operation depends not only on the actual value of N, but on all the preceding changes (and particularly on the immediately preceding changes) of N, and by properly manipulating those changes, any value of M within more or less wide limits may be found associated with a given value of N.

§ 2. The effects of stress on magnetism and on thermoelectric quality will be recurred to at length further on. Meanwhile I shall describe experiments which were begun with the view of searching for hysteresis in the relation of magnetisation to magnetising force, without reference to stress.§ The presence of this peculiar action in the several parts of these researches forms a connecting link between them, but other lines of inquiry which suggested themselves in the progress of the experiments have also been followed up at considerable length.

* 'Proceedings of the Royal Society,' No. 214, 1881, p. 399.

† This observation had, I afterwards learnt, been previously made by E. Cohn (Wied. Ann., VI., p. 385).

‡ 'Proceedings of the Royal Society,' No. 216, 1881, p. 22, and No. 228, 1883, p. 123.

§ The results to be described below were all obtained before I became acquainted with the recent work of Warburg on the same subject (Wied. Ann., XIII., p. 141), or with the less closely related observations of Fromme and Auerbach, to which fuller reference will be made later (§ 32 below).

The fact that iron possesses retentiveness, or, in other words, that it remains magnetic after a magnetising force has been applied and removed, is of itself sufficient proof that there must be hysteresis in the relation of magnetism to magnetising force, when the changes of the latter are such that forces of opposite signs are alternately applied. But it does not follow that hysteresis is necessarily present when the changes of magnetising force are restricted to one direction and to one sign. The case may be supposed similar to that of a strained solid. When a stress along any axis is alternately applied and *reversed*, we know that (provided the resulting strain exceeds the body's limit of elasticity) there is hysteresis, of a simple and obvious character, in the relation of strain to stress. But if, instead of being reversed, the stress is merely applied and removed periodically, the greater part, if not the whole of the resulting change of strain (after the first application) is purely elastic, and we have no ground for asserting that there will then be hysteresis in the relation of strain to stress. In fact, if the process of magnetisation is strictly analogous to the straining of a solid whose limits of elasticity are exceeded by the strain, we should expect to find little or no hysteresis in the changes of magnetism which occur when after applying a magnetising force we (wholly or partially) remove and reapply it.

§ 3. The point under consideration has an important bearing on the theory of WEBER, which endeavours to explain the process of magnetic induction by supposing that the molecules of iron and other paramagnetic substances are always magnets, whose axes point indifferently in all directions, until, in consequence of applied magnetising force, they are turned more or less towards the direction in which the force acts. WEBER supposes that each magnetic molecule, when deflected from its initial position, tends to return to that position with a force which is the same as that which a magnetising force D, acting in the initial direction of its axis, would produce. If then deflection of the molecule be produced by the action of an applied magnetising force X, the direction which the molecule takes up while the force is at work is such that its magnetic axis points along the resultant of X and D. The force X exerts on it (per unit of its magnetic moment) a couple $X \sin \theta$, where θ is the angle made by its axis, in its deflected position, with the direction along which X acts. And the assumed restoring force D exerts a couple $D \sin \beta$, where β is the angle through which the molecule has been revolved. The molecule remains in equilibrium under these, and only these, forces, and when the magnetising force X ceases to act, it returns to its initial position through the action of D. Thus, the theory, in this form, takes no account of residual magnetism, and fails to explain retentiveness.

To remedy this defect, CLERK MAXWELL ('Electricity,' ii., Part 3, chap. vi.) has suggested a further assumption, based on the analogy of magnetism to mechanical strain. He supposes that when a molecule is deflected by a magnetising force X, it returns to its primitive position on the removal of X, provided the angle of deflection β has been less than a certain limit β_0; but if the deflection has exceeded β_0, then when X is removed the molecule does not completely return, but remains deflected through

an angle $\beta - \beta_0$, which he calls the "permanent set" of the molecule. MAXWELL has developed the consequences of this supposition at length, but into these it is not at present necessary to follow him, further than to point out that (following up the analogy to a strained solid whose elasticity is perfect up to a certain limit, and which, when strained beyond that limit, acquires a new limit of elasticity at or near to the greatest value of the previously applied stress) we should expect to find, after the application of any force X, a purely elastic series of magnetic changes when we subsequently remove and reapply X. The movements of each molecule would then lie between the angle of previous permanent set $\beta - \beta_0$ and the angle β, and between those limits we should find each molecule taking up one and the same direction for each value of X, whether that was reached by increment from a lower, or decrement from a higher value. In other words, this modified theory leads us to expect no hysteresis in the relation of magnetic intensity to magnetising force when, after its first application, a magnetising force is subsequently removed and reapplied.

§ 4. To test this point, I have examined experimentally the cyclic changes of magnetisation which accompany cyclic changes of magnetising force, both when the force is confined to one direction, and also when its sign is reversed. The results do not support MAXWELL's extension of WEBER's theory, for they show that *all* changes of magnetism caused by changes of magnetising force exhibit that lagging action which I have called hysteresis. The word "retentiveness," commonly restricted to name that property in virtue of which the magnetic metals retain a portion of their magnetism when the magnetising force is removed, might with propriety be extended so as to designate the resistance to any change of magnetic state (whether increase or decrease) which they exhibit whenever the magnetic field in which they are placed suffers any change. The existence of residual magnetism when the field is reduced to zero is, in fact, only one case of an action which occurs whenever the field is varied in any way, namely, a tendency to persistence of previous magnetic state. And my experiments show that when the magnetism of iron is altered by varying, not the field, but the state of stress of the metal, the same tendency again manifests itself. The word retentiveness might be extended so far as to cover this case of lagging also; and if hysteresis were found only in these and similar magnetic phenomena, there would be no need to invent a name for it. But, as will be shown later, there are cases of hysteresis which have, as far as can be seen, nothing to do with magnetic condition, and which the quality of magnetic retentiveness, even if understood in the widest possible sense, cannot account for. I have therefore found it convenient and even necessary to employ a new term, which merely designates this peculiar action without implying any theory as to its cause.

§ 5. As regards the hysteresis which occurs when the magnetism of soft iron is changed, my experiments confirm the idea already suggested by other observers, that when the molecular magnets of WEBER are rotated they suffer, not first an elastic and then a partially non-elastic deflection as MAXWELL has assumed, but a kind of

frictional retardation (resembling the friction of solids), which must be overcome by the magnetising force before deflection begins at all. Again, as the magnetising force begins to be withdrawn, this friction must be overcome by the quasi-elastic restoring force before each molecule can begin to return towards its primitive position; and the molecule finally takes up a position determined by the equilibrium of the restoring force and the frictional resistance to further return. It seems even possible that the residual magnetism which is found in soft iron when an applied magnetising force is completely removed, may be due entirely to this frictional sticking of the molecules. Hard iron and steel, on the other hand, in which the residual magnetism is more permanent, behave in a way which suggests the combination of this frictional sticking with something else of the nature of MAXWELL'S permanent set.

The frictional resistance must resemble the friction of solids, not the viscosity of liquids, since it appears to be independent of the speed at which the changes of magnetic condition occur; and since the magnetisation which any field produces is reached almost at once on the application of the field, and, so long as the field is kept constant, suffers little or no change with lapse of time.

§ 6. The following notation, adopted from MAXWELL, will be used throughout this paper:—

$\mathfrak{H}$ is the magnetising force.

$\mathfrak{B}$ is the magnetic induction within the metal. It is the number of "lines of force" per square centimetre which would be found if we were to cut an indefinitely narrow crevasse perpendicular to the direction of magnetisation.

$\mathfrak{I}$ is the intensity of magnetism, or the magnetic moment of the metal per cubic centimetre.

$\kappa, = \frac{\mathfrak{I}}{\mathfrak{H}}$, is the coefficient of induced magnetisation.

$\mu, = \frac{\mathfrak{B}}{\mathfrak{H}}$, is the magnetic permeability, or magnetic inductive capacity.

$$\mathfrak{B} = 4\pi\mathfrak{I} + \mathfrak{H}$$

$$\mu = 4\pi\kappa + 1$$

The values of $\mathfrak{H}$, $\mathfrak{B}$, and $\mathfrak{I}$ will be given in c.g.s. units.

In representing graphically the results of experiments, I have generally plotted either $\mathfrak{B}$ or $\mathfrak{I}$ in terms of $\mathfrak{H}$. This is the most direct and, for most purposes, the most useful mode of representation. Another plan, used with good effect by ROWLAND, is to plot either κ or μ in terms of $\mathfrak{I}$ or $\mathfrak{B}$, and this has been adopted in a few instances.

§ 7. When a bar of finite length is magnetised by the action of an electric current in a surrounding solenoid, the magnetising field is due partly to the solenoid and partly to the ends of the magnet itself. During magnetisation the field due to the ends of the magnet opposes the action of the solenoid and reduces the resultant field,

by amounts which vary from point to point along the axis of the bar, while after the magnetising current is reduced to zero there remains a demagnetising field, also unequal along the bar, due to the magnet itself. The magnetism which remains is therefore only that part of the true residual magnetism which is not removed by this self-demagnetising force: by "true residual magnetism" is meant the magnetism which would be found if after being magnetised the metal were left entirely free from all magnetising force. The influence of the ends has been recognised by many writers. To secure uniform magnetisation Von Quintus Icilius experimented with ellipsoids of revolution in place of cylindrical bars. Stoletow* and Rowland,† by using endless magnets, have given their determinations of magnetic susceptibility and the maximum of magnetisation a value incomparably greater than that possessed by previous observations made on short bars. It is clear that the question which my experiments were, in the first instance, specially directed to examine, namely, whether hysteresis occurs in the relation of magnetism to field when the field is varied *without change of sign* (§ 2), could not be answered by any experiments with short bars, since in these the ends produce a magnetising force of reversed sign when the external force due to the solenoid is withdrawn. To answer this question we must use methods of experiment of the same rigorous character as are necessary in examining magnetic permeability or the coefficient of induced magnetisation.

To eliminate the action of the ends, and so secure the conditions suitable for exact experiment in the relation of $\mathfrak{H}$ to $\mathfrak{B}$, Stoletow (on the suggestion of Kirchoff) and Rowland, used magnets whose form was that of a closed ring, and Rowland also used rods so long that the influence of the ends became negligible. With closed rings, as he has remarked, we can do no more than observe sudden changes in magnetism by using an induction coil, wound on the ring, in circuit with a ballistic galvanometer. With rods, on the other hand, we can examine the actual magnetic state at any instant, either ballistically, by drawing off a short induction coil from the middle portion of the length, or by direct magnetometric measurement. My own observations show that it is only when the length of the rod (if of iron) is about 300 or 400 times its diameter that the effect of length becomes insensible. The principal drawback to the use of very long rods is the difficulty of forming them (when the section is at all considerable) so as to be of sufficient length and at the same time homogeneous and uniform in section. By using a ballistic galvanometer whose needle, although sufficiently heavy to act ballistically, was comparatively light, I have been able to obtain good results with rods of very small section—*wires* in fact—which by the operation of drawing are scarcely less uniform in section than turned rods, and are in

* Phil. Mag., xlv. (1873), p. 40.

† Phil. Mag., xlvi. (1873), p. 140, and xlviii. (1874), p. 321. An admirable summary of the work of Stoletow and Rowland, as well as the earlier researches of Wiedemann, Von Quintus Icilius, Thalén, and others, and the later contributions of many other observers, has been lately published by Professor Chrystal in the article "Magnetism," Enc. Brit., ninth edition, vol. xv.

general much more homogeneous. Their small diameter also permits a solenoid of many turns to be wound on them without introducing much resistance, so that a strong magnetising force is obtainable without difficulty. With the single exception of an observation of cast-iron (§ 24 below), all the experiments to be described were made with wires. In some cases the wire has been bent into a ring and welded to form an endless magnet, but generally I have used long straight wires, which, as regards convenience of winding and other particulars, have many practical advantages over rings. A good form of magnet, especially appropriate in the examination of cast-iron and cobalt (where very long rods are impracticable), would be a nearly closed ring, with an opening between the ends just wide enough to allow a short induction coil to be slipped off. So far as the magnetisation of the middle portion was concerned, this would be nearly equivalent to a closed ring, while it would enable the actual magnetic state, and not sudden changes merely, to be measured.

§ 8. *Ballistic Method of Experiment.*—In the ballistic method the changes produced by sudden changes of $\mathfrak{P}$ were determined by the throw of the needle of a galvanometer in circuit with an induction coil which was wound over the middle part of the rod, or over the whole or any convenient part of the ring. The galvanometer was a short coil THOMSON'S, made for lecture purposes, and consequently with an unusually heavy mirror. Its period was moderately long, and the retardation, although sufficient to bring the needle to rest without special appliances, in the interval between observations, was not so great as to unfit it for ballistic use. To test its accuracy in this kind of work a preliminary experiment was made, in which a wire in circuit with the galvanometer was wound into a coil of ten turns, which was slipped over the end of a long permanent bar-magnet. Readings were taken of the throws given when this coil was suddenly drawn off the bar, and again, at each stage, while the number of turns in the coil was progressively reduced to one. The throws were found to be proportional to the number of turns, to a degree of accuracy as great as was attainable in the actual observations, and throughout the whole range of the scale.

To reduce the ballistic readings to absolute measure, I adopted the method used by ROWLAND.* A coil of ten turns of thick wire, which will be called the earth-coil, having a total area of 10×1216 square centimetres, was wound on a light rectangular wooden frame, and was kept always in circuit with the ballistic galvanometer and the induction coil. It was laid flat on a horizontal table, and at the end of each experiment it was quickly turned over, thereby cutting twice the vertical component of the earth's magnetic force, and the throw of the galvanometer was observed. This "earth-coil reading" was the throw corresponding to $2 \times 12160 \times 0{\cdot}34$ lines of induction, 0·34 being the assumed local value† of the vertical component of the earth's force.

§ 9. Magnetising force was given by the current from a battery of gravity DANIELL

* *Loc. cit.*, p. 148.

† In the Physical Laboratory of the University of Tokio, Japan, where the experiments were made.

cells, through a solenoid of moderately fine wire wound closely round the rod or ring. Even when the solenoid consisted of more than one layer, as it frequently did, the correction for the air space in evaluating $\mathfrak{B}$ was negligible, unless $\mathfrak{H}$ was unusually high. The force was varied, in the ballistic observations, by having in circuit with the magnetising solenoid a set of resistance coils which could be plugged up or inserted step by step so as to give a series of sudden increments or decrements in the value of $\mathfrak{H}$. The magnetising current was measured by another mirror galvanometer, very strongly shunted, and its value in absolute measure was determined by seeing what deflection a given number of gravity cells in good condition gave through a given total resistance. In reducing the observations I have assumed that a gravity DANIELL cell in good condition, when put in circuit with a total resistance of n British Association units gives a current of $\frac{1{\cdot}1}{n}$ ampères. This agrees well with Dr. C. R. A. WRIGHT'S recent determination;* and an experiment of my own, in which the current from one cell was adjusted by a variable resistance until it just sufficed to neutralise, in a vertical solenoid, the vertical component of the earth's force, gave a closely accordant value.

To have aimed at any very high degree of exactness in the reduction of magnetisation and magnetising force to absolute measure would have been a waste of time. The magnetic differences between different specimens of iron are so great that no useful purpose is served in determining absolutely the magnetic quality of any one specimen with extreme precision. On the other hand, even a somewhat rough reduction to absolute measure is so exceedingly serviceable in making results intelligible and definite in themselves, as well as capable of comparison with the work of other observers, as to justify fully the very considerable amount of arithmetical labour it involves. In dealing with measurements in which comparison of results was to be made, as, for instance, in the magnetisation of the same piece in different states of stress, I have been careful to make the reduced values accurately comparable with each other.

§ 10. *Magnetisation of an Iron Ring.*—In the following experiment a welded ring of moderately soft iron wire was subjected to a magnetising force, which was applied by a series of sudden steps up to 9·14 c.g.s. units, then reduced by steps to 0, and then reapplied, also by steps—with the view of testing the question raised in § 2, as to whether hysteresis occurs in the relation of $\mathfrak{B}$ to $\mathfrak{H}$ during the removal and reapplication of magnetising force. In this, the first example to be quoted, it may be well to give the numerical data and to exhibit the reduction in full: in subsequent examples a graphic representation of the reduced results, with occasionally a numerical statement of them, will suffice.

* Phil. Mag. xiii. (1882), p. 265.

Diameter of wire	0·248	centim.
Mean circumference of ring	31·4	centims.
Number of turns in magnetising coil	474	
Number of turns in induction coil	167	
Deflection of battery galvanometer with 3 cells and 6·85 ohms total resistance	362	
Earth-coil reading	42·9	
Area of earth-coil	1216	sq. centims.
Number of turns in earth-coil	10	
Earth's vertical force	0·34	

Hence magnetising force for one scale division of the battery galvanometer

$$=\frac{4\pi\times 3\times 1{\cdot}1\times 10^{8}\times 474}{6{\cdot}85\times 10^{9}\times 362\times 31{\cdot}4}=0{\cdot}02525.$$

And value of $\mathfrak{B}$ for one scale division of the ballistic galvanometer

$$=\frac{1216\times 10\times 2\times 0{\cdot}34}{167\times 42{\cdot}9\times\frac{\pi(0{\cdot}248)^{2}}{4}}=23{\cdot}89.$$

In the following table the actual readings are given, and also the values of $\mathfrak{H}$, $\mathfrak{B}$, $\mathfrak{I}$ and κ calculated from them. The ballistic readings at the beginning of the magnetisation are too small to allow κ to be determined for the early points with any approach to accuracy, and its early values are therefore omitted. It has been assumed that the ring was initially free from magnetism, which can have been no more than approximately true, for it had been moved about in the terrestrial field before the readings were taken.

The same experiment is shown graphically in Plate 57, fig. 1. It will be observed that although the changes of magnetism which occur during the removal and re-application of $\mathfrak{H}$ are small compared with the whole magnetisation, they exhibit hysteresis very decidedly with respect to their argument $\mathfrak{H}$.

MODERATELY soft iron ring, Plate 57, fig. 1.

Battery galvanometer reading.	$\mathfrak{H}$.	Ballistic throw.	Sum of throws.	$\mathfrak{B}$.	$\mathfrak{I}$.	κ.
5·3	0·13	1·1	1·1	26	2·1	
10·2	0·26	1·1	2·2	53	4·2	
12·0	0·30	0·5	2·7	65	5·1	
16·0	0·40	0·8	3·5	84	6·7	
21	0·53	1·0	4·5	107	8·6	
28	0·71	2·1	6·6	158	12·5	
37	0·93	2·9	9·5	227	18·0	
52	1·31	3·9	13·4	320	25·4	19
67	1·69	9·2	22·6	540	42·9	25
75	1·89	6·9	29·5	705	56·0	30
110	2·78	77·5	107·0	2,560	203	73
133	3·36	78·7	185·7	4,440	353	105
159	4·01	82	267·7	6,400	509	127
196	4·95	91·5	359·2	8,580	683	138
232	5·86	57	416·2	9,940	791	135
285	7·20	57	473·2	11,300	899	125
321	8·10	23·5	496·7	11,870	944	116
362	9·14	24	520·7	12,440	989	108
310	7·83	− 4·4	516·3	12,330	981	
246	6·21	− 6·7	509·6	12,170	968	
188	4·75	− 7·1	502·5	12,000	955	
107	2·70	−14·0	488·5	11,670	929	
0	0	−33·2	455·3	10,880	866	
110	2·78	15	470·3	11,240	894	
196	4·95	14·2	484·5	11,570	921	
246	6·21	11·9	496·4	11,860	943	
317	8·00	14·5	510·9	12,170	971	
362	9·14	10	520·9	12,440	990	

§ 11. Plate 57, fig. 2, shows the changes of magnetism of another ring, of very soft iron wire, when the magnetising force was (1) raised to 7·07, (2) reversed to −7·07, (3) restored to +7·07, (4) reduced to 0, (5) restored again to +7·07, all by a series of numerous sudden steps. Also, during the second operation two small loops were formed by reducing the force by steps to zero, reapplying it, and continuing the operation. In the curve of first magnetisation the greatest value of κ is 180, corresponding to $\mathfrak{H}=3$. During the reversal, as soon as the magnetising force changes its sign a very rapid demagnetisation begins, and a force of about −1·9 suffices to remove completely the residual magnetism. The reversal curves are steepest when the magnetisation is zero: at that point the value of $\frac{d\mathfrak{B}}{d\mathfrak{H}}$ is about 14,500. That is to say, for every line of force (per sq. centim.) which is then entering or leaving the field, 14,500 lines of induction are entering or leaving the substance of the metal. Complete double reversal has the effect of slightly reducing $\mathfrak{B}$ below the value reached in the first application of $\mathfrak{H}$, but removal and reapplication of the force raises $\mathfrak{B}$ slightly above its primitive value. Every loop in the diagram shows that

when we reverse the *change* of magnetising force from increment to decrement, or *vice versâ*, the magnetisation begins to change very gradually relatively to the change of $\mathfrak{H}$, no matter how fast it may have been changing, in the opposite direction, before. So much is this the case that the curves, when drawn to a scale such as that of the figure, appear in all cases to start off tangent to a line parallel to the axis on which $\mathfrak{H}$ is measured, whenever the change of $\mathfrak{H}$ is reversed in sign.

§ 12. In a large number of other experiments I have examined the effects of varying the field $\mathfrak{H}$ in all possible ways. The accompanying changes of $\mathfrak{B}$ always exhibit static hysteresis with respect to the variations of $\mathfrak{H}$. The curves connecting these quantities always form loops as in fig. 2, and the characteristic mentioned in the last sentence of § 11 appears to be quite general. It is scarcely necessary to point out that this is not what we should expect from the theory of retentiveness suggested by MAXWELL (§ 3). On the other hand, it is just what we should expect if we suppose that there is a static frictional resistance to the rotation of WEBER'S magnetic molecules.

An interesting point may be noticed with regard to the small loops, two of which are shown in the diagram (fig. 2). Suppose that either on the ascending or the descending branch of the main curve we begin to form such a loop by removing the magnetising force, taking as the starting point a place in the main curve such that when $\mathfrak{H}$ becomes zero $\mathfrak{B}$ is also zero. Evidently this could be done by selecting a suitable starting point either in the positive part of the ascending branch of the main curve or in the negative part of the descending branch. Then when $\mathfrak{H}$ is reduced to zero the piece combines entire freedom from magnetisation with absence of magnetising force, but its condition is widely different from that of a previously unmagnetised piece. In particular, it is unsymmetrical as regards susceptibility to magnetisation in the two longitudinal directions, being much more ready to take magnetism of an opposite sign from that which it last possessed than to take magnetism of the same sign. The curves of $\mathfrak{J}$ and $\mathfrak{H}$ in the two quadrants, if we imagine them both to be drawn, would meet in a sharp angle at the origin, instead of being continuous, as they would be in a previously unmagnetised specimen. This is indeed only one example, though a very striking one, of the fact that in consequence of hysteresis a condition of no magnetism, in a field of no force, is capable of being reached by many processes, some of which will leave the metal ready to show a startling want of neutrality when it is subsequently magnetised in one or the other direction.

§ 13. *Residual Magnetism in Soft Iron.*—A feature in these and other early experiments which caused me much surprise was the largeness of the residual magnetism. In fig. 1, 87 per cent., and in fig. 2, 81 per cent. of the total induced magnetism remains when the magnetising force has been completely removed; and in other examples I have found the residual magnetism of soft iron to be 90 and even 93 per cent. of the induced magnetism. It is generally stated in the best text-books that the magnetic condition of soft iron disappears almost wholly when the inducing field

is withdrawn. This entirely erroneous opinion has probably been established by experiments on rods whose length was insufficiently great to prevent them from demagnetising themselves more or less completely. In fig. 2 we see how great is the effect of a small amount of reversed force in reducing the residual magnetism. To illustrate the extent of the self-demagnetising action which occurs in even a moderately long rod, we may take the following ideal case. Suppose that an infinitely long, straight, round rod of radius r has been uniformly magnetised in the direction of its length, and the magnetising force withdrawn, leaving it with a uniform residual magnetisation whose intensity is $\mathfrak{I}$. Then imagine a length $2a$ to be suddenly cut out of it, and the ends to be instantaneously removed. At the instant of separation the magnetisation may still be supposed to be uniform, and the self-demagnetising force which then acts at any point in the axis of the rod at a distance x from the centre is

$$-2\pi\mathfrak{I}\left\{2-\frac{a-x}{\sqrt{r^2+(a-x)^2}}-\frac{a+x}{\sqrt{r^2+(a+x)^2}}\right\}.$$

Such a distribution is of course self-destructive; but it is interesting to examine the numerical values of the demagnetising force it would cause, with a given value for $\mathfrak{I}$ and for the ratio of length $2a$ to radius r. Let the rod's length be fifty times its diameter, and suppose that it consists of the same soft iron as the ring of fig. 2, and has been similarly treated before cutting, so that the residual value of $\mathfrak{B}$ at the instant of cutting is 9000. The following numbers show the resulting values of the demagnetising force due to the ends at various points along the axis of the bar:—

Distance from centre expressed as a fraction of the length.	Self-demagnetising force in c.g.s. units.
0·5	4500
0·495	2488
0·49	1318
0·48	475
0·45	87
0·4	22·7
0·35	10·3
0·3	6·0
0·25	4·6
0·2	2·9
0	1·8

Comparing these with fig. 2 we see that throughout very nearly its whole length the rod would be subjected in this ideal case, by its own magnetism, to a force more than sufficient to remove that magnetism altogether. It need not therefore cause surprise that there is little residual magnetism in the equilibrium distribution which is arrived at after any applied magnetising force is withdrawn from a rod whose length is even as much as fifty times its diameter.

§ 14. Another cause contributes to produce the erroneous opinion that soft iron has little retentiveness, namely, the enormous influence which the slightest mechanical disturbance has in removing the residual magnetism. The soft iron ring of fig. 2 loses nearly all its residual magnetism if gently tapped after the magnetising force has ceased to act. I have frequently removed in this way all but one or at most two per cent. of the originally large residue. So susceptible indeed is soft iron to the effect of mechanical disturbance, that when the field is removed the lightest touch by the fingers suffices to destroy a large part of the residual magnetism, although it appears that so long as the iron is left perfectly undisturbed the residual magnetism does not suffer loss with lapse of time. I shall recur to the effects of vibration later; meanwhile they are mentioned in order to explain that we may take advantage of this property to reduce a piece of soft iron to a very nearly neutral state after magnetisation, so that we may study the comparative effects of again magnetising the same piece under different conditions. Simple tapping in a field of no force reduces the piece to a state differing very little from that in which it was after annealing and before any magnetisation had taken place. This I have found to be the case by taking curves of magnetisation of a very soft iron wire after annealing, and again after shaking out the residual magnetism: such curves turn out to be sensibly coincident.

§ 15. *Magnetisation of long Rods of various lengths.*—Taking advantage of this property, in virtue of which we can reduce a given specimen of soft iron after magnetisation to a state nearly identical with its primitive state, I made the following group of observations to illustrate the influence of length on the magnetisation of straight rods of circular section. A long straight wire of the same very soft iron as the ring of fig. 2 was well annealed, wound with a magnetising solenoid and placed on a horizontal table in the E.-W. position. After the wire was tapped to get rid of its initial magnetism a curve of $\mathfrak{B}$ and $\mathfrak{H}$ was taken by the ballistic method. $\mathfrak{H}$ was then reduced to zero and the residual magnetism was shaken out by tapping the wire on the table, still keeping the E.-W. position, and the permanent residue (which was very small) was determined by slipping off the induction coil. The induction coil was about 5 centims. long, and was placed at the centre of the rod's length. The process of magnetisation was repeated three times, giving sensibly the same results each time. The wire's diameter was 0·158 centim., and its original length was 47·5 centims., or 300 diameters. Then the length was reduced to 31·6 centims., or 200 diameters, by cutting off equal portions from both ends, and a curve of magnetisation was taken. The residual magnetism was again tapped out, the length again reduced in the same way to 150 diameters, and another curve of magnetisation taken. The same process was repeated with the length equal to 100, 75, and 50 diameters successively. The peculiarity of the method lay in this, that throughout the series of observations the magnetic induction was determined through precisely the same piece of material. The influence of length *per se* was therefore exhibited in a way which would have

been impossible had the successive tests involved the unknown and not inconsiderable differences of magnetic quality which different specimens would have possessed or different annealings given.

In each case the magnetising force due to the solenoid was raised by steps to the same value, 34·2 c.g.s. units, and lowered by steps to zero. The curves of Plate 57, fig. 3, show its relation to $\mathfrak{B}$ at the centre of the rod in the various cases. The full lines are the "on" curves, or curves showing the relation of $\mathfrak{B}$ to the solenoid's magnetising force during its application: the dotted lines are the "off" curves, and show the values of $\mathfrak{B}$ during decrease of the solenoid's magnetising force. The number affixed to each curve gives the ratio of length to diameter in the test it belongs to. The abscissas give the magnetising force due to the solenoid, which of course differs from $\mathfrak{H}$ by the amount of the variable field which the rod exerts upon itself. To avoid confusion in the figure, a portion only of each pair of curves is drawn, except in the case of the longest and shortest rods, for which the curves are drawn in full. The greatest magnetisation reached was sensibly the same for the longest rods and for those of intermediate lengths. At the beginning of each "on" curve a small amount of initial magnetism will be noticed, which is the part of the previous residual magnetism not shaken out by tapping.

§ 16. It appears that the rod 300 diameters long differs little in its magnetic behaviour from an indefinitely long rod or from a ring. Its residual magnetism is 85 per cent. of the total, and its greatest value of μ is about 3500. The "on" curve of the 200 diameters rod falls not much below that of the 300 diameters rod, but its "off" curve is notably different near the conclusion, where the demagnetising influence of the ends becomes sharply apparent. The result is to reduce the residual magnetism to 60 per cent. of the total. In the 150 diameters rod the residual magnetism falls to 39 per cent., in the 100 diameters rod to 20 per cent., in the 75 diameters rod to 9 per cent., and in the 50 diameters rod to about 6 per cent. The "off" curves of the shorter rods are distinguished by a long straight descent towards the axis of $\mathfrak{B}$, showing a sensibly uniform rate of demagnetisation during the later part of the withdrawal of the externally applied force.

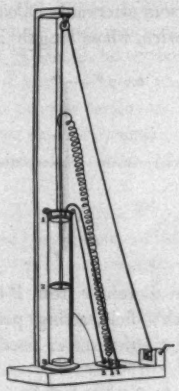
The comparative difficulty of magnetising a short rod is shown by fig. 3 in a way which requires no comment. If we were to infer values of μ and κ from the test of the 50 diameters rod, neglecting the fact that the magnetising force due to the solenoid is not the whole of $\mathfrak{H}$, we should find for the maximum of μ 656, and for that of κ 52; and these coefficients are much more nearly uniform for various values of the magnetising force than when we determine them in a legitimate manner by observing the magnetisation of very long rods or of rings.

§ 17. From this and other experiments, I concluded that even when dealing with the softest iron, we may take a rod whose length is not less than 300 diameters as giving results scarcely different from those given by a ring or a longer rod, and in subsequent experiments rods were almost exclusively used whose length was from 300 to 400

times their diameter. In hard iron and in steel a smaller ratio of length to diameter would no doubt give an equally good approximation to the condition of endlessness.

It should not be forgotten that want of perfect homogeneity introduces a self-demagnetising force even in the longest rod or in a ring, by causing some of the lines of induction to escape from the substance of the metal at intermediate points, and it does not appear that we can easily, if at all, approximate more closely to the ideal condition of uniform magnetisation than by the use of a straight piece of wire in which the ratio of length to diameter is (say) 400. If it be desired to deal with a larger section of metal than a single wire, a good plan is to form a multiple ring by winding a long continuous wire into a coil of as many turns as may be wished, and cutting the ends so that they abut against one another. This gives a more homogeneous structure than is got by welding a rod into a ring, or even by turning a ring out of a solid forged piece. One or two experiments made with multiple rings have only served to confirm the general accuracy of the results got by the use of straight wires.

§ 18. *Direct Magnetometric Method of Experiment.*—In addition to the ballistic method, a direct magnetometric method has been employed in many cases in examining



the magnetisation of long wire rods. The wire was placed in the vertical position, with its upper end nearly level with, and magnetically east of a small mirror magnetometer, formed by suspending the mirror of a THOMSON galvanometer by a silk fibre about 8 centims. long. Two coils were wound round the rod: through one of these a constant current was maintained which just sufficed to neutralise the vertical component of the earth's magnetic field. The other formed the magnetising solenoid, and the current in it was varied not by sudden steps as in the ballistic method, but

continuously by means of a "slide" of very simple construction, which is shown in the annexed figure. This consisted of a tall cylindrical glass jar filled with a dilute solution of sulphate of zinc, with a block of amalgamated zinc fixed at the top, and another at the bottom, the blocks being connected by the terminals 1 and 3 to the two poles of the battery. A third block of zinc was raised or lowered between these, through the fluid, by means of a cord and pulley, and formed the sliding terminal of the magnetising solenoid, of which one end was connected to it by the terminal 2, and the other end was connected through a commutator to one of the fixed blocks. This liquid slide gave a remarkably effective and convenient means of varying the magnetising current, and no difficulty was experienced in keeping the current constant when it was desired to do so.

The position of the magnetised wire with respect to the magnetometer was such that any change in *distribution* of magnetism produced minimum effect, the displacement of the upper "pole" being at right angles to the line joining it with the magnetometer, while the lower pole was comparatively inoperative. The reduction of the observations to absolute measure presents no novel features. $\mathfrak{I}$ was first evaluated, the deflection due to the magnetising solenoid alone being separately determined, and subtracted from the observed magnetometer readings before these were reduced to find $\mathfrak{I}$. $\mathfrak{B}$, where it is given, was afterwards calculated from $\mathfrak{I}$ and $\mathfrak{H}$.

Let PP′ be the magnetised wire, whose length is l and moment lm. Let M be

the position of the magnetometer, distant r from P and r' from P′, and let H be the horizontal component of the earth's field, acting perpendicular to the plane MPP′. Then if θ be the deflection of the magnetometer needle,

$$\mathrm{H}\sin\theta=\left(\frac{m}{r^2}-\frac{m}{r'^2}\cdot\frac{r}{r'}\right)\cos\theta,$$

$$m=\frac{\mathrm{H}r^2\tan\theta}{1-\left(\frac{r}{r'}\right)^3}.$$

Or, calling d the diameter of the wire,

$$\mathfrak{I}=\frac{4lm}{\pi d^2}=\frac{4l\mathrm{H}r^2\tan\theta}{\pi d^2\left\{1-\left(\frac{r}{r'}\right)^3\right\}}.$$

In this simple form the direct magnetometric method, as a means of determining magnetic qualities in absolute measure, is not entirely free from objection, but it allows certain points to be investigated which the ballistic method is unable to deal with, notably the influence of time in magnetisation, and the comparative effects of sudden and gradual application or removal of magnetising force, and it is also particularly convenient in testing the effect of stress on magnetic susceptibility.

§ 19. *Demagnetisation by the method of reversals.*—As an alternative to the plan of tapping out the magnetism, which is inapplicable to steel or hard iron, another plan has more frequently been used to remove residual magnetism when it has been desired to repeat the magnetisation of a given specimen without change of its physical state. This consists in subjecting the piece to a succession of magnetising forces with opposite signs and of gradually diminishing intensity. By means of the liquid slide described above, the current was slowly reduced to zero, while a rapid reversing key between the slide and the solenoid was kept in action. The earth's vertical force was, as has been said, balanced by a steady current in a separate solenoid, and when the strength of this last was properly adjusted, the process of demagnetising by reversals was very successful in removing all traces of residual magnetism.* It is clear that this process leaves the piece symmetrically conditioned as regards subsequent magnetisation in either of the two longitudinal directions. The condition of a piece demagnetised in this way is no doubt different from that of a piece never before magnetised (indeed the curves of $\mathfrak{Z}$ and $\mathfrak{H}$, taken before and after the process, show that it is somewhat, though not very, different as regards susceptibility); but the neutral state produced by this process forms an exceedingly convenient starting point, to which we may recur over and over again, when we wish to determine the susceptibility or the retentiveness of the same piece under different conditions of stress, temperature, &c.

§ 20. *Cyclic Magnetisation of Wrought-iron Wires.*—A few experiments will now be detailed in which well annealed† specimens of soft iron wire were subjected to a complete cycle of magnetisation by the successive application of equal and opposite magnetising forces. In each case the length of the specimen was so great as to realise, as nearly as possible, the condition of endlessness. In the following experiment, the results of which are given numerically below, and graphically in Plate 57, fig. 4, the specimen was a very soft iron wire 0·158 centim. in diameter, and 64 centims. or 400 diameters in length; and the method used was the step-by-step ballistic method (§ 8). The steps by which $\mathfrak{H}$ and $\mathfrak{B}$ were varied were those shown in the table below. The magnetising force was first raised to 17·26, then reversed

* A process similar in principle was applied some years ago by Mr. H. S. Maxim to cure magnetised watches.

† The process of annealing consisted in drawing the wire through a blowpipe flame so slowly that each part of the length in turn became bright red and then cooled gradually as it moved away from the flame. For wires of less than 1 mm. diameter an ordinary spirit-lamp flame was generally used, without a blowpipe.

to −17·26, reversed again to +17·26, reduced to 0, and restored to +17·26, all by steps. At the beginning of the operation the wire had a small amount of initial magnetism, which was determined by sliding off the induction coil.

VERY Soft Annealed Iron Wire, Plate 57, fig. 4.

ℌ.	𝔅.	ℌ.	𝔅.	ℌ.	𝔅.	ℌ.	𝔅.
0	166 (initial)						
0·05	166	− 0·05	10,970	0·05	−10,795	0·05	10,845
0·19	183	− 0·19	10,790	0·19	−10,620	0·19	10,850
0·37	216	− 0·37	10,460	0·37	−10,268	0·37	10,860
0·71	393	− 0·71	9,530	0·71	− 9,270	0·71	10,890
1·08	941	− 1·08	7,360	1·08	− 6,780	1·08	10,930
1·84	4,088	− 1·84	20	1·84	− 642	1·84	11,040
3·30	8,684	− 3·30	− 7,840	3·30	8,190	3·30	11,480
5·62	11,380	− 5·62	−11,100	5·62	11,270	5·62	12,210
7·63	12,320	− 7·63	−12,080	7·63	12,170	7·63	12,650
10·54	12,950	−10·54	−12,690	10·54	12,770	10·54	13,040
14·19	13,280	−14·19	−13,030	14·19	13,110	14·19	13,300
17·26	13,450	−17·26	−13,190	17·26	13,280	17·26	13,440
14·19	13,370	−14·19	−13,110	14·19	13,220		
10·54	13,250	−10·54	−13,000	10·54	13,100		
7·63	13,100	− 7·63	−12,900	7·63	12,940		
5·62	12,970	− 5·62	−12,800	5·62	12,830		
3·30	12,680	− 3·30	−12,520	3·30	12,570		
1·84	12,240	− 1·84	−12,080	1·84	12,150		
1·08	11,850	− 1·08	−11,700	1·08	11,790		
0·71	11,590	− 0·71	−11,430	0·71	11,510		
0·37	11,277	− 0·37	−11,130	0·37	11,200		
0·19	11,100	− 0·19	−10,970	0·19	11,030		
0·05	11,000	− 0·05	−10,820	0·05	10,880		
0	10,980	0	−10,800	0	10,840		

Here the residual magnetism is 82 per cent. of the induced magnetism, both on the positive and on the negative side. An opposite force of about −1·9 serves to remove it completely. The general form of the curves in fig. 4 should be noted, since, as many other experiments have shown, this is a thoroughly typical example of the behaviour of annealed iron.

§ 21. In the next example the method of experiment was different. The magnetising force was varied continuously and very slowly by means of the liquid slide described in § 18, occasional pauses being made to allow current and magnetometer observations to be taken. The magnetisation was measured by the direct magnetometric method of § 18. The wire was a piece of moderately soft iron, well annealed, of 0·077 centim. diameter, and 30·5 centims. or 400 diameters in length. The magnetising force was raised to 22·3, reversed to −23, and restored to +22·3, all very gradually. Whenever, as was frequently the case, the magnetometer deflection appeared to be creeping up or down after the alteration of magnetising force had ceased, time was given to allow the magnetisation to become sensibly steady before a

reading was taken. The following corresponding values of $\mathfrak{H}$ and $\mathfrak{B}$ are reduced from the observations; they are also exhibited graphically in Plate 58, fig. 5.

ANNEALED Iron Wire, Plate 58, fig. 5.

Current in magnetising solenoid,—galv. reading.	$\mathfrak{H}$.	Magnetometer reading.	$\mathfrak{B}$.	μ.	Current in magnetising solenoid, galv. reading.	$\mathfrak{H}$.	Magnetometer reading.	$\mathfrak{B}$.	Current in magnetising solenoid, galv. reading.	$\mathfrak{H}$.	Magnetometer reading.	$\mathfrak{B}$.
0	0	0	0		11·5	− 0·41	342	14,140	9	0·32	−348	−14,390
2	0·32	1	41	128	23	− 0·81	329	13,600	19	0·67	−339	−14,010
4	0·35	4	165	194	31	− 1·10	318	13,150	29	1·02	−327	−13,520
39	1·38	10	413	299	41	− 1·45	295	12,200	39	1·38	−306	−12,650
59	2·15	23	1,480	670	51	− 1·80	255	10,480	49	1·73	−274	−11,330
79	2·80	89	3,680	1310	61	− 2·20	166	6,860	59	2·09	−195	− 8,060
99	3·50	173	7,220	2070	71	− 2·51	70	2,890	69	2·44	−103	− 4,260
119	4·21	236	9,880	2350	81	− 2·87	− 12	− 496	79	2·80	+ 4	165
139	4·92	279	11,540	2350	91	− 3·22	− 53	− 2,190	86	3·04	58	2,400
159	5·63	304	12,570	2230	101	− 3·50	−142	− 5,870	99	3·50	130	5,370
179	6·69	327	13,520	2020	121	− 4·28	−226	− 9,340	114	4·03	199	8,230
239	8·46	348	14,390	1700	141	− 4·99	−273	−11,490	140	4·96	279	11,530
289	10·23	359	14,840	1450	169	− 5·73	−306	−12,650	190	6·72	332	13,720
342	12·11	365	15,090	1250	211	− 7·47	−338	−13,970	240	8·50	351	14,510
441	15·61	373	15,420	990	261	− 9·23	−352	−14,550	291	10·30	362	14,970
574	20·32	378	15,630	770	312	−11·05	−361	−14,920	339	12·00	368	15,210
629	22·27	380	15,710	705	411	−14·55	−369	−15,250	340	12·04	374	15,460
464	16·42	379	15,670		511	−18·09	−373	−15,420	579	20·50	381	15,750
239	8·46	375	15,500		659	−23·08	−376	−15,550	630	22·30	383	15,830
139	4·92	372	15,380		536	−18·94	−376	−15,550	0	0	353	14,590
89	3·15	369	15,270		411	−14·55	−376	−15,550				
39	1·38	363	15,010		311	−11·01	−375·5	−15,530				
0	0	350	14,470		211	− 7·47	−375	−15,500				
					111	− 3·93	−372	−15,380				
					61	− 2·16	−368	−15,210				
					23	− 0·81	−361	−14,930				
					11·5	− 0·41	−358	−14,800				
					0	0	−352	−14,550				

The figures show that the wire of this experiment, although less susceptible than the last to low values of the magnetising force, took a greater magnetisation with high values of the force. But the most notable feature of this experiment is the enormously great amount of residual magnetism which this wire retained when the magnetising force was completely withdrawn. On the positive side the residual was 92 per cent. of the induced magnetism, and on the negative side 92·5 per cent. Its absolute value is very remarkable. $\mathfrak{B}=14,500$ (the residual value) corresponds to an intensity of magnetism ($\mathfrak{I}$) of 1150 c.g.s. units of moment per unit of volume. Great as this is, it was even slightly exceeded in another experiment with the same wire, in which 93 per cent. of the induced magnetism remained after the magnetising force was removed, giving a residual intensity of 1200 c.g.s. units of moment per unit of volume. So far as I am aware no steel magnet ever holds so much magnetism as this. In fact, if by retentiveness we mean simply the faculty of remaining magnetic when magnetising force is removed, and in the absence of mechanical or other disturbance, soft iron is far more retentive than either hard iron or steel.

§ 22. One more example may be given of the application of a complete magnetising

cycle to annealed iron. In this a much stronger magnetising force was used than before. The wire was a piece nearly the same as that of the last experiment, diameter 0·078 centim., length 29·3 centims., or nearly 400 diameters. The observations (which were made by the direct magnetometric method) are given in a reduced form below, and are also exhibited in Plate 58, fig. 6.

Annealed Iron Wire, Plate 58, fig. 6.

$\mathfrak{H}$.	$\mathfrak{B}$.	μ.	$\mathfrak{H}$.	$\mathfrak{B}$.	$\mathfrak{H}$.	$\mathfrak{B}$.
0	800 (initial)	—				
2·8	4,830	1725	− 4·35	− 6,300	4·7	11,130
3·7	7,510	2030	− 7·8	−11,540	7·1	11,940
4·8	9,520	1980	−15·5	−14,360	11·7	13,690
6·2	11,130	1795	−46·6	−15,200	15·5	14,360
7·8	12,340	1580	−89·8	−16,040	23·3	14,770
9·3	13,140	1410	−26·7	−14,910	38·8	15,590
10·9	13,550	1240	−19·5	−14,630	66·8	16,160
14·6	14,220	970	−15·5	−14,490	89·8	16,310
23·3	14,900	640	−11·2	−14,360		
31·1	15,180	490	− 7·9	−14,080		
46·6	15,700	340	− 6·2	−13,950		
69.9	16,160	230	− 4·6	−13,680		
89·8	16,450	183	− 4·0	−13,410		
0	12,740		0	−12,070		
15·5	14,760		1·5	− 8,580		
31·0	15,310		2·4	− 2,680		
62·1	16,010		2·9	670		
89·9	16,450		3·1	1,880		
62·1	16,150		3·6	4,020		
31·0	15,480		3·7	4,830		
23·3	15,310		4·3	6,970		
15·5	15,030		4·8	8,320		
10·9	14,890		5·1	8,850		
7·8	14,620		5·4	9,390		
4·7	14,480		7·1*	12,080		
0	12,740		6·1	11,940		
			4·6	11,800		
			2·5	11,400		
			0	10,460		

§ 23. *Characteristics of the Curve of Magnetisation.*—The examples which have been cited exhibit very clearly the behaviour of annealed iron when subjected to magnetising forces without mechanical disturbance. It matters little whether the changes of magnetising force are caused to occur suddenly or gradually: there are slight differences in the two cases which will be discussed later, but the general form of the curve is scarcely changed. A comparison of the figures already given will show that the curve of first magnetisation has the following characteristics. At first the magnetisation takes place very gradually with respect to the increment of $\mathfrak{H}$,—how gradually these experiments do not enable me to say, but the appearance of the

* A small loop was formed here by reducing $\mathfrak{H}$ from 7·1 to zero, and reapplying it. See the figure.

curves is not inconsistent with the idea that $\frac{d\mathfrak{I}}{d\mathfrak{H}}$ is initially zero. By the time the magnetising force reaches the value of 1 c.g.s. unit or so, a pretty sharp upward bend takes place, and the magnetisation proceeds rapidly and at a nearly uniform rate for a considerable length of the curve. Then this rate changes gradually, as the region of what is popularly known as saturation is approached, in which region we again find a nearly straight portion of the curve, where a continuous though much less rapid increase of magnetism is still taking place. With magnetising fields whose strength is 100 units or so (my experiments did not deal with stronger fields) the values both of $\mathfrak{B}$ and of $\mathfrak{I}$ show no sign of reaching a limit.

During the withdrawal of the magnetising force, if that has been strong, the magnetism stays well up in the region of so-called saturation until the force is almost wholly removed. As soon as a small force of opposite sign is applied the residual magnetism begins to disappear, and opposite magnetism to take its place, with great rapidity. The rate of this change with respect to the change of $\mathfrak{H}$ is however very uniform during the greater part of its progress. Other characteristics, such as the hysteresis which occurs when a magnetising force is removed and reapplied, have been already described, and others still will be evident on inspection of the curves.

One feature which deserves special notice is the large amount of magnetism which even a very weak field will induce in annealed iron, when the specimen is endless, or long enough to prevent its ends from materially modifying the field. Take for instance the experiment last cited. With a field of 90 c.g.s. units the value of the induction $\mathfrak{B}$ is 16,500, but to produce an induction of 10,000 a field of less than 5 units is sufficient. And this susceptibility, high as it is, is greatly exceeded when during the application of the magnetising force the iron is subjected to mechanical vibration (see § 50).

The curves showing the relation of magnetisation to magnetising force have a very different form when we are dealing with hard-drawn iron, or with a sample which instead of being in the annealed state, has been strained beyond its limit of elasticity before being magnetised. Examples of these will be given later; meanwhile the following experiments with cast-iron and steel may be placed here for convenience of comparison with soft iron.

§ 24. *Magnetisation of a Cast-iron Ring.*—The ring was turned circular, and of circular section, the dimensions being—

	Centims.
External diameter	15·05
Internal „	12·30
Diameter of section	1·378

The method of experiment was the step-by-step ballistic method of § 8. The results are shown in Plate 58, fig. 7. A large loop was formed by applying and reversing and reapplying a magnetising force of nearly 16 units; and then a small loop was formed

by removing and reapplying the same force. This force was insufficient to make the cast-iron approach magnetic saturation. The numerical values of $\mathfrak{H}$ and $\mathfrak{B}$ are given below: the initial value of $\mathfrak{B}$ could not of course be determined, and it is assumed to have been zero. [Owing to an omission in the laboratory note-book some uncertainty attaches to the reduction of the magnetism of this ring to absolute measure. The example serves, however, to show the presence of hysteresis in the magnetisation of this material.]

CAST-IRON Ring, Plate 58, fig. 7.

$\mathfrak{H}$.	$\mathfrak{B}$.	$\mathfrak{H}$.	$\mathfrak{B}$.	$\mathfrak{H}$.	$\mathfrak{B}$.
0	0	− 0·73	2450	0·67	−2100
0·07	3	− 0·95	2420	0·95	−2050
0·20	12	− 1·26	2380	1·50	−1960
0·38	24	− 1·87	2290	2·39	−1790
0·73	48	− 2·37	2200	3·27	−1530
1·13	76	− 3·27	2010	3·94	−1390
1·87	145	− 3·94	1860	4·76	−1080
3·27	344	− 4·76	1600	5·77	− 620
5·54	864	− 5·74	1210	7·34	320
7·34	1490	− 7·34	360	8·86	1350
9·94	2360	− 8·86	− 660	11·28	2390
13·15	3170	−11·28	−1990	13·15	2895
15·75	3680	−13·15	−2670	15·75	3450
13·15	3560	−15·71	−3420	13·15	3330
9·81	3390	−13·15	−3300	9·94	3150
7·34	3200	− 9·94	−3200	7·34	2980
5·41	3080	− 7·34	−2940	5·41	2830
3·27	2900	− 5·41	−2790	3·27	2640
5·83	2760	− 3·27	−2590	1·88	2500
1·13	2680	− 1·87	−2430	1·18	2410
0·73	2640	− 1·26	−2340	0	2280
0	2550	− 0·73	−2320	1·13	2340
		0	−2200	1·83	2370
				3·27	2460
				5·41	2590
				7·34	2710
				9·93	2920
				13·13	3190
				15·71	3460

§ 25. *Magnetisation of Steel Wire in the hard-drawn, annealed, and glass-hard states.*—The next three experiments were made on a piece of steel wire 0·137 centim. in diameter, and 19·7 centims. or 144 diameters in length. This length would have been insufficient to allow of a fair experiment if the material had been soft iron, but steel is so much less susceptible that the effect of the ends is less important, and a condition approaching that of endlessness is reached with a shorter length than would be admissible in the case of iron. In all three experiments the step-by-step ballistic method was used.

The wire was first tested when in what may be called its normal temper, the hard-drawn state in which it was commercially supplied. Plate 58, fig. 8, shows its behaviour when a magnetising force of 57·5 units was applied, reversed, reapplied, removed and

reapplied. The greatest value of $\mathfrak{B}$ reached was between 14,000 and 15,000, and of this 57 per cent. survived the removal of the inducing force.

§ 26. The same wire was then annealed by heating to bright redness and cooling slowly, and observations made on it whose results are given graphically in Plate 58, fig. 9. The curves here have a form much more similar than those of fig. 8 to the curves for soft iron, but the susceptibility to moderate magnetising forces is much less than in iron. The induced magnetism is nearly the same as for the hard-drawn wire, but the residual magnetism is considerably greater for the annealed than for the hard-drawn wire, being now from 76 to 80 per cent.

§ 27. In the next experiment the same piece of wire was again tested, after being made glass-hard by being plunged into water while at a bright red heat. Its magnetic behaviour in this state is shown in Plate 58, fig. 10. The highest magnetising force applied (over 55 c.g.s. units) did not carry the wire past the steep part of the curve of magnetisation, and gave only 9300 as the value of $\mathfrak{B}$. On the removal of this force the residual value was 6360, or 68 per cent. of the induced. This must not, however, be taken as a measure of the percentage of residual magnetism which would have been found had the steel been more nearly "saturated." To remove the residual magnetism a reverse force of over 40 units was necessary. A noticeable feature in this case is the smallness of the hysteresis when the magnetising force was withdrawn and reapplied. The "off" and "on" curves for these operations are almost coincident. In fact, so far as this point is concerned, MAXWELL's extension of WEBER's theory (§ 3) would serve satisfactorily enough to explain the magnetic retentiveness of this material. Figs. 8, 9 and 10 are all drawn to the same scale, which is also the scale of fig. 6.

§ 28. *Experiments with Pianoforte Steel.*—The foregoing experiments on steel showed the desirability of testing that material under greater values of the magnetising force. Accordingly another series were made in which the method used was the direct magnetometric method (§ 18), and the magnetising force was raised to over 90 units. The wire used in this series of tests was pianoforte steel 0·078 centim. in diameter, and 30 centims. or 380 diameters long. Observations were first made with the wire in its normal commercial temper; their results are given below and also in Plate 58, fig. 11, which shows the relation of $\mathfrak{B}$ to $\mathfrak{H}$ while the magnetising force was being applied, reversed, reapplied, removed, and reapplied as in former examples.

PIANOFORTE Steel Wire, Normal Temper, Plate 58, fig. 11.

$\mathfrak{H}$.	$\mathfrak{B}$.	μ.	$\mathfrak{H}$.	$\mathfrak{B}$.	$\mathfrak{H}$.	$\mathfrak{B}$.	$\mathfrak{H}$.	$\mathfrak{B}$.
0	76		− 4·37	11,510	27·30	+3,590	+ 7·80	12,020
4·40	230		− 7·80	10,850	31·20	4,300	15·60	12,240
7·80	460	59	−15·60	8,580	35·20	7,830	31·20	12,690
15·60	1,540	99	−23·40	3,380	39·16	10,190	39·00	12,950
23·40	4,320	185	−31·20	− 4,115	46·80	12,080	46·80	13,230
31·20	7,770	249	−39·00	− 9,810	54·60	12,950	62·40	13,720
39·00	10,630	273	−46·80	−12,000	62·56	13,490	78·16	14,170
46·88	12,110	258	−62·48	−13,520	78·00	14,170	91·89	14,510
54·60	12,920	237	−78·00	−14,220	92·20	14,590		
62·40	13,490	216	−91·57	−14,590				
70·20	13,890	198			70·04	14,360		
78·00	14,200	182	−70·20	−14,320	46·80	13,820		
91·66	14,590	159	−46·80	−14,010	31·20	13,330		
			−31·20	−13,520	15·60	12,840		
70·20	14,300		−15·60	−12,950	7·80	12,400		
46·80	13,840		− 7·80	−12,550	0	11,850		
31·20	13,440		0	−11,970				
15·60	12,870		+ 7·80	−11,070				
7·80	12,420		15·60	− 9,020				
0	11,850		23·40	− 3,250				

§ 29. Plate 59, fig. 12 shows the behaviour, under corresponding variations of magnetising force, of a piece of pianoforte steel wire of the same quality and dimensions as the above, after it had been annealed by heating to bright redness and cooling slowly. The following figures apply to the first application of $\mathfrak{H}$:—

PIANOFORTE Steel Wire, Annealed, Plate 59, fig. 12.

$\mathfrak{H}$.	$\mathfrak{B}$.	μ.
0	−31 (initial)	—
4·99	398	80
7·80	895	115
11·70	2,010	172
15·66	3,585	229
19·50	5,220	268
23·40	6,750	289
27·30	8,040	295
31·20	9,090	291
35·26	9,900	281
39·00	10,530	270
43·06	11,110	258
46·88	11,520	246
54·60	12,300	225
62·56	12,920	207
70·28	13,390	191
78·47	13,850	176
91·73	14,400	157

§ 30. Plate 50, fig. 13, shows the behaviour of a third piece of pianoforte steel wire cut from the same bundle as the two last, but rendered glass-hard by sudden cooling in water. This time the length was 22 centims. or nearly 300 diameters. The numerical values of $\mathfrak{H}$, $\mathfrak{B}$, and μ during the first application of the force are given below :—

PIANOFORTE Steel Wire, Glass-hardened, Plate 59, fig. 13.

$\mathfrak{H}$.	$\mathfrak{B}$.	μ.
0	85 (initial)	—
10·28	670	65
15·42	1,150	74
20·76	1,770	85
25·69	2,500	98
30·83	3,260	106
35·97	4,020	112
41·11	4,710	115
46·25	5,470	118
51·39	6,060	118
56·57	6,690	118
61·97	7,300	117
66·80	7,880	117
72·25	8,460	117
77·08	8,880	115
82·22	9,380	114
87·36	9,780	112
98·11	10,700	109

§ 31. *Effect of permanent strain on the magnetic susceptibility and retentiveness of Iron.*—In the experiments on wrought-iron hitherto cited the metal used was in every instance in the annealed state. If after being annealed the piece be subjected to a longitudinal stress sufficient to cause permanent set, and, the stress having been removed, the magnetic quality be then examined, a very remarkable change will be found to have taken place. Not only is the piece less susceptible, especially to weak magnetising forces, but the form of the curves connecting $\mathfrak{B}$ or $\mathfrak{I}$ with $\mathfrak{H}$ is greatly changed. The residual magnetism is far less than when the material was in the soft annealed state.

In the following experiment with a piece of soft iron wire 60 centims. long and 0·158 centim. in diameter, the direct magnetometric method was used. The wire was annealed and put through a cycle of magnetisation as in former examples. It was then demagnetised by reversals (§ 19) and hardened by loading it with a weight of 60 kilos., which produced a permanent extension of nearly 6 centims. The weight was removed and a cycle of magnetisation again gone through.

The observations have been reduced by calculating $\mathfrak{H}$ and $\mathfrak{I}$ (instead of $\mathfrak{H}$ and $\mathfrak{B}$ as in former examples; $\mathfrak{B}$ may be readily found from the relation $\mathfrak{B}=4\pi\mathfrak{I}+\mathfrak{H}$). Of course in calculating $\mathfrak{I}$ for the wire after stretching, allowance has been made for the

change of dimensions of the piece. For the sake of easy comparison the numerical values of $\mathfrak{H}$ and $\mathfrak{I}$, and their ratio, κ, during the first application of the magnetising force, (1) before stretching, (2) after stretching, are given below. The force was in each case reversed, reapplied, removed, and reapplied as in former examples; but to avoid filling the page with tables of figures it will suffice to refer to fig. 14, Plate 59, where the whole experiment is graphically recorded.

Soft Iron Wire, Annealed and Hardened by Stretching, Plate 59, fig. 14.

Before stretching.			After stretching.		
$\mathfrak{H}$.	$\mathfrak{I}$.	κ.	$\mathfrak{H}$.	$\mathfrak{I}$.	κ.
0	0	—	0	0	—
0·07	2		0·07	0	0
0·09	3		0·14	0	0
0·14	4	29	0·29	2	7
0·28	10	36	0·42	4	9
0·42	16	38	0·70	7	10
0·58	24	41	0·99	13	13
0·70	33	47	1·44	21	15
0·99	62	63	1·73	27	16
1·16	91	79	2·14	41	19
1·30	140	108	2·88	72	25
1·44	195	135	3·58	116	32
1·58	280	177	4·20	167	40
1·76	364	207	4·90	218	44
2·02	468	232	5·76	265	46
2·14	507	237	7·20	359	50
2·28	549	241	10·78	566	53
2·51	614	245	11·90	613	52
2·74	673	245	15·20	751	51
2·88	702	244	17·50	817	47
3·16	764	242	23·61	947	40
3·58	842	235	29·81	1017	34
4·20	926	220	35·71	1078	30
5·76	1020	177	41·90	1114	27
6·46	1050	163			
7·20	1070	149			
8·64	1110	129			
10·26	1130	110			
11·91	1150	97			
17·50	1190	68			
23·61	1195	51			
35·71	1230	34			
45·51	1230	27			

In fig. 14 the curves in full lines, giving the relation of $\mathfrak{I}$ to $\mathfrak{H}$ for the wire before stretching, will be readily distinguished from their resemblance to those which have already been described and shown in earlier figures. The lower and more sloping curves with more rounded outlines, drawn in broken lines, exhibit the behaviour of iron which has been hardened by stretching. The effect of stretching is that the

subsequent magnetisation proceeds much less readily as the magnetising force is increased, but diminishes much more considerably as the force is withdrawn. The residual magnetism of the stretched wire is less than half that of the unstretched wire; but in the former it is much more stable, requiring nearly three times as much reverse magnetising force to remove it from the stretched as from the unstretched metal. The hysteresis in the stretched wire, during removal and reapplication of the magnetising force, is greater than in the case of annealed pieces.

Several other experiments of a similar kind have shown that the sloping curves of fig. 14 are thoroughly characteristic of strained iron. In fact the difference between them and the curves of magnetisation of an annealed specimen is so distinctly marked that it is easy, by applying this magnetising test, to distinguish a piece which has been strained beyond its limit of elasticity from a piece which has not been so strained.

A comparison of figs. 8 and 9 shows the same kind of difference between the curves for a steel wire when annealed (fig. 9), and when in its commercial hard-drawn state (fig. 8). The operation of wire-drawing gives a strain which differs from that to which the iron wire was subjected in the above experiment only in having lateral compression combined with the longitudinal extension.

§ 32. *Interpretation of* $\int \mathfrak{B} d\mathfrak{H}$ *in a Cycle of Magnetisation.*—The hysteresis which occurs in the relation of magnetisation in iron to magnetising force has been observed and commented on by Professor E. WARBURG* in a paper with which I was not acquainted when the preliminary notice of my own work, referred to above (§ 1), was published. He has also anticipated me in pointing out the important physical value possessed by the *area* of curves representing the relation of $\mathfrak{B}$ to $\mathfrak{H}$ or $\mathfrak{I}$ to $\mathfrak{H}$ during cyclic changes of magnetisation.

The quantity $-\int \mathfrak{I} d\mathfrak{H}$ for such a cycle is, in absolute measure, the work spent in conducting the metal through its changes of magnetisation, per unit of volume. This may very readily be deduced from the general expression for the energy of the magnetic field given by MAXWELL (El. and Mag. II., § 636) as

$$-\frac{1}{8\pi} S.\mathfrak{B}\mathfrak{H}.$$

* Wied. Ann., xiii., p. 141. The rods experimented on by WARBURG were, with one exception, scarcely long enough to satisfy the condition of approximate endlessness mentioned in § 7, as needful in experiments intended to answer the question raised in § 2. Reference should also be made in this connexion to recent papers by F. AUERBACH, Wied. Ann., xiv. (1881), and C. FROMME, Wied. Ann., iv. (1878), and xiii. (1881). The papers of AUERBACH and FROMME deal, under the name of "magnetische Nachwirkung," with the influence which previous magnetic state has on the actual magnetisation of iron and steel; but their mode of treating the subject differs considerably from that of WARBURG. AUERBACH distinguishes rightly between the "Nachwirkung" which is static, and that which (like the "elastische Nachwirkung" of strained india-rubber, or the "residual polarisation" of a dielectric solid) depends on time. It is the former of the two with which we are now concerned. Neither FROMME nor AUERBACH used rods long enough to give an approach to uniform magnetisation.

In the present case $\mathfrak{B}$ and $\mathfrak{H}$ have the same direction. For any indefinitely small change of the field

$$dE = -\frac{1}{8\pi}(\mathfrak{H}d\mathfrak{B} + \mathfrak{B}d\mathfrak{H}).$$

When the change is cyclic, $\int\mathfrak{H}d\mathfrak{B}$ and $\int\mathfrak{B}d\mathfrak{H}$ are equal, and

$$\int dE = -\frac{1}{4\pi}\int\mathfrak{B}d\mathfrak{H}$$
$$= -\int\mathfrak{I}d\mathfrak{H}.$$

This is the energy expended in performing the cycle, per unit of volume of the substance magnetised, and it has a positive value for all actual cycles of magnetism in iron or steel in consequence of the hysteresis which exists in the relation of $\mathfrak{I}$ to $\mathfrak{H}$. The energy so expended is a thing quite apart from the further dissipation which occurs when, on account of the changes of magnetism taking place not indefinitely slowly, currents are induced in the substance of the magnet and in neighbouring conductors, and also from that which occurs in consequence of the thermomagnetic properties of the metal.* These latter sources of dissipation are functions of the time, and are absent when the magnetic change occurs indefinitely slowly: their effects do not appear in the cyclic curves of $\mathfrak{I}$ and $\mathfrak{H}$ when we leave a sufficient interval after each change of $\mathfrak{H}$ before taking a reading of $\mathfrak{I}$. The former, however, is independent of the rate of magnetisation, the hysteresis to which it is due being a static phenomenon; and if we accept the idea that the magnetic molecules of WEBER have their rotation opposed by a species of static friction, the dissipation of energy we are now considering is the work done in forcing them to rotate back and forth against this friction.

§ 33. *Numerical Values of* $\int\mathfrak{I}d\mathfrak{H}$ *for various Cycles.*—Measurements have been made of the values of $\int\mathfrak{B}d\mathfrak{H}$ in a number of the cycles of magnetisation in iron and steel which have been already described and figured. These, divided by 4π, give the corresponding values of $\int\mathfrak{I}d\mathfrak{H}$, or the energy expended, in c.g.s. units of work or ergs., in carrying one cubic centimetre of the material through the cycle in question.

	Ergs.
(1) For the large cycle of fig. 4 (§ 20), produced by the double reversal of magnetisation in a piece of very soft annealed iron wire from $\mathfrak{B}=13{,}450$, $\mathfrak{H}=17{\cdot}26$, to $\mathfrak{B}=-13{,}190$, $\mathfrak{H}=-17{\cdot}26$, and back, the value of $\int\mathfrak{I}d\mathfrak{H}$ is	9,300
(2) For the small loop of the same figure, produced by removing and restoring the magnetising force of $+17{\cdot}26$, the value of $\int\mathfrak{I}d\mathfrak{H}$ is . .	490
(3) For the large cycle of fig. 5 (§ 21), produced by the double reversal of magnetisation in a less soft annealed iron wire, from $\mathfrak{B}=15{,}710$, $\mathfrak{H}=22{\cdot}27$, to $\mathfrak{B}=-15{,}550$, $\mathfrak{H}=-23{\cdot}08$, and back, the value of $\int\mathfrak{I}d\mathfrak{H}$ is	16,300

* Sir W. THOMSON, Phil. Mag., vol. v., 1878, pp. 24, 25.

Ergs.

(4) For the large cycle of fig. 6 (§ 22), produced by the double reversal of magnetisation in a piece of annealed iron wire similar to the above, but between higher limits of magnetising force, namely, from $\mathfrak{H}=90$ to -90, and back, the value of $\int \mathfrak{I} d\mathfrak{H}$ is . . . . . * 16,700

(5) For the upper small loop of fig. 6, produced by removing and restoring the magnetising force of $+90$, the value of $\int \mathfrak{I} d\mathfrak{H}$ is . . 350

(6) For the cast-iron ring of § 24, fig. 7, the reversal of magnetisation from the greatest positive value there produced to an equal negative value, and back, gives as the value of $\int \mathfrak{I} d\mathfrak{H}$ 6,100

This, however, is obviously much less than the energy which would be expended in a double reversal of magnetisation in cast-iron, were the magnetisation as nearly complete as in the experiments on wrought iron and steel.

The above values of the energy dissipated in producing cyclic changes of magnetisation are greatly exceeded when the material is steel.

(7) For the hard-drawn steel wire of § 25, fig. 8, the double reversal of magnetisation between the extreme positive and negative limits ($\mathfrak{B}=$ about 14,600, $\mathfrak{H}=57{\cdot}5$), gives for the value of $\int \mathfrak{I} d\mathfrak{H}$ 60,000

And the small loop, for the removal and restoration of $\mathfrak{H}$ in the same experiment, on the positive side, gives 6,300

(8) The corresponding quantities in the case of the same steel wire, annealed (§ 26, fig. 9), are :—

For the double reversal . . . . . $\int \mathfrak{I} d\mathfrak{H} =$ 70,500

For the small loop . . . . . $\int \mathfrak{I} d\mathfrak{H} =$ 1,800

(9) And in the case of the same steel wire, glass-hardened (§ 27, fig. 10):

For the double reversal . . . . . $\int \mathfrak{I} d\mathfrak{H} =$ 76,000

For the small loop . . . . . $\int \mathfrak{I} d\mathfrak{H} =$ 700

In this case, as in the case of cast-iron, the magnetisation was so far from "saturation" that a comparison of the above values of the integral with those of other experiments would be unfair.

In the three next examples the magnetising force was much greater than in the above, and the energy expended in going through the cycles was also greater. The measurements are those of §§ 28–30, figs. 11, 12, and 13, the material being pianoforte steel, and the magnetising force ranged from $+100$ to -100 nearly.

* This value was inadvertently given as 1670 instead of 16,700, in a paper read before the British Association at Southport (Report for 1883, p. 404), and reprinted in the Phil. Mag. for November, 1883, p. 383.

		Ergs.
(10)	Pianoforte steel at normal temper (fig. 11) :—	
	For the double reversal	$\int \mathfrak{B} d\mathfrak{H} = 116{,}000$
	For the small loop	$\int \mathfrak{B} d\mathfrak{H} = 3{,}500$
(11)	The same, annealed (fig. 12) :—	
	For the double reversal	$\int \mathfrak{B} d\mathfrak{H} = 94{,}000$
	For the small loop	$\int \mathfrak{B} d\mathfrak{H} = 5{,}000$
(12)	The same, glass-hardened (fig. 13) :—	
	For the double reversal	$\int \mathfrak{B} d\mathfrak{H} = 117{,}000$
	For the small loop	$\int \mathfrak{B} d\mathfrak{H} = 2{,}500$

The following measurements are taken from the experiment of § 31, fig. 14, in which an iron wire was subjected to a large cycle of magnetisation first in the annealed state, and then again after being stretched considerably beyond its limit of elasticity. The values of $\int \mathfrak{B} d\mathfrak{H}$ (for a double reversal of the greatest intensity of magnetisation) are :—

(13)	For the wire before stretching	10,000 ergs.
(14)	For the same wire after stretching	16,400 „

The difference is considerable, but it is scarcely so striking as the other differences in magnetic behaviour which have been described in § 31 above.

Heating effect of cyclic changes of Magnetism.

§ 34. The energy expended in a cyclic process of magnetisation can take no other form than that of heat, diffused throughout the substance of the metal. Experiments have been made by JOULE and others to determine by direct observation the heating effect of magnetisation in iron.* In most direct measurements of this quantity no distinction is made between the heating effect due to the induction of electric currents and that due to changes of magnetisation *per se*, and so excellent an authority as Professor ROWLAND, writing in 1881, has expressed himself as doubtful whether changes of magnetisation, considered apart from the currents they induce, give rise to any development of heat at all.† But no direct calorimetric measurements are needed to show that a cyclic process of magnetisation does give rise to a certain development of heat, even if it be performed so slowly as to make the dissipation of energy through

* JOULE, Phil. Mag., xxiii., 1843; GROVE, Phil. Mag., xxxv., 1849; VILLARI, 'Nuovo Cimento,' 1870; CAZIN, Ann. de Chim. et de Phys., 1875; TROWBRIDGE, Proc. Amer. Acad. of Arts and Science, 1879. Other references are given by WARBURG in the paper cited in the text.

† "I do not mean to here affirm that no heat can be due to the demagnetisation alone, but that we have at present no experimental proof of such direct transformation. All the experiments hitherto made have merely given us the heating due to induced currents."—ROWLAND, 'Scientific American, Supplement,' Nov. 5, 1881.

the induction of currents vanish. The positive value which the integral $-\int \mathfrak{B} d\mathfrak{H}$ always has is of itself conclusive evidence, and the magnitude of this quantity can be determined in absolute measure with an accuracy which would be unattainable in direct measurements of the heating effect which it represents. WARBURG, who has made determinations of the value of $-\int \mathfrak{B} d\mathfrak{H}$ in certain cases, has (along with L. HÖNIG, Wied. Ann., xx., 1883, p. 814) compared them with the results of direct calorimetric measurements, the difficulty of which is well illustrated by the poor agreement he finds between the two classes of observations.*

From the values given above for the integral $-\int \mathfrak{B} d\mathfrak{H}$ we may easily calculate the rise of temperature which a piece of iron or steel suffers when subjected to the cyclic changes of magnetism to which these values refer. Let us assume that the operation is conducted so slowly that $-\int \mathfrak{B} d\mathfrak{H}$ represents the whole work done, and that the heat produced is prevented from leaving the metal. Then, taking JOULE's equivalent as 41,600,000 ergs. per gramme-degree (centigrade), the specific gravity of the metal as 7·7, and its specific heat as 0·11, we have—

Rise of temperature for every erg. expended per cubic centimetre

$$= \frac{1}{41{,}600{,}000 \times 7{\cdot}7 \times 0{\cdot}11}$$
$$= 2{\cdot}84 \times 10^{-8}$$

in degrees centigrade.

We have seen that the double reversal of a *strong* condition of magnetism in soft iron involves the expenditure of about 10,000 ergs. per cubic centim., and the consequent rise of temperature is therefore 0·000284° C. Nearly 4000 double reversals of a magnetic state approaching saturation would therefore be necessary to raise the temperature of a piece of soft iron by 1° C., if we could eliminate the action of induced currents in the metal.

The largest of the values given above (for the case of hard steel) is 117,000 ergs., which corresponds to a rise of temperature of 0·0033° C. per cycle. The rise in other instances of reversal of magnetism, and that caused by removal and reapplication of the magnetising force, may easily be found from the figures already given.

These developments of heat are so small as to make it apparent that the very considerable thermal effects which reversal of magnetism causes in the revolving cores of some dynamo-electric machines must be due almost wholly to the internal induction of currents, so far as they are not due directly to the current circulating in the coils

* Another direct method of observing experimentally the dissipation of energy involved in cyclic changes of magnetism is to measure the "damping" of a swinging magnet by the induction of magnetism in an iron plate placed near it; but this also depends largely on the induction of currents. *Cf.* WARBURG, *loc. cit.*, also F. HIMSTEDT, Wied. Ann., xiv. (1881). HIMSTEDT has concluded from an experimental examination that the greatest part of this damping is due to what I term statical hysteresis, for the effects were found to be in great measure independent of the frequency of vibration of the needle.

of the armature. The experiments have a practical value in showing that cores which are so thoroughly laminated as to render the induction of currents within them unimportant, do not involve any serious loss of energy, and that the efficiency of a machine with a soft iron core or cores whose magnetism is periodically reversed need not, on that account, be materially less than that of a machine which has no such cores. The absence of iron from the armature has been claimed, on the score of efficiency, as an important advantage possessed by some types of machine, but unless the claim has some other basis it appears to me to be illusory. Magnetic reversal does involve some loss of energy, but if the cores are properly laminated so that the loss consists almost entirely of the quantity $-\int \mathfrak{H} d\Phi$, it is so small as to be practically insignificant.

I shall show later that when a cycle of magnetisation is performed while the iron is kept in a state of mechanical vibration the value of $-\int \mathfrak{H} d\Phi$ is much less than when the same cycle is performed with the metal in a state of rest, and indeed almost vanishes in soft iron. Hence, in a dynamo, where vibration occurs to a greater or less degree whenever the machine is running, the energy dissipated through changes of magnetisation is even less than these experiments on still metal might lead us to expect.

[Note added March 25, 1886.—The heating effect of cyclic changes of magnetism, especially of reversals, is important from a practical point of view not only in relation to dynamo-electric machines, but also to "secondary generators," or induction coils for the distribution of electrical energy by electromagnetic induction. In § 52 (below) mention will be found of occasional evidence which these experiments furnished that there is a true *time lag* in the magnetisation of iron: in other words, that there is a certain degree of viscous as well as static hysteresis in the relation of magnetism to magnetising force. Soft iron, especially in early stages of magnetisation, exhibits a sluggishness in assuming the magnetic state proper to the magnetising current—a sluggishness which does not appear explicable as an effect of the induction of currents within the substance of the iron, nor as due to the self-induction of the magnetising circuit. The result of this magnetic viscosity is to augment the value of $-\int \mathfrak{H} d\Phi$ in any rapidly performed cycle, and consequently to increase the evolution of heat. It seems highly probable that in the comparatively slow reversals of magnetism which the core of a dynamo armature undergoes the additional dissipation of energy due to this cause is scarcely sensible. In the case of "secondary generators," however, where reversals of magnetism occur far more rapidly, there is nothing to show that this source of loss is not a sensible portion of the whole; but the high efficiency which this kind of apparatus has been proved capable of makes it at least clear that the magnetic viscosity of the core gives rise to no very serious loss of power.

In connexion with "secondary generators" and induction coils generally, the bearing of the first part of this paper should be noted, as showing the enormous

advantage which a ring-shaped core, or core forming a complete magnetic circuit, possesses over a short bar core with ends. In an ordinary induction coil, so long as the current in the primary circuit is merely made and broken, a short core is necessary, since a ring core would lose but a small percentage of its magnetism at each break; but where reversal of the magnetising current takes place a core approximating to the condition of endlessness has an advantage, in respect of power, which fig. 3 makes obvious.]

§ 35. *Heating Effect of Lower Cycles.*—In most of the examples cited above (§ 33), in which the energy expended in cyclic changes of magnetisation has been determined, the magnetisation was strong, lying in the region of so-called saturation. It seemed desirable to measure the energy expended in carrying iron and steel through double reversals of weaker magnetisation, and accordingly the two following special experiments were made.

In the first a piece of annealed iron wire, 0·078 centim. in diameter and 29 centims. long, was tested by the direct magnetometric method. Starting initially with a state of no magnetisation, the magnetising force was slowly raised to 1·5, then slowly reversed to −1·5, and then slowly re-reversed to 1·5, numerous intermediate observations of $\mathfrak{I}$ and $\mathfrak{H}$ being made. The curves showing the relation of $\mathfrak{I}$ to $\mathfrak{H}$ in this process were afterwards plotted, and the area enclosed by them was measured. Then the magnetising force was further increased to 1·95, and another cycle performed by reversing and re-reversing it. Next it was raised to 2·56, and another cycle performed by double reversal. Next to 3·01, when another cycle was performed, and so on, until the last double reversal took place between the values + 75·2 and −75·2 for $\mathfrak{H}$. In this way 10 areas were measured, giving 10 values of the energy expended in reversing and re-reversing the magnetisation of the wire at different grades of intensity. The whole process is shown graphically in Plate 60, fig. 15, which is a very much reduced version of the diagram by help of which the areas were measured. It would take too much space to reproduce here all the readings taken in determining these curves, as no fewer than 338 corresponding values of the magnetisation and the magnetising field were measured. The following are the values of $\mathfrak{I}$, $\mathfrak{B}$, and $\mathfrak{H}$ at the successive points at which the magnetism was reversed in sign, and the corresponding values in ergs of $-\int \mathfrak{I} d\mathfrak{H}$ for the operation of double reversal. The last column gives the rise of temperature caused by one complete operation of double reversal of magnetism.

GRADED Cyclical Magnetisations of Soft Iron, Plate 60, fig. 15.

$\mathfrak{H}$.	$\mathfrak{B}$.	$\mathfrak{I}$.	$-\int\mathfrak{I}d\mathfrak{H}$.	Calculated rise of temperature.
			ergs.	deg. C.
1·50	1,974	157	410	0·000012
1·95	3,830	304	1,160	0·000033
2·56	5,950	473	2,190	0·000062
3·01	7,180	571	2,940	0·000083
3·76	8,790	699	3,990	0·000113
4·96	10,590	842	5,560	0·000158
6·62	11,480	913	6,160	0·000175
7·04	11,960	951	6,590	0·000187
26·5	13,700	1090	8,690	0·000247
75·2	15,560	1230	10,040	0·000285

If the relation of $\int\mathfrak{I}d\mathfrak{H}$ for double reversals of $\mathfrak{H}$ to the value of $\mathfrak{H}$ at which reversal takes place be plotted, it will be seen that the values of the integral appear to be approaching a limiting value, and the general form of the curve is not very different from that of a curve of $\mathfrak{I}$ and $\mathfrak{H}$ or $\mathfrak{B}$ and $\mathfrak{H}$. By plotting $\int\mathfrak{I}d\mathfrak{H}$ and $\mathfrak{I}$ it may, however, be shown that (except at the very beginning of the curve) the values of the integral, as the loops become larger, increase less rapidly than the values of $\mathfrak{I}$ between which reversal is made to take place.

§ 36. *Graded Cyclical Magnetisations of Steel.*—A similar experiment was made, also by the direct magnetometric method, with a piece of annealed pianoforte steel wire, 0·078 centim. in diameter, and 32 centims. in length. Seven double reversals were effected with magnetising forces graded up to 82 c.g.s. units. The resulting relations of $\mathfrak{I}$ and $\mathfrak{H}$ are shown in Plate 60, fig. 16. The loops formed by double reversal of $\mathfrak{H}$ were, especially with low values of $\mathfrak{H}$, far from closed (the same characteristic is noticeable in the curves for steel which have been already described), and for that reason the experiment scarcely allows comparative measurements of the successive areas to be made.

A noticeable feature in this diagram is the want of symmetry between the positive and negative sides. The positive values of $\mathfrak{I}$, coming as they did *first*, are greater than the negative values induced by an opposite and equal $\mathfrak{H}$. This characteristic is very conspicuous in the early parts of the operation, but disappears when the magnetisation becomes strong. The same feature is present in many other diagrams.

Ratio of Residual to Total Induced Magnetism in Iron and Steel.

§ 37. The large fraction of the whole magnetisation, which was found to survive the removal of magnetising force, especially in annealed iron, has already been alluded to. A series of special experiments were made to determine the amounts of residual magnetism in various specimens of iron and steel, when the induced magnetism was varied within widely different values.

These experiments were in most cases made by the direct magnetometric method. A straight piece of wire, long enough to approximate as nearly as was practicable to the condition of endlessness, was hung vertically behind the magnetometer in the manner described in § 18. The magnetising force was applied by means of a solenoid wound close to the wire, while the vertical component of the earth's field was neutralised by maintaining a constant current of the necessary strength in another solenoid wound on a tube which enclosed the first. The correct value of the current in the second solenoid was calculated with reference to the known value of the earth's force, and then verified experimentally by subjecting to the process of demagnetising by reversals (§ 19) a long piece of soft iron in the interior. This process gives complete demagnetisation when the piece subjected to it lies in a perfectly neutral field, but leaves a large residue when the field is not fairly balanced. It thus affords an accurate test of the exact neutralisation of the earth's vertical component.

In some instances the ballistic method was used, and then the rods experimented on were laid at right angles to the terrestrial magnetic field. A few experiments were also made by the ballistic method on rings. The results obtained from long rods by the two methods, and those from rings, were in substantial agreement.

Plate 60, fig. 17, shows an actual example of the direct magnetometric method of investigating the ratio of residual to temporary magnetisation. Starting from a state of no magnetism, a force $\mathfrak{H}$ was applied and removed, then again applied and carried to a somewhat higher value, again removed, again reapplied and carried to a somewhat higher value, again removed, and so on. The application and removal of the force was conducted very gradually, and in this particular example intermediate readings of the magnetism during the application and removal were taken, from which the curves in fig. 17 have been plotted. Generally, however, only the induced and the residual magnetism for each stage in the increase of $\mathfrak{H}$ were determined. The following table gives the successive values of the applied force $\mathfrak{H}$, the intensity of induced magnetism, and the residual magnetism which survived the removal of the force, and finally the ratio of the residual to the induced or temporary magnetism. This ratio increases with increased strengths of magnetisation, and in this case does not pass a maximum even at the highest value of $\mathfrak{H}$. The piece tested here was the soft annealed iron wire of fig. 5 (§ 21), 0·077 centim. in diameter, and 30·5 centims. or 400 diameters in length.

INDUCED and Residual Magnetism in Soft Iron, Plate 60, fig. 17.

$\mathfrak{H}$.	$\mathfrak{B}$ induced.	$\mathfrak{B}$ residual.	Ratio of $\mathfrak{B}$ residual to $\mathfrak{B}$ induced.
1·34	413	165	0·400
3·33	6,690	5,660	0·846
4·88	11,910	10,790	0·906
6·80	14,060	13,060	0·921
8·43	14,800	13,640	0·922
13·74	15,670	14,550	0·926
22·37	16,080	15,010	0·933

The ratio in this example is slightly greater than I ever observed it in other cases, though values of 0·87 and even 0·90 were not uncommon, and in one other example 0·92 was exceeded. It appears to be approaching a maximum. In many other instances, which will be cited below, this maximum was actually passed, by using sufficiently great magnetising forces.

§ 38. The series of double loops described in § 35, and shown in fig. 15, give another set of values of residual and temporary magnetism corresponding to a greater set of values of $\mathfrak{H}$, in an annealed iron wire (length = 29 centims., diameter = 0·078 centim.). The following are the values they yield of $\mathfrak{H}$, $\mathfrak{I}$ (induced) and the ratio of the residual to the induced value of $\mathfrak{I}$. The figures given refer to the positive side of the diagram.

INDUCED and Residual Magnetism in Soft Iron, Plate 60, fig. 15.

$\mathfrak{H}$.	$\mathfrak{I}$ induced.	Ratio of residual to induced.
1·50	157	0·651
1·95	304	0·781
2·56	473	0·824
3·01	571	0·842
3·76	699	0·837
4·96	842	0·836
6·62	913	0·825
7·04	951	0·805
26·5	1090	0·753
75·2	1230	0·683

Here a distinct maximum is passed, when the ratio is about 0·84, after which it falls very considerably as the magnetising force is strengthened.

§ 39. As an example of the ballistic method the case may be cited of a *ring* of iron wire 0·248 centim. in diameter, the diameter of the ring being 8·3 centims. The following values were obtained :—

Φ.	Ratio of residual to induced.
0·70	0·165
0·95	0·172
1·36	0·183
1·68	0·353
2·31	0·537
2·75	0·577
3·21	0·671
3·78	0·729
4·62	0·774
5·35	0·803
6·41	0·818
7·15	0·825
8·10	0·829

§ 40. The following series of determinations of total and residual magnetism was made by the direct magnetometric method with another annealed iron wire, 0·078 centim. in diameter, and 28·7 centims. in length. The results are given numerically in the table below. Here the maximum in the ratio was 0·906, but the subsequent falling off in its value was less than in the former case (§ 38), as the magnetising force was not raised so far.

Φ.	$\mathfrak{I}$ induced.	$\mathfrak{I}$ residual.	Ratio of residual to induced.
0·86	26	6	0·250
1·48	67	26	0·381
1·98	164	96	0·598
2·66	478	378	0·793
3·24	680	574	0·843
3·78	802	696	0·868
4·29	888	786	0·884
4·86	956	859	0·899
5·40	991	898	0·906
6·05	1036	918	0·890
6·81	1067	946	0·883
7·56	1100	966	0·881
9·29	1142	997	0·873
11·20	1166	1014	0·868
12·64	1180	1023	0·868
14·59	1196	1033	0·863
15·49	1203	1034	0·861
17·24	1212	1042	0.859

In another experiment the ratio of residual to induced magnetism in an annealed iron wire passed a maximum of 0·885 with $\Phi=6$, and fell to 0·779 with $\Phi=90$.

§ 41. Another very extensive series of observations of this ratio was made on the annealed iron wire of § 31, both before and again after it was hardened by stretching,

as described in that paragraph. The influence of stretching in reducing the retentiveness as well as the magnetic susceptibility of the iron is very remarkable. Here, again, the method was the direct magnetometric one.

Soft Iron Wire, Annealed and Hardened by Stretching, Plate 59, fig. 18.

Before stretching.				After stretching.			
$\mathfrak{H}$.	$\mathfrak{Z}$ induced.	$\mathfrak{Z}$ residual.	Ratio of residual to induced.	$\mathfrak{H}$.	$\mathfrak{Z}$ induced.	$\mathfrak{Z}$ residual.	Ratio of residual to induced.
0·42	16	3·9	0·24	0·42	3·6	0	0
0·58	24	6·6	0·27	0·99	13·1	2·9	0·22
0·70	33	9·9	0·30	1·44	21·1	6·5	0·31
0·99	62	24	0·40	1·73	26·9	11·8	0·38
1·16	91	46	0·50	2·14	41	15·3	0·38
1·30	140	85	0·61	2·88	72	32·7	0·46
1·44	195	133	0·68	3·58	116	61·7	0·53
1·58	280	209	0·74	4·20	167	98	0·59
1·76	364	283	0·78	4·90	218	132	0·61
2·02	468	380	0·81	5·76	265	167	0·63
2·14	507	418	0·82	7·20	359	225	0·625
2·28	549	455	0·83	10·78	566	327	0·58
2·51	614	513	0·84	11·90	613	348	0·57
2·74	673	568	0·85	15·20	751	381	0·51
2·88	702	598	0·85	17·50	817	399	0·49
3·16	764	650	0·85	23·61	947	414	0·44
3·58	842	711	0·85	29·81	1017	417	0·41
4·20	926	783	0·85	35·71	1078	419	0·39
5·02	984	832	0·84	41·90	1114	419	0·38
5·76	1020	848	0·83				
6·46	1050	864	0·82				
7·20	1070	877	0·82				
8·64	1110	897	0·81				
10·26	1130	910	0·80				
11·91	1150	913	0·80				
17·50	1190	929	0·79				
23·61	1195	929	0·78				
35·71	1230	933	0·76				
45·51	1230	933	0·76				

These results are also shown in Plate 59, fig. 18, where the full lines express in terms of $\mathfrak{H}$ the values of the induced and residual magnetism, and their ratio, *before* stretching, and the dotted lines show the corresponding quantities *after* stretching. The maximum which the ratio of residual to temporary magnetism passes is even more distinctly marked in the hardened than in the annealed condition. The curves of fig. 18 show very clearly that the residual magnetism approaches saturation much sooner than the induced magnetism does, as $\mathfrak{H}$ is increased. In the region of strong force, while the curves of induced magnetism are still rising considerably, the curves of residual magnetism are running very nearly parallel to the axis of $\mathfrak{H}$, and it is to this that the maximum in the ratio is due.

§ 42. Several experiments with other pieces of iron wire, first annealed and then hardened by stretching, have fully confirmed the results of the preceding paragraph. Plate 61, fig. 19, shows the relations of induced and residual magnetism, and of their ratio, to magnetising force in another piece of stretched iron wire, which was much more strongly magnetised than the wire in the former example. Here the ratio, after touching a maximum of 0·6 with $\mathfrak{H}=9$, fell to 0·32 when the magnetising force was raised to 92.

Other experiments on the same subject will be found below in §§ 109–112, where the influence of stress on the ratio of residual to induced magnetism is considered.

§ 43. My experiments were better adapted for finding the proportion of residual to temporary magnetism when the magnetisation was moderately strong than when it was very feeble. So far as can be judged from the curves and figures given above, the ratio is initially zero, that is to say, the earliest developments of magnetism, induced by a magnetising force rising from nothing, are entirely temporary, and disappear altogether when the force is withdrawn. But these experiments are inconclusive on this point, though they show distinctly that, in any case, very weak magnetisation is very slightly retained when the magnetising force is withdrawn, even by specimens of iron which will retain as much as 90 per cent. of stronger magnetisation. In steel the absence of retentiveness at low magnetising forces is still more marked. It is clear from this that if we are to ascribe retentiveness to a frictional sticking of the displaced molecules, we must complicate the hypothesis by believing that all or some of the molecules have a certain range of possible displacement within which this friction does not act, or that some of them are free from frictional retardation.

§ 44. *Ratio of Residual to Induced Magnetism in Steel.*—Corresponding observations were made on pianoforte steel, in the three states already mentioned, namely, in the normal commercial temper, annealed, and glass-hardened by sudden cooling. The wires tested were in each case 0·78 mm. diameter and about 30 centims. long.

For pianoforte steel in its normal temper we have the following values of induced and residual magnetism for a graded set of values of $\mathfrak{H}$. The ratio here passes a distinct maximum, and (in conformity with what has just been said) is, in the early part of the experiment, exceedingly small. The following are the actual magnetometer readings, corrected for the electromagnetic action of the magnetising solenoid. To reduce them to $\mathfrak{I}$ they have only to be multiplied by 6·06.

PIANOFORTE Steel Wire, Normal Temper, Plate 61, fig. 20.

$\mathfrak{H}$.	Magnetometer readings.		Ratio of residual to induced.
	Induced.	Residual.	
9·4	5·0	0·6	0·120
10·9	7·1	1·5	0·211
12·5	10·1	3·3	0·327
14·0	13·8	6·0	0·435
15·6	17·8	9·0	0·505
17·2	24·3	14·1	0·580
18·9	30·6	19·7	0·644
20·3	37·5	26·4	0·704
23·4	54·9	41·0	0·747
25·0	64·0	48·8	0·762
26·5	72·2	56·9	0·788
28·2	82·9	66·0	0·796
29·6	90·3	73·0	0·808
31·2	98·2	80·3	0·818
32·8	106·4	87·8	0·825
34·3	113·5	94·4	0·832
35·9	120·2	100·5	0·836
37·4	126·2	106·0	0·840
42·1	140·5	120·2	0·856
46·9	150·4	127·5	0·848
54·6	161·1	135·7	0·842
62·4	167·5	140·9	0·841
70·2	172·6	143·9	0·830
78·0	176·0	146·0	0·829
90·9	182·1	149·0	0·818

This experiment is shown graphically in fig. 20, where, as in the former cases, curves are given showing the relation of $\mathfrak{H}$ induced, $\mathfrak{H}$ residual, and their ratio, to $\mathfrak{H}$.

§ 45. Plate 61, fig. 21, gives in the same way the corresponding results for a piece of the same pianoforte steel wire, annealed. Here the ratio of residual to temporary magnetism touches a maximum of 0·805 with $\mathfrak{H}=23$, and falls to 0·71 with $\mathfrak{H}=97$.

§ 46. Finally, Plate 61, fig. 22, gives corresponding results for a piece of the same steel wire, glass-hardened. Here even the strongest magnetising force used did not saturate the wire sufficiently to allow the maximum in the ratio of residual to induced magnetism to be very distinctly passed. It appears, however, to have been reached, and its value was 0·69.

§ 47. *Summary of Conclusions as to the Ratio of Residual to Induced Magnetism in Iron and Steel.*—The results of this section of the experiments may be summarised as follows :—

(1) When the induced magnetisation of iron or steel is very weak it almost all disappears on the removal of the inducing field.

(2) When it is increased its capability of being retained also increases, very rapidly, and the ratio of residual to induced magnetism reaches a maximum.

(3) Roughly speaking, the intensity of magnetisation at which this maximum in the ratio appears is in the region of what WIEDEMANN has called the "Wendepunct," or point at which the ratio of $\mathfrak{I}$ to $\mathfrak{H}$ is a maximum (the point at which a tangent drawn through the origin meets the curve connecting $\mathfrak{I}$ and $\mathfrak{H}$.)

(4) The value of the maximum in the ratio of residual to induced magnetism is from 0·84 to 0·93 in annealed iron, nearly as great in annealed, hardened, or tempered steel, but much less in hard-drawn iron, where its value is more like 0·6.

(5) As the magnetisation is further increased the residual magnetism approaches saturation more rapidly than the induced, and consequently this ratio diminishes—slightly in soft iron, more in steel, and much more in hard-drawn iron.

By induced magnetism, in the above statements, is meant the value of $\mathfrak{I}$ which is induced when a magnetising force $\mathfrak{H}$ is applied gradually, and without mechanical disturbance, the piece under test being initially in a neutral state.

By residual magnetism is meant the value of $\mathfrak{I}$ which remains when, after being so applied, the magnetising, force is removed gradually, and without mechanical disturbance.

No magnetising force must act on the piece except the assumed force $\mathfrak{H}$, and consequently the conditions of the experiment are realised only when we use either endless magnets or magnets of such length that the field produced by the ends is sensibly inoperative throughout the greater part of their length. These conditions were fulfilled in the experiments which have been described, as far as it seems practically possible to fulfil them. Any actual sample of iron must be only imperfectly homogeneous, and consequently its magnetisation, in a uniform field, cannot be uniform, apart from the action of the ends. Probably no form of specimen can be found which is more nearly homogeneous, and at the same time uniform in section, than the form chosen in these experiments—a drawn wire with a diameter of about one millimetre.

At the same time, the magnetisation of these wires, though perhaps as nearly uniform in the central part of their length as is practicable in any experiment, could not have been absolutely uniform, nor were they quite free from self-demagnetising forces when the field due to the solenoid was removed. When we consider that under the necessarily imperfect conditions of the experiments as much as 90 and even 93 per cent. of the magnetisation of soft iron was retained on the removal of the solenoid's magnetising force, it seems far from unlikely that soft iron, if tested under ideally perfect conditions, would be found to have perfect retentiveness, that is to say, that none of the induced magnetisation would disappear on the gradual removal of all inducing force. This view does not admit of experimental verification or disproval: the experiments show that it is at least not untenable.

Effects of Vibration on Induced and Residual Magnetism in Iron.

§ 48. It is well known that mechanical disturbance makes iron take up more magnetism under an inducing force than it would take up if undisturbed, and that after the withdrawal of an inducing force mechanical disturbance reduces the residual magnetism. I was, however, unprepared to find these effects exhibit themselves so markedly as they did in certain specimens of annealed iron.

It has been already mentioned (§ 14) that after an annealed wire of soft iron had been magnetised, and the magnetising force removed, gentle tapping had the effect of almost completely removing the residual magnetism, great as that was. In fact, in dealing with very soft annealed iron, I generally found mechanical vibration—produced by gently beating against the table—to be the most convenient way of restoring the wire under test to its original neutral condition after magnetisation. The residual magnetism of soft iron is so sensitive to disturbance that in experiments made with the view of observing the retentiveness of the metal, the utmost care must be used to avoid accidental errors due to this cause. For example, a long rod of soft iron, to which a magnetising force had been applied and withdrawn, leaving about 90 per cent. of the induced magnetism as residual, was lying on a table in the laboratory, an induction coil round the rod being in circuit with a ballistic galvanometer. A person walking quickly along a neighbouring corridor caused by each footstep a distinct transient current in the galvanometer, due to the shaking out of successive parts of the rod's residual magnetism. The lightest tap or even rub by the fingers made the magnetism leave the rod with a rush; and during the application of magnetising force, the most apparently insignificant disturbance, occurring when the magnetisation was slight, would give rise to an enormous increase of magnetism, making it suddenly rise twenty-fold or more. For these reasons it is necessary, in experiments such as those which have been described in the preceding part of this paper, to guard very rigorously against accidental disturbance of the piece under test. Methods of measurement which involve the removal of the piece from the magnetising solenoid, or its movement in any way, before the magnetisation is determined, are quite out of the question. All the observations already described were made after I had become convinced of the necessity of preventing vibration, by supporting the rods and rings of iron on stone columns, detached from floors, and well out of the way of passers-by.

A number of separate experiments were carried out with the view of testing more particularly the effects of mechanical vibration on magnetic susceptibility and retentiveness. The results are given below.

§ 49. (March 17, 1882.) A long wire of soft iron, annealed, 0·158 centim. in diameter and 64 centims. or 400 diameters in length, was examined as follows by the ballistic method. The magnetising force was raised by steps to a certain value, at which the magnetisation was determined by summing up the ballistic throws due

to the successive steps by which the magnetising force was increased. Up to this point there had been no disturbance. Then the wire was tapped, or rather beaten smartly against the table, the magnetising force being kept constant. The magnetisation was then, after tapping, measured by slipping off the induction coil. Next, the induction coil was replaced and the magnetising force again raised by steps, without disturbance, to a new value, the magnetisation being determined as before by summation of throws. Then the wire was again tapped, and the resulting magnetism again determined by slipping off the induction coil, and so on. A similar series of observations were made at various stages during the removal of the magnetising force. The results are given below, and graphically in Plate 61, fig. 23, where the full lines show the magnetic changes which occurred while the rod was left undisturbed, and the broken lines those which were produced by vibration. At the outset the wire had a magnetism ($\mathfrak{B}$) of 170, not extractable by tapping.

EFFECTS of Vibration on the Magnetism of Soft Iron, Plate 61, fig. 23.

	$\mathfrak{H}$.	$\mathfrak{B}$.	Change of $\mathfrak{B}$, caused by vibration under constant value of $\mathfrak{H}$.
On. .	0	170	
	0·32	190	+6430
	0·32 (with vibration)	6,620	
	1·08	6,720	
	1·61	7,120	+4480
	1·61 (with vibration)	11,600	
	2·00	11,620	
	2·9	11,770	
	3·4	11,940	+1020
	3·4 (with vibration)	12,960	
	5·3	13,100	
	6·9	13,360	
	7·8	13,660	+ 580
	7·8 (with vibration)	14,240	
	11·2	14,330	
	13·3	14,420	
	16·5	14,560	+ 80
	16·5 (with vibration)	14,640	
Off. .	11·6	14,600	
	8·9	14,530	
	7·2	14,470	−1430
	7·2 (with vibration)	13,040	
	5·2	12,970	
	3·1	12,910	
	1·77	12,790	−1350
	1·77 (with vibration)	11,440	
	1·16	11,320	
	0·77	11,290	
	0·33	11,260	−4380
	0·33 (with vibration)	6,880	
	0·23	6,880	
	0	6,880	−6560
	0 (with vibration)	320	

It is in the early part of the curve, the part where (under ordinary conditions) the magnetism has not yet begun to rise rapidly, that the effects of vibration are most remarkable. At later points the effects are still considerable, until we reach the region of approximate saturation, where, as might be expected, vibration produces little change.

The form should be noticed which the curve takes at each place when, after tapping, the process of increasing or decreasing $\mathfrak{H}$ is resumed without mechanical disturbance. Like the initial part of a normal curve of magnetisation these portions seem to start off tangent to the direction along which $\mathfrak{H}$ is measured, and it is only after a considerable change of $\mathfrak{H}$ has taken place that the magnetisation begins to change at all rapidly. This characteristic of the curves affords strong confirmation of the idea that retentiveness in soft iron is chiefly due to a resistance to the rotation of WEBER'S molecular magnets of a kind resembling the static friction of solid bodies.

§ 50. *Magnetisation of soft Iron with and without Vibration.*—In the next experiment the same piece of soft iron was again tested by the ballistic method. A pair of "on" and "off" curves were taken in the ordinary way without vibration, $\mathfrak{H}$ being raised by steps to 16·8 and reduced by steps to zero, while $\mathfrak{B}$ was measured by the summation of throws. Then, after the residual magnetism had been removed by tapping, a second pair of "on" and "off" curves were taken, in which the wire was vigorously tapped at each value of the magnetising force, and the resulting magnetism determined by slipping off the induction coil. The former pair are shown by full lines, the latter by broken lines (— · — · —), in Plate 61, fig. 24. Finally, after the magnetisation was again raised to 8000, by tapping under a force of 0·6, the process of magnetising was continued *without vibration*. This last part of the operation is shown by the dotted line (·····) in the figure.

It will be seen that the "on" and "off" curves with vibration are nearly coincident. The following readings refer to the "on" curve with vibration :—

$\mathfrak{H}$.	$\mathfrak{B}$.
0	240 (initial)
0·04	840
0·15	3,370
0·31	5,370
0·62	8,260
0·96	9,540
1·60	10,740
2·92	12,040
5·04	13,140
7·00	13,460
16·8	14,750

The enormously great magnetic susceptibility which soft iron exhibits when vibrated is worthy of special remark. In the above example a magnetising force of

only 0·5 c.g.s. unit gave an induction ($\mathfrak{B}$) between 7000 and 8000, or say half the greatest magnetisation which it is practicable to produce by the strongest forces. The susceptibility is a maximum at or close to the beginning of the curve. There the value of μ or $\frac{\mathfrak{B}}{\mathfrak{H}}$ is not less than 20,000, and that of κ or $\frac{\mathfrak{I}}{\mathfrak{H}}$ is about 1600.

The slow beginning and subsequent rapid rise of magnetism which is characteristic of the process of magnetisation when performed without mechanical disturbance is entirely absent from the curve taken with vibration, but reappears when, at any point in the vibration curve, we stop vibrating and continue the change of $\mathfrak{H}$ with the metal in a state of rest. The great retentiveness shown by iron at rest has completely vanished. At the highest magnetising force reached the value of $\mathfrak{B}$ given with vibration is only a little greater than that given without vibration. This difference would probably diminish and ultimately disappear (as it should do by WEBER'S theory) if a very strong magnetising force were applied.

§ 51. *Effects of Vibration in hard-drawn Iron and in Steel.*—It is only very soft annealed iron that exhibits the influence of vibration in so marked a way. A piece of iron wire cut from the same bundle as the above, but *not annealed*, was subjected to the same set of changes of $\mathfrak{H}$, and behaved in the way shown in Plate 61, fig. 25. The processes were the same as in the example just quoted, and the figure explains itself. Here the retentiveness, though much reduced by vibration, is by no means destroyed, and a decided difference is found between the "on" and "off" curves with vibration. The former, too, shows a certain amount of inflexion near its starting-point. As the metal is here in the hard-drawn state its susceptibility and retentiveness (without vibration) are much less than when annealed (*cf.* § 31).

In steel, whether drawn, annealed, or tempered, the effects of vibration are similar in kind to those shown in fig. 25.

[Note added March 25, 1886.—A point of practical interest in this connexion is the circular magnetisation of iron telegraph lines by the signalling currents which traverse them, and the consequent retardation of signals. It was pointed out by FLEEMING JENKIN as early as 1865 * that the experiments of GUILLEMIN showed a greater retardation of signals on land lines than could be accounted for, on THOMSON'S theory, by the electrostatic capacity and resistance of the lines; and it is now generally recognised that this additional retardation is due to the large amount of self-induction which a telegraphic circuit exerts on account of the circular magnetisation of the iron wire. The results of the present experiments may be applied to estimate the magnitude of the effects in question. In circular magnetisation we have, of course, the condition of endlessness very perfectly realised. Moreover, the almost incessant vibration to which telegraph lines are subject must have the effect of increasing the magnetic permeability to values not far short of those found in the above experiments, where the wires were tapped during the application of magnetising

* "On the Retardation of Electrical Signals on Land Lines," Phil. Mag., June, 1865.

force. We may, as an extreme case, applicable to a very soft iron wire strongly vibrated, take μ as 20,000 for any magnetising force of small intensity. Apply this to the case of a wire of ordinary size, say 4 mm. in diameter, and carrying a current of one-tenth of an ampère or 0·01 c.g.s. unit. At the circumference of the wire, where the magnetising force is greatest, its value is

$$\frac{2C}{r},$$

where C is the current and r the radius of the wire; and this value is independent of the distribution of the current throughout the section, so long as that is symmetrical with regard to the axis. The magnetising force is therefore equal to

$$\frac{2\times 0{\cdot}01}{0{\cdot}2}=0{\cdot}1 \text{ c.g.s. unit.}$$

With $\mu=20{,}000$ this gives $\mathfrak{B}=2000$ as the value of the magnetic induction in the outermost layer of the metal—a magnetisation nearly half as great as that of the armature of a good modern dynamo! Along with this circular magnetisation there is combined, in wires which run more or less north and south, a longitudinal magnetisation due to the earth's field, of an amount comparable with the above, so that the actual lines of induction form helices whose pitch, besides decreasing from the axis towards the circumference of the wire, shortens and lengthens with every fluctuation of current.

In a circuit consisting of two long parallel straight aërial conductors, each of radius r and permeability μ, separated by a distance b, MAXWELL's formula ('Electricity and Magnetism,' ii., § 685) gives for the self-induction per unit of length of the system the value

$$\frac{L}{l}=2\log\frac{b^2}{r^2}+\mu.$$

It is interesting to notice the comparative values which this quantity assumes when the conductors are, on the one hand, of non-magnetic metal, such as copper or phosphor-bronze, and on the other hand (again as an extreme case) of very soft annealed iron, kept in a state of vibration. To make a numerical comparison, we may take wires of 4 mm. diameter, separated by a distance of 20 centims. Then, for non-magnetic metal,

$$\frac{L}{l}=2\log\frac{20^2}{0{\cdot}2^2}+1=19{\cdot}4.$$

And for soft iron, under vibration,

$$\frac{L}{l}=2\log\frac{20^2}{0{\cdot}2^2}+20{,}000=20{,}018.$$

Hence, with the assumed data, the iron circuit has one thousand times the self-induction of the non-magnetic circuit. This throws light on the fact * that a notable

* PREECE, British Association (Aberdeen), 1885.

increase in the speed of signalling becomes possible when copper is substituted for iron in a line whose electrostatic capacity and resistance are kept unaltered; and that the much more rapid electric impulses which traverse a telephone circuit often become muffled beyond recognition when iron takes the place of phosphor-bronze.]

§ 52. *Time Lag in Magnetisation.*—A few miscellaneous points will now be noticed which presented themselves in the course of these experiments.

I have already said that the hysteresis which has been described as occurring in the relation of magnetisation to magnetising force is of a static character. Some evidence was, however, given that in addition to much static hysteresis there is a small amount of viscous lagging in the changes of magnetism which follow changes of magnetising force. I repeatedly observed that when the magnetising current was applied to long wires of soft iron, either gradually or with more or less suddenness, there was a distinct *creeping up* of the magnetometer deflection after the current had attained a steady value, as measured by the deflection of the galvanometer through which it passed. This action was sometimes so considerable as to oblige me to wait for some minutes before taking the magnetometer reading. The amount of this creeping is, however, very small compared with the static hysteresis. It occurs most conspicuously in the softest iron and at points near the beginning of the steep part of the magnetisation curve, and is much more marked in wires which are being magnetised for the first time after annealing, than in wires which have been previously magnetised, and demagnetised by the method of reversals. The action goes on too long to be ascribable to the self-induction of the circuit, and indeed occurs most noticeably when the changes of the current are effected gradually by means of the slide; and I do not think that it can be regarded otherwise than as evidence of true magnetic viscosity.

§ 53. *Comparative Effect of Sudden and Gradual Changes of Magnetising Force, and Resultant Effects of Cyclic Changes of Magnetising Force.*—It has long been well known from the experiments of VON WALTENHOFEN * and others that when a magnetising force is suddenly applied the induced magnetism is greater than when the force is gradually applied, and that when a magnetising force is suddenly withdrawn the residual magnetism is less than when the force is gradually withdrawn. My own experiments afford many confirmations of these facts, but they show that in very long uniformly magnetised rods the difference between the effects of sudden and the effects of gradual changes of magnetising force are not great. Thus, in the ballistic method, the difference in effect is not very material whether we produce a change of magnetising force in a series of numerous steps, or in a single step. The former process gives slightly, but very slightly, less total change of magnetism than the latter. And in the magnetometric method the magnetisation produced by raising the magnetising current continuously by means of a slide was found to differ but little from that given by a sudden establishment of the same current. The differences,

* Pogg. Ann., 1863.

such as they are, are of the kind which Von Waltenhofen's experiments would lead us to expect, but their magnitude is not such as to make results got by the ballistic method be materially different from results got by the magnetometric method.

In the following experiments the comparative effects of slow and sudden changes of magnetising force are involved, along with certain other phenomena which will now be described, and which are in part a mere confirmation of results already published by C. Fromme.*

§ 54. *Effects of Repeated Application and Removal of Magnetising Force.*—When a magnetising force is first applied, then removed, and then re-applied, whether suddenly or gradually, the resulting value of $\mathfrak{I}$ is somewhat higher than that reached by the first application. A third application gives a somewhat higher value, and so on, the effects apparently approaching an asympotic limit. This has been already shown by the experiments of Fromme. At each removal of the magnetising force the residual magnetism is also left somewhat greater than before. And this second action (the increase of the residual magnetism) exceeds the increase of the induced magnetism, with the result that the changes of magnetism between residual and induced *diminish in range* with successive removals and re-applications of the magnetising force.

The following observations were made by the ballistic method on a long piece of soft annealed iron wire. The readings are given without reduction to absolute measure; they relate to a point which falls early in the steep part of the curve of magnetisation.

Magnetising Current.	Throw of Ballistic Galvanometer.	Magnetism.
First made	203	203
broken	−53·6	149·4
made	+54·2	203·6
broken	−47·8	155·8
made	+48·7	204·5
broken	−45·7	158·8
made	+46·6	205·4
broken	−44·9	160·5
made	+46·1	206·6
broken	−44·0	162·6
made	+45·6	208·2
After many makes and breaks—		
broken	−42·6	
made	+43·1	
After many more makes and breaks—		
broken	−39·5	
made	+39·8	

Similar results were repeatedly obtained, both with freshly annealed wires and wires from which a previous strong magnetism had been shaken out by tapping. In curves

* Pogg. Ann., Ergbd. vii., 1875, and Wied. Ann., iv., 1878.

showing the relation of $\mathfrak{B}$ or $\mathfrak{I}$ to $\mathfrak{H}$, the same thing exhibits itself in what may be called the over-closing of loops formed by removing and re-applying a given value of $\mathfrak{H}$. A good example of this is furnished by Plate 60, fig. 17, which shows how much more considerable the action now spoken of is at early than at late stages of the magnetisation.

The following experiment (also on annealed iron) shows that the same kind of action occurs when the current is slowly changed by the slide of § 18, and the magnetism is determined by a magnetometer.

Magnetising Current.	$\mathfrak{H}$.	Magnetometer Deflection.	$\mathfrak{I}$.
Gradually raised to 70	2·46	93	298
,, reduced to 0	0	65	208
,, raised to 70	2·46	97	310
,, reduced to 0	0	70	224
Then 100 sudden makes and breaks—			
Suddenly raised to 70	2·46	103	330
,, reduced to 0	0	80	256

§ 55. *Effects of Repeated Reversal of Magnetising Force.*—When a magnetising force is applied and then repeatedly *reversed*, the changes of magnetism, instead of being strictly cyclic, form what may be termed unclosed loops. Instances of this are given by a number of the preceding figures, especially by Plate 60, fig. 16, which shows a series of these unclosed loops in the magnetisation of steel wire. The result is, as in the former case, that successive repetitions of the process give a gradually diminishing range of magnetic change. This action, like the last, occurs most conspicuously at points in the early part of the curve of magnetisation. The following observations were made specially to exhibit it, on a piece of annealed iron wire, 400 diameters long, by the magnetometric method.

Magnetising Current.	Magnetometer.	
0	0	
Gradually raised to . . . +190	+146	
,, reversed to . . −190	−141	
,, ,, ,, . . +190	+127	Here there is gradual diminution of range.
,, ,, ,, . . −190	−133	This part of the operation is shown in fig. 26.
,, ,, ,, . . +190	+120	
,, ,, ,, . . −190	−132	
Suddenly ,, ,, . . +190	+124	Here there is an increase of range due to the
,, ,, ,, . . −190	−136	suddenness of these reversals.
,, ,, ,, . . +190	+123	
Fifty double reversals, then		
Suddenly reversed to . . +190	+111	But after repeating the sudden reversals often
,, ,, ,, . . −190	−127	enough the range becomes smaller than ever.
Then gradually ,, . . +190	+108	And a *gradual* repetition of the cycle causes
,, ,, ,, . . −190	−126	still a further reduction of range.

In the first part of the above operations, during the five gradual reversals of magnetising force, intermediate readings were taken, which enabled the curves shown in Plate 61, fig. 26, to be drawn. These show at a glance the manner in which the range of magnetic change diminishes. Sudden reversals, following on these, cause at first an increase of range, thus illustrating the comparative effects of gradual and sudden change of $\mathfrak{H}$, but on being repeated many times they reduce the range to a lower value than before.

§ 56. The same piece of wire was next subjected to a magnetising force about five times greater than the above, and was then demagnetised by reversals (§ 19). Experiments similar to the above were then made on it, when it was found that *the tendency to a diminution of range with repetition of a cyclic alteration of magnetising force had disappeared.* The diagram, Plate 61, fig. 27, shows the effect of applying, reversing, and reapplying the same magnetising force as in the former case, after the wire had been demagnetised by reversals. It shows that the changes of magnetism are now cyclic. The same result was given by other specimens, which when freshly annealed gave much diminution of range, but when demagnetised by reversals after the magnetising force had been raised to a high value, were found to have lost this property. In this respect, then, a wire demagnetised by reversals differs from the same wire in its primitive annealed state. It will be seen, too, by comparing figures 26 and 27, that the unsymmetrical susceptibility with respect to forces of opposite signs which exists in the annealed wire has given place to a very perfect symmetry after demagnetisation by reversals. Re-annealing the wire restored all the characteristics of the primitive state.

§ 57. Similar experiments with other specimens of wire, and at other points in the curve of magnetisation, gave results confirmatory, in every particular, of the above. When the magnetisation is strong, however, the differences produced by repetitions of the same process are insignificantly small: and it is in the early parts of the curve that we find the most striking effects. One or two other examples may be given very briefly.

Soft iron wire, freshly annealed.

Magnetising Current.	Magnetometer.	
Gradually 100	+130	
" . . . . −100	−125	
" . . . . +100	+117	
" . . . . −100	−116	
Suddenly . . . . +100	+109	Here the repetition of the reversal, although sudden, still causes diminution of range.
" . . . . −100	−114	
Fifty more double reversals, then—		
Suddenly . . . . +100	+106	
" . . . . −100	−106	
Gradually . . . . +100	+103	Here even after many sudden reversals a *gradual* reversal still causes a decided lessening of the range.
" . . . . −100	−104	

The following observations, made with another piece of annealed iron wire at a part of the curve very sensitive to the actions now spoken of, show well the reduction of range by reversals and then the rise of magnetism, induced and residual, produced by successive removals and re-applications of $\mathfrak{H}$. This last occurs in a very marked way after the range of magnetic change has been reduced by reversals of $\mathfrak{H}$. The two directions of the current will for brevity be distinguished as A and B. The changes were sudden, and the magnetism was determined by the direct magnetometric method. A want of symmetry is very noticeable here between the positive magnetisation due to the current A, which is first applied, and the subsequent negative magnetisation due to the equal and opposite current B.

	Magnetometer.	
Made A	+232	
„ B	−110	Diminution of range by reversals.
„ A	+180	
„ B	−101	
„ A	+172	
„ B	−100	
Twenty reversals, then—		
Made B	− 95	
„ A	+158	
Broke A	+150	Rise of magnetism (induced and residual) by successive removals and re-applications of $\mathfrak{H}$.
Made A	+200	
Broke A	+193	
Made A	+206	
Broke A	+201	
Twenty makes and breaks, then—		
Broke A	+205	
Made A	+209	
Then reversals again—		
Made B	−105	The diminution of range by reversals is again conspicuous.
„ A	+178	
Forty reversals, then—		
Made A	+163	
„ B	−105	
Broke and remade B	−136	
Ditto twenty times	−175	

Further experiments with the same wire showed, after the range was reduced by reversals, that a gradual removal and re-application of $\mathfrak{H}$ produced, though to a much less degree, the same effect as a sudden removal and re-application; the effect, namely, of increasing the magnetism.

§ 58. The magnetisation of steel exhibits, even more than that of iron, reduction of range with successive reversals of $\mathfrak{H}$, and want of symmetry between the values of $\mathfrak{I}$ induced by successively applied + and − values of $\mathfrak{H}$. Plate 61, fig. 28, shows the changes of magnetism which were undergone by an annealed steel wire when a magnetising force of 15 c.g.s. units was applied, removed, re-applied, reversed, and re-reversed twice. The want of symmetry between the positive and negative values

of $\mathfrak{B}$ is very marked in this example: the steel has acquired a strong *set* towards the side of the first magnetisation.

Values of the Coefficients μ and κ.

§ 59. It follows from the form of the magnetisation curve, as has been pointed out by STOLETOW and others, that the coefficient of magnetic susceptibility κ, or $\frac{\mathfrak{I}}{\mathfrak{H}}$, and therefore also the permeability μ, or $\frac{\mathfrak{B}}{\mathfrak{H}}$, rises, as magnetisation proceeds, from a comparatively low value to a maximum, and then decreases continuously as $\mathfrak{H}$ is further raised, at least within such limits of $\mathfrak{H}$ as have ever been reached in actual experiments. The changes in κ and μ which occur during the process of normal magnetisation are best seen by ROWLAND'S method of plotting κ in terms of $\mathfrak{I}$, or μ in terms of $\mathfrak{B}$. Plate 62, fig. 29, gives two examples of this last mode of representation. One of the curves shown there refers to the annealed iron wire of § 21. In it the values of μ (which are given numerically in § 21) pass from 128 for a very low value of $\mathfrak{B}$ to a maximum of 2370 for $\mathfrak{H}=4{\cdot}5$, and down again to 705 for $\mathfrak{H}=22{\cdot}27$. In the other, which refers to the annealed iron wire of § 22, the early values of μ are not given, but the magnetisation has been pushed further by raising $\mathfrak{H}$ to 89·8, with the result of reducing μ to 183 and of giving the curve an upward inflection in its descending limb. That such an inflection occurs when the magnetisation is sufficiently increased has been already noticed and commented on by FROMME (Wied. Ann., xiii., p. 695). It makes the formula given by ROWLAND (Phil. Mag., xlviii., p. 339) fail to serve as an equation to the curve in this extreme portion.

§ 60. Plate 62, fig. 30, gives two curves showing, in the same manner, the relation of κ to $\mathfrak{I}$ in the iron wire of § 31, first in its annealed state, and again after it had been hardened by stretching. They are drawn to the same scale, and bring out well by contrast the great influence which permanent set has in reducing magnetic susceptibility. They are plotted from the observed values stated in § 31. Other curves of the same character may easily be drawn by aid of the data contained in previous paragraphs.

§ 61. Curves of this kind suggest several interesting theoretical questions regarding the results which we should find if we were able to extend indefinitely the range of observations, both towards vanishingly low and towards indefinitely high values of $\mathfrak{H}$.

It seems exceedingly probable, to judge from the portion of the curve of κ and $\mathfrak{I}$ which we can actually determine, that, if produced backwards, the curve would cut the axis of κ at a finite positive value of κ.* This, in other words, would imply that the susceptibility is not indefinitely small at the very beginning of magnetisation.

To reconcile such a result with the idea of a static frictional resistance to the

* ROWLAND has so produced his curves of μ and $\mathfrak{B}$ backwards, and has given values of μ corresponding to $\mathfrak{B}=0$. These, however, are merely hypothetical.

rotation of the magnetic molecules, we should be obliged to assume either that this friction was vanishingly small for some of the molecules, or that a certain range of motion was possible, to all or some of them, before the frictional resistance began to be felt. We have seen already that one of these assumptions would be required to account for the indefinitely small retentiveness which hard iron and steel seem to possess when exceedingly feebly magnetised.

But though the observed values of κ extend to very low values of $\mathfrak{B}$, between these and the zero of magnetisation there is room for a complete change in the form of the curve, and it would be rash to infer anything respecting the initial value of κ from the way in which the curve is trending at the finite magnetisations for which the lowest observed values of κ have been determined. The curves, indeed, give some slight evidence that a downward bend occurs near the origin, and, in any case, they afford no positive proof that κ is not initially zero, or even negative.

§ 62. If we could suppose that the frictional sticking of the molecules was at first complete, so that none of them budged from their primitive positions when a very low value of $\mathfrak{H}$ was applied, then we should expect to find the initial value of κ not zero, but a negative quantity. For in that case the cause producing paramagnetism (namely, the rotation of the molecules) would be entirely inoperative, and the application of $\mathfrak{H}$ might be expected to induce Ampèrian currents in the stationary molecules in the same manner as in other substances, with the result of making the magnetisation opposite in direction to the magnetising force, until the latter became large enough to overcome friction and begin turning the molecules. Thus, if none of the molecules could begin to turn until $\mathfrak{H}$ reached a certain finite value, we should expect to find iron *diamagnetic* with respect to lower values of $\mathfrak{H}$.

[Note added April 9, 1886.—It is possible that substances which are diamagnetic in fields of ordinary intensity are so, not because the molecular currents are absent or weak, but because the molecules are held so fast that their alignment (which would give paramagnetisation) is produced with difficulty. So long as they do not turn, or turn very slightly, diamagnetic induction is the resultant effect of the applied field.

If the resistance to the turning of the molecules resembles that found in iron, we should expect the diamagnetic induction which occurs when the field is moderate to give way to paramagnetic induction as the field is increased; also that a residual *paramagnetic* polarity would be left after a strong field has been applied and removed,

even if the substance has remained diamagnetic during the action of the field. This view is supported by the experiments of LODGE ('Nature,' 1886, March 25, p. 484; also April 1, p. 512), which have shown residual paramagnetic polarity in copper and other diamagnetics after exposure to a strong field. According to this view the curve of $\mathfrak{I}$ and $\mathfrak{H}$ for substances in general is (in its earlier portion) of the form sketched. If the region ab extends so far as to include fields of considerable intensity, we call the substance diamagnetic. In iron, b comes so near the origin that we are always dealing with the further portion of the curve.]

§ 63. The extension of the curves of κ and $\mathfrak{I}$, or of μ and $\mathfrak{B}$, at the upper end is subject to a like uncertainty. As ROWLAND has remarked in the introduction to his second paper, we do not know whether $\mathfrak{I}$ or $\mathfrak{B}$, or either of them, attains a maximum; and MAXWELL* has given a reason (similar to the above) for supposing that, after passing a maximum, $\mathfrak{I}$ becomes negative. For, during the application of $\mathfrak{H}$, we may suppose that the induction of Ampèrian currents goes on in the molecules, giving a component of magnetisation opposite in sign to that which is produced by the turning of the molecules. The latter reaches a limit, the former increases indefinitely as $\mathfrak{H}$ is increased, and therefore ultimately exceeds the latter. Hence, iron may be diamagnetic under a very strong magnetising field.

In that case we should find that as $\mathfrak{H}$ is raised, $\mathfrak{I}$, after passing a positive maximum, decreases to zero, becomes negative, and then goes on always increasing negatively. The curve of κ and $\mathfrak{I}$, if produced by increasing $\mathfrak{H}$, would bend back, towards the origin, in its descending limb, pass through the origin and over to the negative side, κ never exceeding a small negative value as $\mathfrak{I}$ is increased negatively without limit.

§ 64. On the other hand, even if iron be ultimately diamagnetic under strong magnetising forces, we have no reason to expect that $\mathfrak{B}$ will pass a maximum. Since $\mathfrak{B}=4\pi\mathfrak{I}+\mathfrak{H}$, $\mathfrak{B}$ cannot pass a maximum with increase of $\mathfrak{H}$ unless $\frac{d\mathfrak{I}}{d\mathfrak{H}}$ attains a negative value greater than $\frac{1}{4\pi}$. This is a greater coefficient of magnetisation than any diamagnetic substance is known to possess, and, moreover, is too great to be consistent with WEBER'S theory of diamagnetism, according to which μ cannot be negative.

These considerations, if they serve no other useful purpose, show the futility of drawing conclusions as to the initial and ultimate values of the magnetic susceptibility of iron in indefinitely low and indefinitely high fields, from observations made, as all observations must be made, in fields of finite magnitude. For this reason it appears that an empirical formula, such as ROWLAND applies to the curves of μ and $\mathfrak{B}$, must be misleading when pushed beyond the range of actual experiment. If we do not know whether $\mathfrak{I}$ or $\mathfrak{B}$, or either of them, attains a maximum, it is a truism to say that there is no use in assigning numerical values to that maximum. The results obtained by extending a formula past the limits of experience

* MAXWELL, 'Treatise on Electricity and Magnetism,' ii., §§ 843, 844.

have scarcely even a speculative interest if the basis of the formula is not an intelligible physical theory and its terms are not capable of physical interpretation.

§ 65. WEBER'S theory of the magnetisation of iron is incomplete, because it omits to take account of retentiveness. MAXWELL'S extension of it, which ascribes retentiveness to something resembling the permanent set of an overstrained solid fails, it appears to me, to explain the hysteresis which is found in all changes of magnetism brought about by changes of the magnetising force. The idea of a resistance to rotation of the molecules resembling the friction of solids, which has been suggested by several writers, suffices to explain hysteresis, and is supported by other phenomena which I have already noticed in passing. It accords remarkably well with the low values of κ which we find in the early part of the magnetisation curve; also with the effects which vibration has in increasing the susceptibility of iron to small forces and in destroying its retentiveness. In fact, for soft iron it requires little modification to make it cover the facts. In hard iron and steel, on the other hand, where the residual magnetism is of a far stabler character, retentiveness seems partly due to some such action as MAXWELL has suggested, though effects resembling those of friction are also present. Both notions—that of permanent set and that of frictional resistance—seem needful for anything like a full account of the phenomena, and if we are to attempt to form a mechanical conception of the process on such lines as these, we must assume: (1) An elastic tendency on the part of the molecules to recover their primitive position when displaced; (2) a static frictional resistance to their displacement and to their return, removable by vibration; (3) a limit for each, such that if the displacement of the molecule exceeds it, a permanent displacement, not removable by vibration, results; (4) probably a viscous resistance to the displacement and return of the molecules; (5) an unequal distribution amongst the molecules of the frictional resistance, such that in some of the molecules it is very (perhaps indefinitely) small. An alternative to this last supposition has already been mentioned, but it is in any case necessary to assume that the friction varies somewhat widely amongst the molecules, in order to prevent the susceptibility from changing discontinuously.

§ 66. A theory involving so many arbitrarily adjustable constants evidently admits of being brought without difficulty into general harmony with what we are able to observe of the process of magnetisation. Any examination of it by comparison with observed magnetic values could scarcely be conclusive. We might, however, by applying these ideas, construct, or at least imagine, a mechanical model in which the pieces would move in such a manner as to give results closely imitative of those observed when we magnetise and demagnetise iron. It must be admitted that any mechanical analogue of this kind is probably a very crude representation of molecular movements. One cannot suppose that if we could obtain an insight into the real nature of magnetic induction we should find anything at all resembling miniature drums turning under brake-straps; nevertheless, it is certain that the internal movements which are

produced in iron by changes of magnetising force (and, as I shall show later, by other things besides magnetising force) do occur in ways which are not only extremely suggestive of the movement of solid bodies against frictional resistance, but are exactly analogous to such movements in some of their effects.

§ 67. In the molecular theory of magnetisation, as developed by WEBER and MAXWELL, the equation expressing the equilibrium of a magnetic molecule when deflected by a magnetising force, X, is

$$X \sin \theta = D \sin (\alpha - \theta),$$

where α is the original inclination of the molecule's axis to the line of action of X, θ is its inclination to the same line when deflected, and D is an assumed directive force which tends to keep the molecule in its primitive position.

We may introduce the idea of frictional resistance by writing this—

$$X \sin \theta = D \sin (\alpha - \theta) + \rho,$$

where ρ is a statical couple due to friction (to be reckoned per unit of magnetic moment of the molecule). In this form the equation will apply to cases where the deflection of the molecule is being or has just been increased by application of X. When the deflection is being diminished by reduction of X the term expressing the friction is to have its sign reversed.

The most easily affected molecules are those whose inclination to the axis of X is $\frac{\pi}{2}$. Hence, on beginning to apply magnetising force there is no deflection of any molecule until $X = \rho$. Again, after magnetisation has been produced and the applied force X begins to be removed, no return of the molecules (that is to say, no loss of induced magnetism) occurs until the amount by which X is reduced exceeds 2ρ. The subsequent loss of magnetism as X is further reduced will depend on the relation of D to ρ.

Now, in soft iron the retentiveness is so nearly complete that D must be very small. We may examine the comparatively simple extreme case which we should find if D were equal to zero. Every molecule which is being turned by X must then satisfy the equation $X \sin \theta = \rho$, and the action is always limited to those molecules for which $\sin \alpha > \sin \theta > \frac{\rho}{X}$. For any given value of X, during the increment of X, the only molecules affected are those whose original inclinations lie between the values $\alpha_1 = \sin^{-1} \frac{\rho}{X}$ and $\alpha_2 = \pi - \alpha_1$. Molecules lying outside these limits do not contribute to the resultant magnetisation of the piece.

Using the notation of MAXWELL ('Electricity and Magnetism,' vol. ii., § 443), in which m is the magnetic moment of a molecule, and n the number of molecules per unit of volume, we have

$$\mathfrak{I} = \int \frac{mn}{2} \cos \theta \sin \alpha \, d\alpha,$$

The limits are α_1 and α_2 as above, and $\cos\theta = \sqrt{1-\frac{\rho^3}{X^3}}$.
Hence,

$$\mathfrak{I} = mn\left(1-\frac{\rho^3}{X^3}\right)$$

This applies from $X=\rho$ to $X=\infty$. For values of X below ρ, $\mathfrak{I}=0$. mn is of course the magnetism of saturation. The following values of $\mathfrak{I}$ are calculated from this equation :—

X	$\mathfrak{I}$
ρ	0
$1{\cdot}25\,\rho$	$0{\cdot}360\,mn$
$1{\cdot}5\,\rho$	$0{\cdot}556\,mn$
$2\,\rho$	$0{\cdot}750\,mn$
$3\,\rho$	$0{\cdot}889\,mn$
$4\,\rho$	$0{\cdot}937\,mn$
$5\,\rho$	$0{\cdot}960\,mn$
$10\,\rho$	$0{\cdot}990\,mn$
$100\,\rho$	$0{\cdot}999\,mn$

By supposing that ρ is different for different molecules we can avoid the discontinuity which occurs at $X=\rho$; and the relation of $\mathfrak{I}$ to X deduced from the formula agrees fairly well with actual curves of $\mathfrak{I}$ and $\mathfrak{H}$ in specimens of soft annealed iron. Removal of X would of course give a straight line ($\mathfrak{I}$=constant), and application of X with sign reversed would give a rapid fall as soon as the value of the reversed force exceeded ρ. The results correspond so nearly with those found in soft annealed iron that it may be concluded that this extreme supposition (namely, that D is an insignificantly small quantity, and that retentiveness is due to ρ), although doubtless not very exact, does represent fairly well the behaviour of this material. No such simple theory will answer in dealing with hard iron or steel.

When we subject soft iron to vibration during the application and removal of the magnetising force, we cause ρ to vanish more or less completely, and the action which we observe agrees fairly well with the original theory of WEBER as developed by MAXWELL (*loc. cit.*, equations 5–8), and shows that D, though finite, is a very small quantity.

§ 68. *Numerical Values of the Maximum of μ and κ.*—The following table shows in a collected form, and in round numbers, the maximum values (deduced from these experiments) of μ and κ during initial magnetisation, and the value of $\mathfrak{H}$ at which they were found.

Specimen.	Maximum μ.	Maximum κ.	Value of δ, giving maximum μ.
Moderately soft iron ring (of fig. 1, § 10)	1740	138	5·1
Soft iron ring (of fig. 2, § 11)	2300	183	2·6
Very soft annealed iron wire, 300 diameters long (fig. 3, § 15)	3500	279	2·6
Soft annealed iron wire, 400 diameters long (fig. 4, § 20)	2670	212	2·8
Annealed iron wire, 400 diameters long (fig. 5, § 21)	2370	189	4·5
Another similar piece (fig. 6, § 22)	2040	162	4·1
Steel wire, hard-drawn (fig. 8, § 25)	320	25	30
ditto annealed (fig. 9, § 26)	470	37	18
Pianoforte steel wire, normal temper (fig. 11, § 28)	273	22	39
ditto annealed (fig. 12, § 29)	295	23	28
ditto glass-hard (fig. 13, § 30)	118	9·3	55
Soft annealed iron wire (fig. 14, § 31)	3080	245	2·7
The same, after being hardened by stretching (fig. 14, § 31)	670	53	10·7
Annealed iron wire (fig. 17, § 37)	2470	196	4·5
Iron wire hardened by stretching (fig. 19, § 42)	450	36	17
Pianoforte steel (*bis*), normal temper (fig. 20, § 44)	258	20·5	38
ditto „ annealed (fig. 21, § 45)	415	33	20
ditto „ glass-hard (fig. 22, § 46)	126	10	55
Very soft annealed iron wire (fig. 24, § 50)	3500	279	2·1
The same wire, *with vibration during the application of magnetising force* (fig. 24, § 50)	20,000 (about)	1600 (about)	From 0 to 0·2

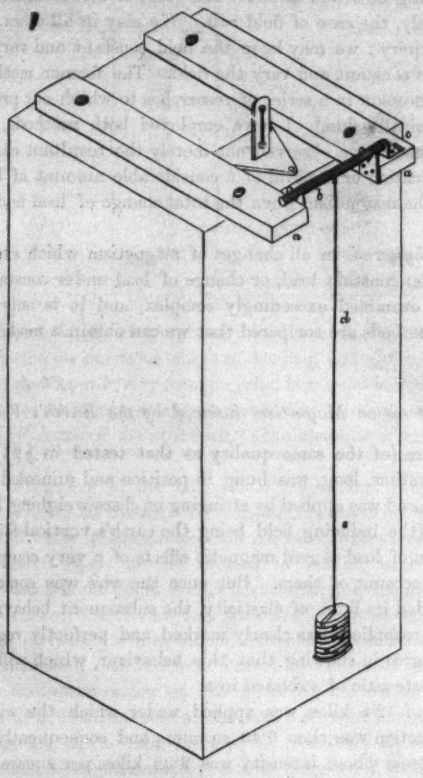
Effects of Stress on the Magnetic Qualities of Iron.

§ 69. *Method of Experiment.*—In all the experiments already described, in which an iron or steel wire formed the subject of test, the wire hung vertically under the action of no other stress than that due to its own weight. I shall now describe a series of experiments which were designed to study the variations of magnetic quality caused by pull applied in the direction of the wire's length, which was also the direction of magnetisation.

The wire to be tested was hung vertically with its upper end on a level with and due east of a mirror magnetometer. In my early experiments the desired longitudinal stress was applied by running mercury into a tank which hung from the lower end of the wire, but this plan was soon abandoned in favour of the much simpler one of stringing discs of lead on a copper wire which formed a continuation of the iron wire under test. Both in accuracy and convenience solid lead weights are preferable to a tank containing fluid, and I found it practicable to apply and remove the discs of lead with much less mechanical disturbance than was produced by running mercury into and out of a tank. This is an important point, for, as will be shown later, the influence of stress on magnetism is much affected by even the slightest vibration. Lead weights formed the load in all the experiments given below.

To enable the magnetometer zero to be tested from time to time it was desirable to have an easy means of removing the wire and replacing it in exactly the same position. The method, shown in the sketch, by which the wire was fixed, admitted of this.

The magnetometer stood on the top of a solid stone pier, which rose, clear of the floor, to a height of about 3 feet above the floor. To the top of it, and on the two sides of the magnetometer, two stout brackets, *a a*, were very rigidly fixed. To these a pair of brass plates with V notches cut in them were secured, which served to carry a cross piece, *b*, consisting of a cylinder of brass. This cylinder was prevented from moving in the direction of its own length by an end plate, *c*. The iron wire under examination,



d, had brazed to it at its upper end a short piece of thicker brass wire which was rivetted into the middle of the cylinder, *b*. At the bottom of *d* a copper wire *e* was attached on which the lead weights were strung.

In some of the experiments the vertical component of the earth's magnetic field

formed the magnetising force.* More usually magnetising force was applied by means of a solenoid which completely enclosed the wire: the strength of the current in the solenoid was measured by a mirror galvanometer, and was generally varied by means of the liquid slide described in § 18.

§ 70. The study of the effects of stress on the magnetic quality of a material consists in observing relations between the intensity of magnetisation and the two independent variables, magnetising field and stress. The effect of stress on residual magnetism is a special case, namely, the case of field $=0$. We may in all cases use one or other of two methods of inquiry: we may keep the field constant and vary the stress, or we may keep the stress constant and vary the field. The former method has been used by Sir WILLIAM THOMSON in a series of researches to which our present knowledge of this subject is principally due.† I have employed both methods, and where I have used the former plan I have observed not merely the resultant change of magnetism caused by the application or removal of a considerable amount of load, but the intermediate values of the magnetism when the total change of load is divided into a large number of steps.

The presence of *hysteresis* in all changes of magnetism which are caused either by change of field under constant load, or change of load under constant field, makes the phenomena to be examined exceedingly complex, and it is only when the results obtained by both methods are compared that we can obtain a moderately clear view of the subject.

Effects of Stress on Magnetism induced by the Earth's Vertical Force.

§ 71. An iron wire (of the same quality as that tested in § 21), 0·079 centim. in diameter and 29 centims. long, was hung in position and annealed there by the flame of a spirit lamp. Load was applied by stringing on discs weighing 1 kilo. each, and the magnetic changes (the inducing field being the earth's vertical force) were observed. The first application of load caused magnetic effects of a very complex character; and I shall reserve the account of them. But once the wire was somewhat stretched by a load which exceeded its limit of elasticity, the subsequent behaviour, when the load was removed and reapplied, was clearly marked and perfectly regular. Earlier and later experiments agree in showing that this behaviour, which will now be described, is thoroughly characteristic of *stretched* iron.

A total weight of 12·6 kilos. was applied, under which the wire stretched about 0·7 centim. Its section was then 0·48 sq. mm., and consequently every kilogramme of load caused a stress whose intensity was 2·08 kilos. per square millim. The load of 12·6 kilos. was applied and removed four times, during which the changes of magnetism gradually became nearly cyclic. Then, during the fifth application and

* The value of this in Tokio, Japan, where the experiments were made, is 0·34 c.g.s. units.

† "Electrodynamic Qualities of Metals," Phil. Trans., 1856, 1875, 1879.

removal of the load the following observations were taken. The readings given are the deflections of the magnetometer needle from its zero: they represent on an arbitrary scale the intensity of magnetisation of the wire.

STRETCHED Iron Wire (March 2, 1882), Plate 62, fig. 31.

	Load in kilos. 1 kilo. = 2·08 kilos. per sq. mm.	Magnetometer reading.		Load in kilos.	Magnetometer reading.
Loading	0	252	Unloading		
	1	259		11·8	426
	2	273		11	428
	3	294		10	432
	4	323		9	435
	5	355		8	438
	6	389		7	440
	7	413		6	438
	8	430		5	431
	9	437		4	413
	10	438		3	385
	11·8	434		2	350
	12·6	429		1	304
				0	254

The same experiment is shown graphically in Plate 62, fig. 31, where the arrows distinguish between the *on* curve, or curve of loading, and the *off* curve, or curve of unloading. The whole action is very nearly cyclic, but there is a remarkable difference between the values of the magnetism corresponding to equal values of the stress during the *on* and *off* parts of the operation. The changes of magnetism lag behind the changes of stress.

This *hysteresis*, or lagging of the changes of magnetism behind the changes of stress, is of a purely static character. The magnetism does not change *after* a load has been applied, and while the load remains constant, except to a very insignificant extent. The successive steps in the process of loading and unloading may be performed at intervals of some hours even without affecting the form of the curves.

Examining now the *on* curve, we see that the effect of moderately loading a stretched wire is to increase its magnetism, under the conditions we have here, but that a maximum is passed, and the later stages of the loading diminish the magnetism. A similar maximum occurs on the *off* curve, but owing to the presence of hysteresis it occurs at a smaller value of the load than the maximum point in the *on* curve. Each maximum is, in fact, shifted by hysteresis to a later part of the operation than it would otherwise occupy, and if there were no hysteresis we should expect it to take a position between the two actual maximums, or at about 8 kilos. of total load. Another characteristic of the curves, obviously attributable to hysteresis, is the comparatively easy gradient at the beginning of the *on* curve and again at the beginning of the *off* curve.

§ 72. It is evident as a further result of hysteresis, that the relation of load to magnetism may, without any change either in the field or in the initial magnetic condition of the piece examined, have any one of the indefinite number of values defined by points lying between the *on* and *off* curves. For example, it is easy with a load of 4 kilos. to reach a value for the magnetometer reading anywhere between 323 and 413. The value of the magnetism depends not only on the actual load, but on the preceding states and changes of the load, and most particularly on the *immediately* preceding changes. To illustrate this more fully the following observations were made, on the same wire, after it had again been subjected to the cycle of loading 0—12·6—0. (That is, after a load of 12·6 kilos. had been again applied and removed.)

The successive changes of load passed through in this experiment were: 0—5—0—8—3—12·6—9—12·6—3—8—0, with intermediate steps, and the resulting magnetometer readings are given below :—

Stretched Iron Wire, Plate 62, fig. 32.

	Load.	Magnetometer.		Load.	Magnetometer.
	0	245		12·6	425
	1	255		11·8	427
Loading (*ab*)	2	270	Unloading (*fg*)	11	430
	3	293		10	434
	4	322		9	437
	5	352		10	434
	4	349	Loading (*gh*)	11	431
Unloading (*bc*)	3	338		11·8	428
	2	319		12·6	425
	1	290		11·8	427
	0	249		11	430
	1	256		10	433
	2	271		9	436
	3	292	Unloading (*hi*)	8	439
Loading (*cd*)	4	321		7	441
	5	351		6	440
	6	384		5	433
	7	413		4	415
	8	430		3	388
	7	430		4	392
	6	427		5	400
Unloading (*de*)	5	419	Loading (*ij*)	6	410
	4	405		7	423
	3	381		8	432
	4	385		7	432
	5	393		6	430
	6	405		5	424
	7	419	Unloading (*jk*)	4	411
Loading (*ef*)	8	430		3	390
	9	438		2	355
	10	440		1	306
	11	436		0	252
	11·8	431	After half-an-hour	0	252
	12·6	425			

The same observations are plotted in Plate 62, fig. 32, which shows the loops formed in consequence of hysteresis by the *on* and *off* curves of subordinate cycles of stress, as well as by those of the main cycle. At each change from loading to unloading, or from unloading to loading, the new branch of the curve starts off in a direction tangent, or nearly so, to a line parallel to the axis along which stress is plotted. The subordinate cyclic operations 0—5—0, 8—3—8, 12·6—9—12·6, and 3—8—3 give rise to magnetic changes which are very nearly cyclic, and consequently produce scarcely any effect on the form of the main curves.

§ 73. *Influence of Vibration on the Effects of Stress.*—The indications of hysteresis in magnetic changes caused by stress disappear almost entirely when we submit the piece under test to mechanical vibration either during or after the changes of stress. The *on* and *off* curves then become nearly coincident. The range of magnetic change corresponding to any change of load is considerably increased. A maximum point is still found in the curve of magnetism and stress, but it is nearly the same for both (on and off) operations, and its position lies, as regards stress, between the positions which the maximums occupy when the operations of loading and unloading are performed without vibration. These results will be seen in the following observations, made on the same wire as the foregoing, and without any change of conditions, except that after each change of load the wire was vigorously tapped before the magnetometer reading was taken.

STRETCHED Iron Wire with Vibration, Plate 62, fig. 33.

	Load.	Magnetometer (after tapping).		Load.	Magnetometer (after tapping).
Loading .	0	181	Unloading .	12·6	431
	1	237		11·8	434
	2	290		11	435
	3	348		10	440
	4	384		9	443
	5	415		8	445
	6	434		7	444
	7	447		6	429
	8	451·5		5	416
	9	451		4	389
	10	447		3	353
	11	442		2	306
	11·8	436·5		1	250
	12·6	431		0	192

The same observations are shown graphically in Plate 62, fig. 33. The on and off curves are not quite coincident: they still exhibit some hysteresis, but far less than when there is no vibration.

§ 74. An instructive method of studying the influence of vibration in destroying, or rather in greatly reducing hysteresis, is to partially load or unload in the ordinary way, and then, pausing at any stage in the process, to tap the wire. An immediate,

often very large, change of magnetisation is the result. If the operation of loading or unloading be then resumed, hysteresis again becomes conspicuous.

The following observations form a good example of this method of procedure. They are plotted in Plate 62, fig. 34, where the full lines refer to the parts of the operation during which there was no vibration, and the lines drawn thus —·—·—· show the changes of magnetism produced by tapping. The wire was the same as that of §§ 71–73, and in the same conditions. A cycle of loading without tapping was gone through first. This brought the magnetometer reading, with no load, to 252. On tapping the wire this fell to 180. Loading was then begun, and carried on up to 5 kilos. without tapping. The wire was then tapped, when the magnetometer reading changed from 337 to 405. Loading was then resumed and carried on to 8 kilos. and the wire again tapped—and so on, as will be clear by inspection of the table:—

STRETCHED Iron Wire, with and without Vibration, Plate 62, fig. 34.

Load.	Magnetometer.		Load.	Magnetometer.	
0	252		12·6	423	after tapping
0	180	after tapping	11·8	425	
1	191		11	429	
2	213		10	433	without tapping
3	251	without tapping	9	437	
4	294		8	441	
5	337		8	450	after tapping
5	405	after tapping	7	452	
6	411		6	451	without tapping
7	425	without tapping	5	443	
8	440		5	425	after tapping
8	453	after tapping	4	419	
9	452		3	400	
10	451		2	361	without tapping
11	447	without tapping	1	322	
11·8	442		0	265	
12·6	436		0	181	after tapping

The agreement of points on the *on* and *off* curves, with tapping, is scarcely so good as in the experiment described in the last paragraph, probably because the vibration was scarcely vigorous enough. It is very interesting to notice how hysteresis reappears at each continuation of the loading or unloading, after the vibration has ceased, manifesting itself by the way in which each curve corresponding to such a continuation starts off tangent, or nearly tangent, to a line parallel to the axis of loads. Finally, in the descending limb of the off curve, the accumulated effect of hysteresis during the reduction of the load from 5 kilos. to 0 leaves the magnetometer reading higher by more than 80 divisions than the value which is reached by vibration at the zero of load. Nothing could illustrate better than this the manner in which hysteresis causes a diminution of the total range through which magnetism changes under varying stress, as compared with the range which we find when the variations of stress are accompanied by vibration.

§ 75. *Initial form of Curves of Magnetism and Load.*—It has been mentioned above that when a change from loading to unloading, or from unloading to loading, occurs, the new curve starts off tangent, or nearly tangent, to the direction in which loads are measured. In other words, the initial change of magnetism appears to be indefinitely small relatively to the change of stress whenever the operation is reversed from loading to unloading, or *vice versâ*. In the experiments which have been already cited the steps by which the load was increased or diminished were too large to allow the initial form of the curve to be very well defined, and as the matter seemed important as throwing light on the character of hysteresis, a set of observations were made in which the process of loading or unloading was conducted by very gradual stages at the places where the precise form of the curve was to be examined. A number of small weights were prepared, which could be added to or removed from the other weights hanging from the wire, wherever it was desired to have a number of closely consecutive points in the curve. Great care was taken in this experiment to avoid the slightest disturbance of any kind, since it was evident that the least trace of vibration would vitiate the results.

The wire, which was the same as before, and still hanging vertically in the earth's field, was loaded in the usual manner up to nearly 5 kilos.—a place where the *on* curve is very steep. The load was then removed, at first by very small parts, in order to determine as precisely as possible the initial form of the *off* curve. The following are the observations. The magnetometer scale was altered before this experiment, but the readings, although different from those taken in former experiments with the same wire, are still proportional to the total magnetisation. The wire had been slightly shaken before the loading began, and consequently the operation was not cyclic.

	Load.	Magnetometer.
Loading	0	89
	1	92·5
	2	100
	3	114
	4	132
	4·87	147
Unloading	4·78	147
	4·67	147
	4·43	146·5
	4	145·3
	3	141
	2	133
	1	118
	0	104

Here, then, it is clear that the *off* curve, beginning from a steep part of the *on* curve, starts off as nearly as can be observed tangent to the line of loads.

Similar observations were made at two other points, namely, at the upper end of

the cycle (12·66 kilos.) and at a steep part of the *off* curve (2 kilos.). The results were as follows:—

Load.	Magnetometer.	Load.	Magnetometer.
11	188	7	191
11·78	186	6	192
12·22	184·7	5	189
12·66	183	4	183·6
12·57	183	3	172·6
12·46	183	2	155·7
12·22	183·1	2·09	155·7
11·78	184	2·20	155·8
11	185	2·44	156·1
10	186·7	2·87	157·8
9	188·5	&c.	&c.
8	190		

These three observations are plotted in Plate 62, fig. 35. The results render the conclusion very probable that, if we call $\mathfrak{I}$ the magnetism and P the stress, $\frac{d\mathfrak{I}}{dP}$ is initially zero when the *change of stress* is altered in sign (from loading to unloading, or *vice versâ*), provided the operation be performed without mechanical disturbance.

§ 76. In all the above experiments on the effects of stress (§ 71 to § 75) the same wire was used—a wire which had been previously stretched beyond its limit of elasticity. To examine the effects of stress within the limit of elasticity on the magnetism of iron *in the annealed state*, and to trace the changes which occur during the passage from the annealed to the hardened state when the stress is carried up to and beyond the limit of elasticity, the following series of observations were made (March 3–4, 1882) with another piece of iron wire cut from the same bundle as the last. As before, the magnetising field 0·34 was in operation throughout.

The wire, after being hung in the vertical position behind the magnetometer which it was to occupy during the experiment, was annealed twice,—once with a small amount of weight hanging from it, which served to straighten the wire when it was heated to redness, and a second time after this weight had been removed. A weight of 1 kilo. was then applied and removed several times, during which the magnetisation rose somewhat. When the effects became cyclic the following readings were taken. As before, the magnetometer readings are proportional on an arbitrary scale to the total magnetism of the piece.

Load.	Magnetometer.
0	291
1	292
0	291
1	292

In this, the first, stage of loading, and with this small amount of stress, *on* gave *increase*, and *off* gave *decrease* of magnetism.

Then the loading was continued, with the result that the application of load up to 2 kilos. carried the magnetism through a maximum down to a value below the initial value, while unloading left it permanently raised :—

Load.	Magnetometer.
0	291
1	292
2	290
1	292
0	293

And another application and removal gave :—

Load.	Magnetometer.
0	293
1	292
2	290
1	291
0	293

Here, then, the effects having become cyclic, *on* gives *fall*, and *off* gives *rise* of magnetism, and the presence of hysteresis is quite distinct. This last action is repeated for all loads up to the limit of elasticity.

The cycles of loading and unloading were now progressively extended, by going 1 kilo. higher each time, but still keeping well within the limit of elasticity. The resulting magnetometer readings showed that, besides the alternate changes (*on* giving *fall*, and *off* giving *rise* of magnetism), there was a slight gradual *reduction* of the magnetism, due, doubtless, to some hardening of the metal; for the wire, although not visibly stretched, could scarcely fail to be permanently affected by the application of even these moderate amounts of load :—

Soft Annealed Iron Wire in Earth's Vertical Field, Plate 62, fig. 36.

	Load.	Magnetometer.
Loading	0	293
	1	292
	2	290
	3	286½
Unloading	2	288½
	1	290½
	0	292½
Loading	1	291
	2	289½
	3	286½
	4	281
Unloading	3	283
	2	285½
	1	288
	0	291½
Loading	1	290
	2	287½
	3	284
	4	280
	5	274½
Unloading	4	277
	3	279
	2	282
	1	286
	0	290
Loading	1	288
	2	285½
	3	282
	4	278½
	5	274
	6	266
Unloading	5	268
	4	271
	3	274
	2	277
	1	281
	0	287

	Load.	Magnetometer.
	0	287
Loading	1	285½
	2	282½
	3	279
	4	275
	5	270½
	6	266
	7	261
Unloading	6	263
	5	265½
	4	268½
	3	272
	2	275
	1	280
	0	286
Loading	1	284
	2	281
	3	277½
	4	273
	5	269
	6	265
Unloading	7	259½
	8	251½
	7	254
	6	256½
	5	259½
	4	262
	3	266
	2	270
	1	275½
	0	282

These and the whole of the following observations detailed in this paragraph are shown graphically in Plate 62, fig. 36.

At this point a variation of the process was introduced. The wire was vibrated by tapping, which caused the reading to rise to 295, and the cycle of loads 0–8–0 was again applied with vibration before each reading of the magnetometer. The readings were as follows; they are also shown by the curves drawn thus —·—·— in fig. 36.

	Load.	Magnetometer (after vibration.)
	0	295
	1	296½
	2	289½
	3	281
Loading	4	275½
	5	267
	6	258½
	7	249½
	8	236
	7	239
	6	243
Unloading	5	246½
	4	253
	1	265
	0	258½

It should be noticed here that a distinct maximum is passed, near the origin, before the fall of magnetism caused by increase of load occurs, and a similar maximum occurs on the *off* curve. The *off* curve lies quite clear of, and below the *on* curve, and the whole operation has resulted in a considerable reduction of magnetism. This is because the combined effect of load up to 8 kilos. and vibration has been to harden the wire somewhat, although no drawing out was noticed.

Next the cycle of loads 0–8–0 was again applied, this time without vibration, and it is interesting to notice that a passage through a maximum occurred with a load of about 2 kilos., a thing which would not have happened if the starting-point had not been altered by the previous tapping (see also fig. 36) :—

	Load.	Magnetometer.
	0	258½
	1	261½
	2	263½
	3	262
Loading	4	259
	5	254
	6	248½
	7	242½
	8	236
	7	237
	6	239½
	5	242
Unloading	4	245
	3	247
	2	250½
	1	254
	0	259

Then the application of load was repeated and carried a step further, to 9 kilos., and then (after removal) to 10 kilos. At that point the wire began for the first time to draw slightly, and a marked change occurred in the form of the curves of magnetism and stress. This will be apparent from the figures given below, as well as from the corresponding part of fig. 36.

Load.	Magnetometer.	Load.	Magnetometer.
0	259	0	255
1	257	1	253
2	255	2	251
3	252½	3	249
4	250	4	246½
5	247	5	243½
6	243½	6	240
7	239	7	236
8	234	8	232
9	229	9	227
8	231	10	218½
7	233	9	220½
6	236	8	223½
5	238½	7	225½
4	241	6	228½
3	244	5	231
2	248	4	234
1	251	3	237
0	255	2	239
		1	240½
		0	239½

The first of these two cycles agrees in character with the preceding ones; we are, in fact, still dealing with *annealed*, not *drawn* wire. But the cycle 0–10–0 begins the wire's passage from the annealed to the hard state, and the curves of magnetism exhibit a corresponding transition. The next stage in the loading, up to 11 kilos., caused a decided drawing out, and with it a complete transformation of the curves into the form which has been already described (§ 71) as characteristic of a stretched piece, and substantially the same form was retained with further successive additions of load. The load was progressively raised to nearly 15·8 kilos. Each step caused some additional stretching, and the act of stretching was accompanied each time by a fall of magnetism. The curves of fig. 36, showing the effects of successive cycles of loading, consequently became lower and lower as the operation was continued. The position of the maximum point, either on the *on* or the *off* curve, shifts out to the right—that is, it occurs with a higher value of the load, as the cycle of loads is extended. The broken lines in the figure refer to the changes which took place while the wire was in the act of stretching. At each stage in the stretching enough time was given for the wire to draw until the magnetometer reading became stationary. The

stretching took place in several steps, and its final effect was to change the diameter of the wire from 0·79 to 0·76 mm. The highest load reached was 15·8 kilos., after which a second application of the cycle 0–15·8–0 was made. The observed readings are all given below, but to avoid confusion the final *off* curve after the last loading is not plotted in the figure.

SAME wire, becoming hardened by stretching, Plate 62, fig. 36.

Load.	Magneto-meter.	Load.	Magneto-meter.	Load.	Magneto-meter.	Load.	Magneto-meter.	Load.	Magneto-meter.
0	239½	0	137	0	119	0	112	0	104
1	239	1	140	1	121	1	115	1	106
2	239	2	147	2	127½	4·8	150	2	110½
3	239	3	158	3	137	5·8	164½	5·8	150½
4	238½	4	170½	4	151½	6·8	175½	6·8	161
5	237½	5	185	5	166	7·8	184	7·8	168½
6	235	6	196	6	180½	8·8	188	8·8	172½
7	231½	7	203½	7	191½	9·8	188	9·8	174
8	228	8	206	8	198	10·8	186½	10·8	173½
9	224½	9	206	9	198	11·8	184	11·8	172
10	219½	10	204½	10	197	12·8	181	12·8	169
11	201	11	201	11	195	13·8	178	13·8	166
10	202	11·8	198	11·8	192	14·8	174	14·8	163
9	203½	12·6	187	12·6	189	15·8	158½	15·3	161½
8	205	11·8	188	13·6	179	15·3	158½	15·8	159½
7	207	11	189½	14	174½	14·8	159½	15·3	160
6	208	10	191	13·6	175	13·8	160½	14·8	160½
5	208	9	193	12·6	176½	12·8	162	13·8	162
4	204	8	195	11·8	177½	9	168½	12·8	163½
3	196½	7	196½	11	179	8	170	9	170
2	181	6	197½	10	181	7	171	8	171
1	163	5	196	9	183	6	169	7	172
0	137	4	191	8	184½	5	168	6	171
		3	179	7	186	4	162	5	168½
		2	165	6	186	3	153	4	163
		1	144½	5	183	2	140	3	152½
		0	119	4	176	1	123	2	140
				3	165½	0	104	1	124
				2	152			0	106
				1	134				
				0	112				

§ 77. The foregoing observations show with sufficient distinctness how a wire which has been stretched behaves, as regards magnetism induced by the earth's field, when successively loaded and unloaded. But the behaviour of the wire while still in the soft annealed state, that is before the stress has been allowed to pass the limit of elasticity, requires further elucidation. A number of other experiments have helped in the analysis of the somewhat complex phenomena which are produced by loading and unloading an iron wire while in the annealed state.

When the wire is annealed in the magnetising field, the amount of magnetisation which it possesses when it has become cool is an uncertain quantity, and depends much on the rapidity of the cooling, being greater for slow than for fast cooling. As soon as loading begins the magnetism rises rapidly, especially when it has chanced (perhaps from the cooling having happened somewhat quickly) to have been initially low. In fact, the annealed wire is, magnetically, in an extremely unstable state, and responds with great sensitiveness to the first application of load, every step of which (up to a certain limit) causes a rise of magnetism. This rise is not lost on the withdrawal of the stress; the application and removal of load leaves the wire permanently more magnetic. Indeed, the first application and removal of a small load produces an augmentation of magnetism not unlike that which would be produced by tapping. This continues during several successive loadings, and it is only when the load has been applied and removed many times that the changes of magnetism become sensibly cyclic.

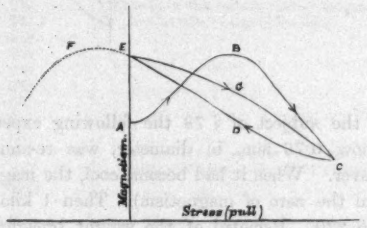
§ 78. Taking now the particular case of the annealed iron wire already dealt with, the character of the effect of load when it becomes cyclic depends on the limit to which loading has been carried. If the total load has been only 1 kilo. (corresponding to an intensity of stress of say 2·1 kilos. per sq. mm.), then its application gives increase and its removal gives decrease of magnetism, when the cyclic state is reached.

But if the load has been 3 kilos., or anything greater (still not exceeding the limit of elasticity), the effect when it becomes cyclic is that *on* gives fall and *off* gives rise of magnetism. This corresponds to the effects shown in fig. 36 for the cycles of load from 0–3–0 up to 0–9–0.

Now it would seem, from the fact that when 1 kilo. only is applied and removed *on* gives rise and *off* gives fall of magnetism, that there is essentially a maximum of magnetisation in this annealed wire corresponding to a very small load, and this view is confirmed by curves taken with vibration, for in these a maximum appears at or near 1 kilo. of load. But this maximum does not appear in cyclic curves given by applying and removing higher values of the load. Its absence is, however, satisfactorily explainable as one of the effects of hysteresis. In fact, it still exists, but hysteresis shifts it to the left of the origin, so that successive applications and removals of pull lie on one side of it. If we could trace the effects of push as well as pull, the presence of this maximum would be immediately detected. The diagram below will make my meaning more clear.

Starting from A (a point reached, say, by tapping the annealed wire, when the load is zero), and beginning to load, we find the magnetism change in the manner represented by A B C. Then on removal of the load, the curve is C D E, E being the value of magnetism corresponding to no load. The *off* curve is still rising at this point, and my explanation of this rise, and of the apparent absence of a maximum point, is that the maximum point is shifted to the left by hysteresis so much as to

fall outside the field of the operations, namely, to the left of the origin. If we could extend the experiment by applying *push* we should expect to find a continuation of the curve such as that sketched in a dotted line, with a maximum point between E and F. But of course on reapplying the load from zero the curve followed is E G C, and any succeeding repetition of the cycle gives a loop like E G C D. On the other hand, if at the beginning we had loaded with 1 kilo. only, *on* would have given rise and *off* would have given fall of magnetism in any repetition of the cycle, since in that case the whole action would have been confined to the ascending branch of the curve, namely, between A and B.



In this way, then, hysteresis causes the maximum which would otherwise occur at or near a load of 1 kilo. to disappear when cyclic loading is continued with greater loads. It also raises the value of the magnetism found with load $=0$ to a value which may fairly be called abnormal (as at E in the figure above), so that subsequent tapping under no load causes a fall of magnetism.

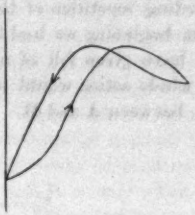
§ 79. If this explanation of the apparently anomalous actions which occur during the loading and unloading of an annealed wire be correct, the principal difference between the curves before and after stretching is in the position of the maximum point. Before stretching the maximum lies so near the origin as to disappear in the manner just described when the loading is repeated more than once. As the range of load is increased it shifts towards the right, and becomes prominently visible when and after the limit of elasticity is reached. It continues to shift towards the right as the wire is drawn out by successive increments in the range of stress. We thus find on careful examination not only continuity between the results obtained before and after stretching, but an essential similarity.

At the same time, experiments both with this and with other specimens of iron have shown that, when loaded and unloaded until the action becomes cyclic, a piece which has not been strained beyond its elastic limit since annealing behaves after the type of curve No. 1 below, and one which has received permanent set behaves after the type of No. 2.

This difference of behaviour appears, in fact, to form a criterion by which we may without fail distinguish a strained from an annealed piece.

No. 1.

No. 2.



§ 80. To illustrate the subject of § 78 the following experiments may be cited. The wire of § 76, now 0·76 mm. in diameter, was re-annealed in the vertical position three times over. When it had become cool, the magnetometer reading was found to be 159 (from the zero of magnetism). Then 1 kilo. was applied, and the reading jumped up to 220. Removal of the weight brought it to 218. The same weight was reapplied and removed several times, each time with the result that *on* gave rise and *off* gave fall of magnetism, while the successive operations gradually increased the magnetism. Then the range of loading was extended. Thus :—

Soft Annealed Iron Wire in Earth's Vertical Field, Plate 62, fig. 37.

Load.	Magnetometer.	
0 (initial)	159	
1 (first application)	220	Sudden rise of magnetism by first application of load.
0	218	
1	222	
0	220½	Here *on* gives rise and *off* gives fall of magnetism.
1	224	
0	222	
1	225½	
2	247	Note here the sudden rise of magnetism caused by this extension in the range of loading.
1	248	
0	248	
1	248	
2	248	
3	249	
2	251	
1	253	
0	254	
1	253½	Here we have distinctly reached the condition in which *off* gives rise and *on* gives fall of magnetism.
2	252	
3	250½	
2	252	
1	254	
0	255½	
1	255	
2	253	
3	251½	
3	246	After tapping the wire.
2	248½	Without tapping.
2	275	After tapping.
1	277	Without tapping.
1	299	After tapping.
0	301	Without tapping — This shows well how hysteresis leaves
0	281	After tapping — the value of the magnetism for zero load abnormally raised.
*		
1	284	Without tapping.
1	295	After tapping.
2	293	Without tapping.
2	289	After tapping.
3	286	Without tapping.
3	272	After tapping.
2	275	Without tapping.
2	286	After tapping.
1	289	Without tapping.
1	304	After tapping.
0	306	Without tapping.
0	279	After tapping.
3	281	Without tapping.
0	287	" "
3	281	" "
0	287	" "

* Up to this point these observations are plotted in Plate 62, fig. 37. The remaining operations have been omitted from the figure to avoid confusing it. The dotted curve showing a maximum at about ¾ kilo. is a curve drawn through the points reached by tapping. The tapping was not violent, in order that the soft wire should be kept from injury, and for that reason the succeeding *on* and final *off* curves with tapping do not nearly coincide.

§ 81. To examine further the state of instability which is reached by annealing in the magnetic field, the wire was again annealed with no load on it; then on cooling—

Load.	Magnetometer.
0	295
1	311
0	309
1 kilo. on and off five times, then	
0	309
1	311

In this instance, perhaps because of slower cooling, the initial magnetism was much nearer its full normal value than in the last case, and a cyclic state was reached after a single application of the load.

The wire was again annealed, by being heated to redness and allowed to cool very slowly. The reading was then 316, and the application of 1 kilo. raised it only to 317.

Finally the wire was again heated to redness, and cooled as quickly as possible (in air). The magnetometer reading was then exceptionally low, and the first application of load caused a great rise.

Then, as before, successive applications and removals of

1 kilo. gave *on* +, *off* −
2 kilos. gave doubtful, scarcely visible effects,
3 kilos. gave *on* −, *off* +
4 kilos. gave *on* −, *off* +.

§ 82. The same wire was again annealed, and the cycle 0—4 kilos. —0 was performed several times, until the changes of magnetism became cyclic. Observations were then taken during the cycle 0—4—0 which gave the curves *a b c d a* of Plate 62, fig. 38. Then the same cycle was repeated, with vibration of the wire before each observation, and the curves *e f g h e* were obtained. Finally, starting at *e*, the cycle 0—4—0 was again performed twice without vibration. This gave the curves *e i j k l m n o*.

These observations confirm what has been already said as to the existence of a maximum point near the zero of stress in the effects of stress on the magnetism of annealed iron. The maximum, which had been obliterated by hysteresis in the cycle *a b c d a* appears again in the vibration curves, and is again seen to disappear when the process of loading and unloading is resumed, without vibration. The curves *i j k l m n o* show the manner of this disappearance very plainly, and support the conclusions of § 78. A few more repetitions of the cycle would restore completely a

simple loop like *a b c d*, but at a higher part of the diagram, since during vibration the magnetisation has been permanently raised.

§ 83. *Effects of Stress on Residual Magnetism.*—Throughout the foregoing examination of the effects of stress the vertical component of the earth's magnetic force (about 0·34 c.g.s. units) was acting longitudinally on the pieces tested. The effects observed were therefore due in part to the influence of stress on the inductive susceptibility of the iron, and in part to the influence of stress on the existing magnetisation. In order to study the effects of stress on residual magnetism a solenoid was placed round the wire throughout its whole length, and a constant current from a gravity DANIELL's cell was kept up in the solenoid, its strength being adjusted, by the introduction of suitable resistance, to a value which produced within the solenoid a field just equal and opposite to the vertical component of the terrestrial field.* This left the wire free from longitudinal magnetising force, and the changes of its residual magnetism, caused by applying and removing loads, were then observed by the magnetometer as before.

A piece of the same iron wire as before (0·79 mm. in diameter and 33 centims. long) was annealed and placed within the solenoid, in which, however, there was at first no current. The wire was loaded several times, up to 8 kilos., with the earth's vertical force acting, and was also tapped, with the result that at the zero of load the magnetometer showed a deflection of 450. Then a current neutralising the field was made in the solenoid, and the magnetism of the wire fell to 440.† This residue diminished quickly when loads were again applied, as follows, but several repetitions of the cycle 0—8—0 brought about a cyclic state of matters, of the kind which has been already described as produced in annealed iron by loading and unloading under the action of a constant field.

§ 84. Then the range of load was extended. Each addition caused a fall of residual magnetism; especially when (at 12 kilos.) the limit of elasticity was reached and the wire began to draw. At 13 kilos., when the wire had drawn about 5 per cent., it was unloaded and the cycle 0—13—0 repeated. The curve already described as characteristic of the effects of stress on a stretched wire in an inducing field reappears here as equally characteristic of the effects of stress on a stretched wire when there is no inducing field, and when the magnetism which is varied is wholly residual. The observations are as follows: they are also shown in Plate 63, fig. 39.

* This adjustment was tested at intervals by removing from the solenoid the wire under examination and substituting in its place a wire of specially soft annealed iron. If tapping reduced the magnetism of this wire to zero the adjustment of the current was correct.

† This proportion of residual to induced magnetism, namely, $\frac{44}{45}$ or 97·8 per cent., is higher than in any of the experiments described in the earlier part of this paper. It must, however, be noted that in this case the wire was tapped while the inducing field was acting, and then the field was removed without tapping.

EFFECTS of Stress on Residual Magnetism (April 11, 1882), Plate 63, fig. 39.

Load.	Magnetometer.*	Load.	Magnetometer.
0	450	9	263
	in earth's field	10	253
0	440	11	244
	residual	12	233
1	436	13	192
2	429		
3	410	Wire drew about 5 per cent.	
4	387	12½	192
5	360	12	193
6	335	11	193½
7	317	10	194
8	302	9	195
7	303	8	195
6	306	7	193½
5	310	6	189½
4	314	5	181
3	318	4	166
2	324	3	146
1	329	2	127
0	330	1	103
Cycle 0—8—0 made six times; then		0	81
		1	84
0	300	2	89
1	298	3	96
2	296	4	107
3	292	5	121
4	288	6	134
5	284	7	147
6	279	8	155
7	274½	9	160
8	270	10	162
7	272	11	162
6	275	12	161
5	278	13	159½
4	282	12	160
3	285	11	161
2	290	10	162
1	294	9	163
0	300	8	163½
1	297½	7	163
2	295	6	161
3	291½	5	157
4	287	4	148
5	282	3	135
6	279	2	118
7	274	1	98
8	269	0	78

* In this series, one division of the magnetometer scale corresponds to $\Im = 0{\cdot}77$ in c.g.s. units before stretching, and $\Im = 0{\cdot}8$ after stretching. The initial value of $\Im$ was, therefore, 346. The final reading of 78 divisions gives $\Im = 62$. Each kilo. of load corresponds to a stress of 2·08 kilos. per sq. mm. before stretching, and 2·18 after stretching.

§ 85. The above operations are shown on the left hand side in fig. 39. The curves on the right hand side of the same figure refer to the following subsequent operations, for which the observed numbers need not be given. After several repetitions of the cycle 0—13—0, still acting only on the residual magnetism, the curves *ab* were taken. Then the earth's force was allowed to act (that is to say, the neutralising current in the solenoid was broken) and the cycle 0—13—0 was applied twice. The results of the second application are shown in the curves *cd*. Comparing *cd* with *ab* we see that in the inducing field the range of change of magnetism caused by change of stress is greater than when we are dealing with residual magnetism only, and the former changes occur of course at a higher position in the diagram than the latter; but the general character of the curves is substantially the same in both cases. At the end of the cycle *cd* the magnetometer reading was 140, and on tapping the wire this fell to 105, showing how, as a result of hysteresis, the value of the magnetism for zero of load was abnormally raised.

Next the neutralising current was again applied, and the wire was tapped (load = 0). The residual magnetism fell to 53. Loading and unloading then gave the curves *ef*, which, in consequence of hysteresis, left the residual magnetism at the end *greater* by 18 scale divisions than it was (after tapping) at the beginning. Tapping again reduced it to very nearly its former value.

Finally, another cycle of loads was applied with vibration before each reading, but the wire was broken at its bottom end by too violent tapping during the removal of load. The curves for this operation are *g* and *h*. Although during the operation the residual magnetism was suffering a great additional reduction by the combined effects of tapping and stress, a maximum of residual magnetism occurred in *g* at an intermediate value of the stress, as the curves taken without tapping lead us to expect.

It appears, then, that the effects of stress on the magnetism of iron, annealed or drawn, are substantially the same, whether the magnetism be induced or residual. In the latter case the effects are complicated by the gradual working out of the residual magnetism by successive loadings, just as in the former case they are complicated by the gradual working in of the induced magnetism, but when any operation is repeated often enough to give nearly cyclic effects this complication is eliminated, and we then find the effects of stress to be substantially identical in the two cases.

§ 86. *Effects of Stress on Magnetism induced by Fields of Various Strengths* (April 13, 1882).—The iron wire of the last paragraph (diameter 0·77 mm.), which had been broken close to one end, was again used, the length being now 30·5 centims. It was not re-annealed, but tested in the drawn condition to which it had been brought by the operations already described. By gradually increasing the current in the solenoid surrounding the wire, the magnetising field was brought to the value 3·33 c.g.s. units; during this time there was no load on the wire. Loading was then begun, and every addition caused at first an enormous increase of magnetism. Thus:—

	Load.	Magnetometer reading.	3.
	kilos.		
Field raised to 3·33	0	173	144
Began loading	1	246	205
	2	341	292
	3	467	389
	4	586	409
	5	678	565
	&c.		

At about 10 kilos. a maximum was passed. The load was increased up to 13 kilos., then removed, and the cycle 0—13—0 was repeated three times, with the result that the changes of magnetism became nearly cyclic. At the third repetition the following readings were taken :—

Load.	Magnetometer.	3.	Load.	Magnetometer.	3.
0	682	569	13	837	697
1	690	575	12	844	703
2	711½	593	11	853	711
3	751	626	10	863	719
4	794	662	9	873	727
5	836	697	8	883	736
6	869	724	7	892	744
7	890½	742	6	901	751
8	899	749	5	905	754
9	897	747	4	900	750
10	887	739	3	879	732
11	873	727	2	835	696
12	856	713	1	774	645
13	837	697	0	690	575

This cycle is shown in Plate 63, fig. 40, where it is marked IV. The dotted line running up through the diagram shows the effect of the loads which were first applied after the magnetising force had been brought to the value 3·33, at which it was kept during the remainder of the operation. Here, as in previous experiments to which attention was drawn in § 81, the first process of loading finds the magnetism in an exceedingly sensitive state, ready to respond by leaps and bounds to the successive increments of stress. The conditions, however, are different in the two cases. The wire of § 81 had previously been annealed in the magnetic field. In the present instance the field had been gradually increased to a constant value before the load was applied. We know already that the induced magnetism so reached is exceedingly sensitive to mechanical vibration (*cf.* § 49), and we now find a similar sensitiveness to application of stress, although unaccompanied by any vibration.

When the action is repeated often enough to bring the changes of magnetism to a

nearly cyclic state, the form of curve with which we are already familiar is given, with a maximum at 8½ kilos. on the *on* branch, and at 5 kilos. on the *off* branch.

The wire being now without load, the magnetising current was next reduced to the value just necessary to neutralise the earth's vertical force—in other words, the field was reduced to zero—and the effects of stress on the residual magnetism were examined. The effect of the first application of the cycle 0—13—0 on the residual magnetism is shown in the curves marked IV*a*. in fig. 40. We see in it the resultant of two actions, viz., the usual cyclic effect and, superposed on it, a rapid reduction due to magnetic instability. On applying a load of 1 kilo. the fall of residual magnetism (resembling the fall which would be caused by tapping, or the rise which either tapping or loading would cause in the magnetism induced by an increasing field) more than counterbalances the small augmentation of magnetism which (in the cyclic condition) would be the effect of this load, and a curious dip at the beginning of the *on* curve is the result. Then the augmentation by stress becomes relatively more influential, until the maximum is passed, after which the two actions combine to give a rapid fall of magnetism as the load is further increased.

The cycle 0—13—0 was again applied twice, and, when the residual magnetism had settled into a nearly steady state, the cycle shown at IV*b*. in fig. 40 was taken.

The same wire was then subjected to several other magnetising fields, and cycles of loading performed in them. The results, in each case after the cycle 0—13—0 had been repeated several times, are shown in fig. 40, where the curves are numbered as follows :—

No. of curve.	Magnetising field (c.g.s. units).
I.	0·34
II.	1·66
III.	2·17
IV.	3·33
V.	5·17

The curves show the values of $\mathfrak{B}$ and the total load, which can be reduced to kilos. per sq. mm. by multiplying it by 2·18.

§ 87. (May 5, 1882.) In the above experiment the range of magnetising force was somewhat limited, and the following experiments were made to test the effects of stress on higher values of magnetisation.

In this and subsequent experiments the method of demagnetising by reversals of a gradually diminishing current was used to bring the wire to an initially neutral state between successive operations. The method has been described in § 19.

The wire was the same piece of iron as before (§§ 83–86), but before these experiments it was re-annealed and then stretched by a load of 15½ kilos. which stretched it from 30·4 to 31·8 centims. The effects of loading up to 15 kilos. were examined in three fields, viz., 0·34, 2·49, and 18·65 c.g.s. units. The operations in each case were as follows :—

(1) Demagnetisation by reversals with no load on.
(2) Application of the magnetising field with no load on.
(3) Loading and unloading (0—15—0) until a nearly cyclic state was reached.

The effects of the first loading in each field are shown by dotted lines in Plate 63, fig. 41, and the cyclic effects reached after several repetitions of the loading are shown by full lines in the same figure.

Thus the point a (fig. 41) is the magnetism reached by applying a force 0·34 without load. The curves ab, bc are the effects of loading to 15 kilos. and unloading in this field. The full lines just above them are the still very imperfectly cyclic effects given by a repetition of the cycle 0—15—0 in the same field.

Similarly the points d and e are the values of magnetism reached by applying the fields 2·49 and 18·65 respectively; the dotted lines starting from these points show the effects of the first loading in those fields; and the full lines above them the cyclic effects after two or three loadings.

The curves fgh show the effects of stress on the residual magnetism left after the removal of the field 2·49. The wire has been more stretched here than in the experiment of § 86, and, probably for that reason, appears to be somewhat more persistently retentive in the present case.

It is interesting to notice here how the susceptibility to the action of stress, as regards both the first general augmentation and the subsequent cyclic changes of magnetism, is greater in the intermediate field of 2·49, than in either the strong or the weak field. In the weak field the magnetic effects are still not cyclic after many repetitions of the cycle of load. In the high field, on the other hand, a sensibly cyclic state is reached after a single loading.

The maximum point occurs at a lower load as the strength of the field is raised.

The general effect of putting *on* 15 kilos. is positive for the low field, more strongly positive for the intermediate field, but negative for the high field. This reversal of the effects of *on* and *off*, by increase of the magnetising field, was noticed first by VILLARI, and afterwards independently by THOMSON, who has since named it the "Villari reversal."*

§ 88. To study more particularly the forms of the curves of magnetism and load during the passage of the effects of stress through a maximum, and the reversal of these effects in high fields, the following series of observations was made, in which a variety of magnetising fields ranging up to 34 c.g.s. units were employed.

(May 8th, 1882.) The same wire was further stretched under a load of 18·5 kilos., suffering an additional elongation of 3 per cent. After demagnetisation by reversals, with no load on, a magnetising field was gradually applied, and then kept constant during the changes of load. Loads were applied, up to 18·5 kilos., removed, reapplied, and so on. The load was then removed, the wire demagnetised by

* VILLARI: Pogg. Ann., CXXVI., 1868; THOMSON: Phil. Trans., 1879, p. 55.

reversals, a new field applied, and the operation of loading and unloading resumed. The results are shown in Plate 63, fig. 42, and it would lengthen this paper needlessly to give all the recorded numerical values. The effects of the first loading after the application of each magnetising field are shown in dotted lines: the effects of a cycle 0—18·5—0 after several repetitions had brought about a nearly cyclic state of matters, are shown in full lines. The wire was demagnetised by reversals before the application of each new magnetising field. The fields were :—

Curve (in fig. 42).	Magnetising field (in c.g.s. units.)
I.	0·34
II.	2·49
III.	5·75
IV.	8·6
V.	11·6
VI.	20·1
VII.	34·0

One kilo. of total load corresponds to 2·3 kilos. per sq. mm.

This series of curves shows well the gradual transformation which occurs as we increase the intensity of magnetisation, before applying cyclic changes of stress. As the region of magnetic saturation is reached the maximum points on the *on* and *off* branches occur with lower and lower values of the load, and the descending branch of the *on* curve (to the right of the maximum) becomes a more and more important part of the whole. In the highest curve (No. VII.) it constitutes, in fact, nearly the whole action, and the *off* maximum point comes very near the zero of load, with the result that the subsequent *on* curve starts from so high a point as to exhibit no maximum at all. To the last, however, the initial curve of loading, after the field has been applied, shows a maximum point, though that also comes nearer the zero of load as the wire becomes more strongly magnetised. The resultant effect of the first loading passes a maximum, but remains positive in all but the highest curve; and after the cyclic state is established the general effect of *on* load is positive up to curve V., and has its maximum in curve II. The following table shows some of the results deducible by a comparison of the curves :—

EFFECTS of Stress on a Stretched Iron Wire in Constant Magnetic Fields, Plate 63, fig. 42.

No. of curve.	(1) Magnetising field, $\mathfrak{H}$.	(2) Initial $\mathfrak{Z}$ value of before loading.	(3) This was changed by first application of 18·5 kilos. to	Difference (3) − (2) showing effect of first *on*.	Cyclic effects of 18·5 kilos.			Value of load in kilos giving *maximum* of $\mathfrak{Z}$.		
					Value of $\mathfrak{Z}$ for *off*.	Value of $\mathfrak{Z}$ for *on*.	Difference *on* − *off*.	During first loading.	During cyclic *on*.	During cyclic *off*.
I.	0·34	7	75	68	110	183	73	14	10·7	7·0
II.	2·49	50	590	540	463	614	151	11	9·8	6·1
III.	5·75	195	740	545	642	748	106	10	9·6	5·9
IV.	8·6	372	842	470	776	845	69	9·8	9·5	5·7
V.	11·6	527	907	380	870	910	40	9·6	9·4	5·4
VI.	20·1	875	1045	170	1115	1045	− 70	8·4	8·2	4·7
VII.	34·0	1185	1140	− 45	1270	1141	−129	4·8	No maximum.	1

The numbers in column (2) are the values of $\mathfrak{Z}$ reached by applying each magnetic field to the previously demagnetised wire. They correspond to the starting points of the dotted lines in fig. 42.

By comparing figs. 40, 41, and 42, as well as from fig. 36, we see that the position of the maximum point is shifted out (that is, towards higher values of the load) the more the wire is stretched. It is not, however, shifted out so far as to prevent the descending (right hand) limb of the curve from becoming relatively larger as the range of loading is extended by successive stretchings. This limb is most prominent in fig. 42, where the range of loading is extended to 18·5 kilos., although the maximum points occur there at higher loads than in figs. 40 and 41, in which the amount of stretching was less.

§ 89. *Similar Experiment with a Soft Annealed Wire.* — Only one more experiment of the same class as the foregoing need be referred to in detail. It was made (May 26, 1882) on another specimen of the same kind of iron wire as before (0·79 mm. diameter and 33 centims. long) to test the effects of load in various magnetic fields when the wire was in the soft annealed state. The wire was carefully annealed and surrounded by two solenoids, in one of which a current was maintained which neutralised the vertical component of the earth's field, while the other was used to give the desired magnetising force. The wire (whose limit of elasticity was pretty sharply defined at 10 kilos.) was never loaded with more than 6 kilos., and instead of reducing the load to zero in each cyclic application, a load of 1 kilo. was kept always on, in order to avoid errors due to local bending and unbending, which are much more liable to occur when we are dealing with annealed wire than with wire which has been well straightened by stretching. The process of experiment was in other respects the same as before. Before the application of each magnetising field the wire was demagnetised by reversals (with 1 kilo. on), then the

field was applied, and then a cycle of loads 1—6—1 was applied several times while the magnetic changes were noted. Except in the region of approximate saturation, the first effects of loading were positive, but in succeeding applications *on* gave diminution and *off* gave increase of magnetism, the curves for *on* and *off* forming a simple loop like that already described in speaking of the effects of stress on an annealed wire exposed to the vertical component of earth's field. (§ 78.)

The action was in this case examined in fields ranging up to 12 c.g.s. units, and the results are shown in Plate 64, fig. 43, where the curves are numbered as follows :—

Curve.	Magnetising field (in c.g.s. units.)
I.	0·68
II.	1·50
III.	2·05
IV.	2·73
V.	3·41
VI.	4·78
VII.	12·0

The line which (in every case but curve VII.) slopes up to the right starts from the value of magnetism reached by applying the field to the (previously demagnetised) wire, and shows the effect of the first loading. The cycle of loads was not applied often enough to make the changes of magnetism cyclic, except at the highest magnetisation, where (as in the stretched wire already referred to) a cyclic state is reached very quickly. In each field the effects of several successive loadings and unloadings are exhibited in fig. 43, in a way which inspection of the figure will make obvious. Curve I*a* was obtained in the field 0·68 by tapping the wire and then applying and removing loads.

Each kilo. of load corresponds in this wire to a stress of 2·04 kilos. per sq. mm.

§ 90. *Curves of* $\mathfrak{I}$ *and* $\mathfrak{H}$ *taken under Constant Load.*—Concurrently with the series of experiments described in §§ 87–88, and represented in figs. 41 and 42, another series of observations was conducted on the same wires and with these in the same states as in the foregoing tests, by a wholly different method. Each wire, after being demagnetised by reversals, had a definite load applied to it. Then magnetising force was gradually applied, and the relation of $\mathfrak{I}$ to $\mathfrak{H}$ was observed. This gave a curve of magnetisation under that particular load. Then, after demagnetisation, a different load was put on, another curve of magnetisation taken by again applying magnetising force, and so on. The magnetising force was applied gradually and continuously by means of the slide described in § 18, and the magnetisation was observed by the direct magnetometric method. In fact, as regards measurement of magnetism and field, the conditions of each experiment were identical with those under which the changes of magnetism were caused by changing loads in a constant field, and the

same factors were used to reduce to absolute measure the results of these two methods of experiment. The results of the two are, therefore, directly comparable with each other.*

§ 91. *Annealed Iron Wire.*—In the group which I shall first describe of observations made in this way the wire of § 87 was used, but in the *soft annealed* state which preceded the stretching by 15½ kilos. there described. Curves of the relation of magnetisation to magnetising force were taken (May 1, 1882) with loads of 1, 2, 4, and 6 kilos., as well as with no load, and after each curve taken with load, the load was removed and a curve without load taken, in order to see whether the loading had produced any permanent change in the quality of the metal. For brevity a curve of $\mathfrak{I}$ and $\mathfrak{H}$, taken with no load on the wire, will be called a "normal" curve. The normal curves after each unloading agreed so very approximately with one another as to show that loading up to 6 kilos. did not sensibly harden the wire. It should be remembered that as the diameter was 0·77 mm. 1 kilo. of load corresponds to a stress of 2·15 kilos. per sq. mm. After each magnetisation the magnetising current was gradually reduced to zero, observations of $\mathfrak{I}$ being made; the wire was then unloaded and demagnetised by reversals, a new load applied, and another curve of magnetisation taken.

The results are shown in Plate 64, fig. 44, where $\mathfrak{I}$ and $\mathfrak{H}$ are given in absolute units. The curves show the effects of gradually removing as well as gradually applying magnetising force, the effects of removal of $\mathfrak{H}$ being shown, however, only towards the end of that operation, in order not to confuse the figure. The general characteristics of the curves are these:—For equal *low* values of the magnetising force the magnetisation is greater in the loaded than the unloaded curves. But each loaded curve crosses the normal or unloaded curve at a value of $\mathfrak{I}$ which is lower the higher the stress, and beyond that the magnetic susceptibility is less when the wire is under load than when it is not loaded. Finally, in the region of saturation, all the curves are converging, and it appears probable enough that with very great magnetising forces the magnetisation would be the same for no load and for all values of the load.

As regards the *off* curves, these preserve the relative places the *on* curves have in the region of saturation, that is to say, the normal curve is highest, and the curves for load lie lower the greater the load is. The distance between the *off* curves augments as the magnetising force approaches zero, and we then find that the loaded wire is much less retentive than the unloaded.

The maximum value of κ or $\frac{\mathfrak{I}}{\mathfrak{H}}$ is greater in the 1 kilo. curve than in any of the others. But the steepest slope or $\frac{d\mathfrak{I}}{d\mathfrak{H}}$ is to be found in the normal curve.

* A few curves of magnetisation under load, similar to those about to be described, have recently been published by Mr. R. Shida, in a paper entitled "Experimental Determinations of Magnetic Susceptibility and of Maximum Magnetisation in Absolute Measure," Proc. Roy. Soc., No. 227, 1883.

The presence of stress in an annealed wire tends to round off the outlines of the curve of magnetisation, so that it resembles somewhat the curve described in § 33 as characteristic of a wire which has been stretched beyond its limit of elasticity, and this happens although the stress is too small to give the wire any permanent set, or to harden it appreciably.

§ 92. Plate 64, fig. 45 represents the results of the same experiment in a different manner. In it the values of magnetisation reached by applying a particular magnetising force while the load was kept constant, are plotted in relation to the constant loads (2, 4, and 6 kilos.) which were kept on during the several magnetisations, and a curve is drawn through the points so obtained. This gives a series of curves, each referring to some particular value of the magnetising force, which is entered in the figure alongside of the curve. The curves may be said to represent the relation which the susceptibility to magnetisation (by the several assigned magnetising forces) bears to the stress upon the wire. They are closely analogous to curves of the type of figs. 31 to 43, but with this important distinction, that in the present case the load was on before the magnetising field was applied, while in them the magnetising field was applied first and then the load. Were there no hysteresis in the changes of magnetisation the curves of fig. 45 should agree exactly with curves obtained by applying to an annealed wire a constant magnetising force first and then varying the stress. But we know that there is hysteresis in the relation of magnetisation to magnetising force, under constant load. We also know that there is hysteresis in the relation of magnetisation to stress, when stress is changed in a constant field. There is, therefore, a double reason why the curves obtained in the way now exemplified should differ from those of fig. 43, where, as here, the iron was dealt with in the soft annealed state.

Fig. 45 shows that at low magnetisations the susceptibility is increased by the presence of longitudinal pull. At higher magnetisations we find a maximum of susceptibility, which occurs with lower and lower values of the stress the stronger the magnetisation is, until finally, when we approach the region of saturation this maximum disappears, as it were, to the left of the figure, and we then find the susceptibility greater without than with the load.

This is quite in agreement with the results already obtained by the other method, —the method, namely, of varying stress in a constant field (see figs. 36–38, and 43, §§ 76–82, 89).

§ 93. *Stretched Iron Wire.*—The effects of the presence of stress on magnetic susceptibility become much more conspicuous when we are dealing with wire which has been stretched beyond its elastic limit.

The wire of the above experiment was stretched by applying a load of 15½ kilos., which brought it to the state already mentioned in § 87. In addition to the experiments of the old type described there, observations were made upon it (May 4, 1882) by the plan of magnetising under constant load. The loads used were 0, 1, 2, 4, 6,

and 8 kilos. After demagnetising by reversals the load was put on, then magnetising force was gradually applied up to 18 c.g.s. units and gradually removed, while magnetometer observations were taken. The results are shown in Plate 64, fig. 46, where, for the sake of comparison, a normal (no-load) curve, taken just before the wire was drawn by 15½ kilos., is also given. The effects of load, especially of 6 or 8 kilos., is enormous in increasing the susceptibility in the first part of the curve: the susceptibility is actually *eight times* greater with than without that load in the early part of the magnetisation. It is particularly interesting to notice that the wire after being stretched and then loaded with 4, 6, or 8 kilos. is actually *more* susceptible to magnetisation by low fields (under $\mathfrak{H}=1$) than it was when in the soft annealed state. The crossing of the 6 and 8-kilo. curves will be noticed.

§ 94. Neither the range of loads nor the greatest magnetising force applied was sufficiently great to make this experiment exhaustive, and accordingly another series of similar observations was made (May 10, 1882) on the same wire after it had been further drawn by 18·5 kilos. Its state was then identical with that in which the experiment of § 88 was made, and the results given below are, therefore, directly comparable (allowance being made for the effects of hysteresis) with those there described as obtained by the other method, and exhibited in fig. 42. Curves of the relation of $\mathfrak{I}$ to $\mathfrak{H}$ were taken under the following constant loads:—0, 1, 2, 4, 6, 8, 10, 12, 14, 16·2, 18·5 kilos., the highest magnetising force reached being 33·6 c.g.s. units. Before taking each curve the previous magnetisation was removed by the method of reversals before the previously applied load was removed. Then that load was removed, and the process of demagnetising by reversals was repeated. Then the desired load was put on, and the wire was magnetised, the relation of $\mathfrak{I}$ to $\mathfrak{H}$ being observed. The results are given numerically below, and are shown in Plate 64, fig. 47, from which it will be seen that the curves taken with load, although lying much higher than the normal or no-load curve in the early part of their course, begin to cross it as the magnetisation becomes strong, the first to cross being the 18·5-kilo. curve—that of greatest stress. In other words, the magnetic susceptibility, although far greater in the stressed than the unstressed wire so long as the magnetisation is moderate, becomes less in the stressed wire as the region of saturation is reached. During nearly the whole of its length the 8-kilo. curve lies higher than any of the others. It is with this load that the *maximum* of susceptibility occurs, except at very low and again at very high values of the magnetisation.

In the lower part of the same figure curves are given which show the relation of $\mathfrak{I}$ to $\mathfrak{H}$ during the gradual removal of $\mathfrak{H}$.

MAGNETISATION of a stretched iron Wire under various (constant) loads.—Plate 64, fig. 47.

$\mathfrak{H}$.	Values of $\mathfrak{B}$ reached by magnetising under the following loads :—											
	0.	1 kilo. or 2·3 kilos. per sq. mm.	2 kilos. or 4·6 kilos. per sq. mm.	4 kilos. or 9·2 kilos. per sq. mm.	6 kilos. or 13·8 kilos. per sq. mm.	8 kilos. or 18·4 kilos. per sq. mm.	10 kilos. or 23·0 kilos. per sq. mm.	12 kilos. or 27·6 kilos. per sq. mm.	14 kilos. or 32·2 kilos. per sq. mm.	16·2 kilos. or 37·3 kilos. per sq. mm.	18·5 kilos. or 42·5 kilos. per sq. mm.	0 repeated.
0	0	0	0	0	0	0	0	0	0	0	0	0
1·15	12	15	20	35	38	..	62	55	45	44	42	11
2·01	27	32	48	92	112	160	163	138	118	110	99	25
2·87	50	62	95	192	250	310	310	258	225	200	180	45
4·31	110	150	215	377	490	542	532	470	425	380	343	108
5·75	195	258	342	525	650	698	688	620	570	522	475	190
7·19	282	362	455	637	760	800	785	725	675	620	572	280
8·62	367	455	553	725	842	875	860	800	755	700	650	365
11·50	525	612	710	860	955	978	962	908	870	813	762	520
14·37	660	748	835	958	1032	1045	1032	982	947	892	845	655
17·25	780	858	933	1030	1088	1092	1080	1038	1005	955	910	770
20·12	882	950	1012	1088	1128	1128	1118	1078	1048	1003	960	873
23·10	970	1025	1078	1130	1160	1155	1145	1107	1085	1042	1000	958
26·0	1048	1088	1130	1165	1185	1178	1168	1133	1112	1072	1035	1035
33·6	1182	1200	1225	1229	1232	1220	1210	1180	1168	1135	1105	1180
23·10	1112	1142	1172	1190	1200	1188	1175	1145	1127	1088	1052	1108
17·25	1028	1078	1120	1158	1175	1162	1150	1112	1090	1045	1002	1023
11·50	885	958	1023	1095	1142	1125	1108	1060	1028	973	920	882
5·75	695	777	862	982	1047	1045	1018	955	905	840	788	690
2·87	585	662	755	890	970	970	935	863	808	745	690	580
0	465	537	622	762	840	838	793	712	660	603	550	455

§ 95. The same results are also exhibited graphically in Plate 65, fig. 48, in the manner explained in § 92, and this figure admits of immediate comparison with fig. 42, because both figures refer to the same piece of wire in the same physical condition. Fig. 48 shows very clearly how there is, for each value of the magnetising force, an amount of stress for which the susceptibility is a maximum, how this maximum occurs at a lower and lower value of the stress as the magnetisation is increased, and, finally, how the resultant effect which the greatest load (18·5 kilos.) exerts on the susceptibility passes from positive to negative as the state of saturation is approached. The magnetising force to which each of the curves relates is given in the figure, and is in several values the same as that at which, in the previous group of observations, the effects of loading and unloading were investigated (§ 88). From what has been said in § 92 it will be at once seen that the curves of fig. 42 would agree with those of fig. 48 if the effects of hysteresis were absent. In that case it would be a matter of indifference whether stress were applied before magnetising force, or magnetising force before stress. Hysteresis affects both experiments, though quite differently, but by examining the two sets of curves together we can see the general relations of magnetic susceptibility to stress in the features which are exhibited by both. From the two together we may confidently conclude that the influence of pull on magnetism is positive until the magnetisation is so much raised as to bring about the VILLARI reversal; that this positive effect is slight at low magnetisations, and increases greatly so as to pass a maximum before reversal; that the amount of this positive effect depends on the amount of stress applied, passing a maximum whose position varies in the way stated above; that the value of the magnetisation at which the VILLARI reversal occurs depends on the value of the stress, being lower the greater the stress. In fact, if we deal only with very small stresses it is doubtful whether any reversal of the positive effect of stress would be reached even at the highest attainable value of the magnetisation.

§ 96. *Residual Effects of Stress Changes occurring when the Wire is free from Magnetism.*—During the progress of some experiments of the same type as those just recorded, but preliminary to them, a very curious phenomenon was noticed, which gave a fresh and interesting example of the presence of hysteresis in a somewhat occult form.

If we take an iron wire which has already been stretched beyond its original limit of elasticity, and which has, therefore, reached a stable state as regards any subsequent applications of lesser pull, we may load and unload and magnetise it as we please, but provided we demagnetise it by the method of reversals after removing the load, we shall, on taking a new "normal" curve of magnetisation, always find its susceptibility sensibly the same. But suppose that after the wire has been completely demagnetised we apply a load and remove it. During this process there has been no magnetisation visible, and there has been no mechanical change of any ordinary kind. Nevertheless, a molecular change of a very decided character has occurred, which is at once detected

when (after applying and removing load) we take a curve of magnetisation. We find the magnetic susceptibility now changed from what it was. The difference of condition is merely this, that in the former case, after removing a previously applied load the wire was passed through the process of demagnetising by reversals before a new curve was taken, while in the latter case the wire was demagnetised before the load was removed, and the load then removed before the curve was taken. So slight a difference of procedure as this might fairly be expected to have no sensible influence on the results of the test, but it was only after recognising that this difference and others like it had an extraordinarily great influence that I was able to obtain the mutually consistent results stated in the above paragraphs, by adhering to a strictly uniform mode in the taking of every curve. I found that after the wire had been demagnetised completely, the application and removal of any load, though producing no immediately visible effect, affected most materially the subsequent behaviour of the wire. Indeed the curve of magnetisation under any given load depends not only on the load actually present, but on any changes of load which have been permitted to take place since the preceding demagnetisation. For example, if a curve was to be taken with (say) a load of 3 kilos. on the wire, and if, after complete removal of all visible magnetisation, the load were accidentally raised to 4 kilos. and 1 kilo. then removed, the resulting curve was very sensibly different from what it would have been if the weight had simply been raised to and kept at 3 kilos., and this, too, in spite of the fact that the wire had been frequently subjected to more than four times that amount of stress, and was therefore in a mechanically stable state.

§ 97. This obscure but very interesting phenomenon of the residual effects of previously applied stress formed the subject of a large number of experiments, made for the most part on the wire of § 94, which had been previously stretched by a load of 18·5 kilos. In describing these experiments, the interest of which lies only in the comparative and not in the absolute values of the induced magnetism, it is needless to reduce the observations to absolute measure, and I shall generally give merely the galvanometer and magnetometer readings, instead of $\mathfrak{H}$ and $\mathfrak{Z}$.

(May 12, 1882.) After many applications of a load of 18·5 kilos. the wire was demagnetised by reversals without load, and then on applying magnetising force the readings in Column I. below were taken.

Next, after removal of the load, the wire was again demagnetised, then loaded to 18·5 kilos., then unloaded, and the observations of Column II. were taken. Lastly, the wire was again demagnetised, loaded to 18·5 kilos., unloaded, *tapped*, and the observations of Column III. taken. In all three cases there was no load on the wire during the process of magnetisation.

Galvanometer readings.	$\mathfrak{H}$.	Magnetometer readings.		
		I. After demagnetisation with no load.	II. After the cycle $0-18\frac{1}{2}-0$.	III. After the cycle $0\text{-}18\frac{1}{2}\text{-}0$ and then vibration.
0	0	0	0	0
20	1·15	5	8	5
35	2·01	11	19	10
50	2·87	19	40	17
75	4·31	44	73	35
100	5·75	78·5	110	70
150	8·62	149	176	150
200	11·50	212·5	230	214
250	14·37	267	278	268
300	17·25	314·5	321	314
350	20·12	355	358·5	354
400	23·00	390	394	388
450	25·87	420	420	422
576	33·12	472	472	471
400	23·00	445	441	443
300	17·25	412	407	409
200	11·50	355	351	352·5
100	5·75	278	275·5	276
50	2·87	234	232	232
0	0	183·5	183	182

The same experiment is shown in the curves of Plate 65, fig. 49, where the dotted curve is No. III., taken after applying the cycle $0-18{\cdot}5-0$ to the previously demagnetised wire, and then vibrating it before magnetising, the full line (I.) is the curve taken after demagnetisation with no load, and the broken line (II.) is the curve taken after the cycle of loads $0-18\frac{1}{2}-0$. No vibration occurred during the taking of the curve.

This experiment (which was confirmed by many others like it) shows that the application and removal of load, after the wire is demagnetised, increases its susceptibility to subsequent magnetisation, but that this increase of susceptibility is removed, and the original condition approximately restored, by vibrating the wire after the application and removal of load, and before beginning to magnetise. Curve No. III., nearly coincides with No. I., proving that the residual effect of the previous loading and unloading, which had raised the susceptibility to the level shown by Curve II., was neutralised in great part by mechanical disturbance.

§ 98. Now it seems from this, that if we apply and remove stress in a wire whose magnetic state is entirely neutral, *we cause some kind of molecular displacement in the relation of which to the applied stress there is hysteresis.* This molecular displacement, whose precise character has yet to be determined, is the cause of the changes of magnetic susceptibility which accompany changes of stress.

When a load is applied and removed the molecular displacement lags behind the changes of stress, and we therefore find on subsequently magnetising the wire that the susceptibility has been changed by the process—that its value is more nearly

equal than before to what it would be with a load actually on the wire. In the present case, to apply and remove 18½ kilos. has raised the magnetic susceptibility above the value it had after the wire was demagnetised by reversals. The action is, in fact, analogous to the augmentation of magnetism which occurs (through the influence of hysteresis) in a *magnetised* wire which has been tapped, by applying and removing a load (see §§ 78, 85).

Here there is no visible magnetism for the stress to act on, but we find evidence of the fact that a load has been applied and removed in the augmented state of susceptibility in which it leaves the wire.

Moreover, just as in the more obvious manifestations of hysteresis already dealt with, the effect of vibrating is to remove in great part the traces of previous operations. The augmented susceptibility, which is found after loading and unloading, disappears if we vibrate the wire before magnetising, just as when a wire hanging in a magnetic field is vibrated and then loaded and unloaded, the abnormally raised value of the magnetism thereby reached is reduced by again vibrating (§ 85).

§ 99. I was thus led to conceive of the magnetic susceptibility of iron—a quality measurable only by magnetising, but easily thought of as existing apart from any actual magnetisation—as changing with changing stress in a manner which involves hysteresis, even when the changes of load take place in the absence of all visible magnetisation. Its value depends on previous as well as on actually present loads. Now the general relation of susceptibility to stress, as tested by magnetising under constant load, resembles that of magnetism to stress when the stress is changed in a constant field (*cf.* figs. 48 and 42), and therefore it seemed probable that the changes of susceptibility which occur when we load and unload a demagnetised wire (and which we now know to be characterised by hysteresis) would, if we could render them visible, be found to follow curves very much like those of figs. 31–42. In those curves we found a very notable difference between values reached with, say, 3 kilos., on the *on* and *off* branches, and this led me to expect a similar kind of difference in the susceptibility of a wire when, after demagnetisation, it was treated in these two ways:—

(1) Load to 18·5, and unload to 3.

(2) Load to 18·5; unload to 0; load again to 3.

The susceptibility in the first case was likely to be *greater* than in the second, except perhaps when the magnetism approached saturation. Further, it seemed reasonable to expect that the difference between these cases would be very nearly eliminated if in each case the wire was vibrated before the curve of magnetisation was taken.

§ 100. That these expectations were completely fulfilled the following experiment (of May 13, 1882) will show. The wire used was the same as before. Curves of magnetisation were taken after the several modes of treatment set forth in the following table, which also gives the recorded observations of current and magnetism:—

Galvanometer readings. (To reduce to ϕ multiply by 0·0575.)	Magnetometer readings.				
	I. Demagnetised with no load. Then no-load (normal) curve.	II. Demagnetised with no load. Then 0 – 18½ – 3. Load = 3 kilos.	III. Demagnetised with no load. Then 0 – 18½ – 0 – 3. Load = 3 kilos.	IV. Demagnetised with no load. Loaded to 18½, unloaded to 3 kilos., and *tapped* before magnetising. Load = 3 kilos.	V. Demagnetised with no load. Loaded to 3 kilos. and *tapped* before magnetising. Load = 3 kilos.
0	0	0	0	0	0
25	6½	22	13	11	10
50	20	70	14	36	34
75	45	139	109	103	100
100	79	198	176	174	168
125	..	242	226	227	219
150	150	276	265	268	259
200	212	328	323	328	320
250	262	..	365	369	365
300	..	398	398	403	400
350	354	424	425	429	427
450	420	461	462	467	466
588	477	491	494	499	498
0	184	274	275	277	276

Curves corresponding to cols. I., II., and III. of the above table are given in Plate 65, fig. 50. The table and curves show that the susceptibility, except when the magnetisation is high, is greater with a load of 3 kilos., reached by unloading from 18½, than with the same load reached by loading from zero. But columns IV. and V. show that when, after each of these processes, we subject the wire to vibration before beginning to magnetise (it must not be supposed that there was any vibration during magnetisation) this difference is so much reduced as to be scarcely sensible. The effect of tapping is, so to speak, to blot out the recollection of previously existing stresses, and to make the susceptibility depend only on the actually present load.

In addition to the observations stated above, a magnetisation curve was taken after the operation 0—18½—0—18½—3, but this, as might be expected, was almost absolutely coincident with No. II. A repetition of the conditions of col. III. gave sensibly the same readings as those of col. III.

In this experiment 3 kilos. was selected as the load under which the wire should be tested, because in the curves of fig. 42 the difference between the *on* and *off* curves is on the whole greater for 3 kilos. than for any other load.

§ 101. In the experiment of the last paragraph the magnetising solenoid was wound directly on the iron wire, and it seemed conceivable that the wire might have been grasped by it so closely as to be prevented from fully responding throughout its whole length to the change of stress caused by loading and unloading. I did not think it possible that this could have occurred to such an extent as to account for the hysteresis of susceptibility with regard to load which had been observed, but to

place the matter beyond doubt, this solenoid was unwound, and another was applied which, instead of being wound on the wire itself, was wound on a glass tube inside of which the wire hung free.

This new set of conditions gave results which entirely confirmed those obtained before, and which may therefore be described only very briefly. The magnetising force was not raised so far as before, as the early parts of the curves were of principal interest.

The results of four series of readings are shown in Plate 65, fig. 51, as follows :—

I. is the normal or no-load curve, taken after demagnetising with no load on.

II. is the curve taken after first demagnetising with no load and then performing the cycle 0—15 kilos.—0 before magnetising.

III. is the curve taken with 3 kilos. on after first demagnetising with no load, and then applying the loads 0—10—3 before magnetising.

IV. is the curve taken with 3 kilos. on after first demagnetising with no load, and then applying the loads 0—10—0—3 before magnetising.

The same differences are exhibited as in the former instance. II. shows greater susceptibility than I., and III. than IV. It is indeed not a little remarkable that such apparently trifling variations of conditions should cause the great differences which these curves manifest.

§ 102. With the same wire and the glass-tube solenoid mentioned in the last paragraph the following additional observations were made.

The wire was demagnetised by reversals, with no load, and then loaded with 14 kilos. and a curve taken. The wire was then demagnetised by reversals *with the 14 kilos. on*, and a second curve taken. Then the load was removed, and similar pairs of curves taken with 12, 10, 8, 6, 4, and 2 kilos. of load. Plate 65, fig. 52, shows the results. In the left-hand portion of the figure we have the curves taken with loads of 14, 12, 10, 8, 6, 4 and 2 kilos., when each of these loads was applied after demagnetisation, and no second demagnetisation took place before beginning to take the curve. In the right-hand portion of the figure the conditions are the same except that after the load was put on a second demagnetisation was performed before the curve was taken.

Each curve in the latter series lies below the corresponding curve in the former, showing that demagnetisation after loading leaves the wire in a less susceptible condition than loading after demagnetisation produces. Comparing the curves of each group amongst themselves we see that with both modes of treatment a stress of 8 kilos. gives, throughout the greater part of the curves' course, a greater value to the susceptibility than any of the other stresses. It should be remembered that each kilo. of total load means an intensity of stress equal to 2·3 kilos. per sq. mm.

§ 103. The parts of these curves near the origin lie so close to one another that their relative positions are not clearly shown in fig. 52, and the results of this experiment are better seen by plotting the same observations in the manner described in

§ 92 and already exemplified in figs. 45 and 48, where the magnetisation reached under each load is plotted in terms of the load. This is done in Plate 66, fig. 53, the full lines of which refer to the observations made when the wire after being loaded in a magnetically neutral state was not again subjected to the process of demagnetising by reversals before its susceptibility was tested, and the dotted lines refer to the corresponding observations made when the wire, after being loaded, *was* again subjected to the process of demagnetising by reversals. The two curves (plain and dotted) start of course from the same point (at the zero of load), but within the range of magnetisation and load here examined each dotted curve lies below the corresponding plain one; in other words the process of demagnetising when loaded reduces the susceptibility. The load giving *maximum* susceptibility is slightly different in the two, being greater in the dotted than in the plain, and the range of difference of susceptibility under different loads is strikingly less in the dotted than in the plain curves, especially when the magnetisation is feeble. The numbers on the right of the plain curves show the magnetising force corresponding to each pair.

§ 104. Up to this point the experiments on the residual effects of previous stress on susceptibility to subsequent magnetisation had all been made on iron wires which had been hardened by stretching beyond their elastic limit. It seemed desirable to examine in the same way a soft annealed wire, and accordingly the one already used in § 89 was also tested (May 24, 1882) by applying magnetising force while the load was kept constant. The glass-tube solenoid (§ 101) was again used, and the load was not allowed to exceed 6 kilos., nor to be less than 1 kilo., as in § 89. Plate 66, fig. 54, shows the curves of magnetisation obtained in the following four conditions:—

First pair.
- I. Demagnetised with 1 kilo. on; tested with 1 kilo. on.
- II. Demagnetised with 1 kilo. on; loaded to 6 kilos. and unloaded to 1 kilo.; tested with 1 kilo. on.

Second pair.
- III. Demagnetised with 1 kilo. on; loaded to 6 kilos. and again subjected to the process of demagnetising by reversals with 6 kilos. on; then tested with 6 kilos. on.
- IV. Demagnetised with 1 kilo. on; loaded to 6 kilos. and the cycle 6—1—6 performed three times, ending with 6 kilos.; then tested with 6 kilos. on.

One kilo. of load gives a stress of 2·04 kilos. per square mm.

We find that the 6-kilo. curve No. III. lies at first higher and afterwards lower than the 1-kilo. curve No. I.—a result which is entirely in agreement with the conclusions deducible from experiments made by changing loads in a constant magnetic field. We find also that the two curves in each pair differ, in virtue of the residual effects of previous operations, in the manner which previous experiments lead us to expect—namely, that the process of applying reversals after the load is on reduces susceptibility.

It is interesting to notice (as was shown by the curves of figs. 44 and 46, &c., and is now confirmed here) that whereas the presence of a moderate stress, such as 12 kilos. per square mm., greatly increases the susceptibility of a previously stretched wire, it greatly reduces the susceptibility of a soft annealed wire, although, as is evidenced by the recovery of susceptibility after the stress is removed, the stress is too small to produce any sensible hardening of the metal.

§ 105. I have now shown that both in the soft annealed and in the hard-drawn state iron exhibits variations of magnetic susceptibility under varying loads which depend not merely on the existing state of stress, but on the states of stress which have preceded that which is selected as a condition of experiment, although these preceding stresses produce no permanent mechanical effects, and, indeed, leave no directly visible traces of their having been applied, and although the piece under test is free from visible magnetism during these changes of stress. But, with respect to this last point, it occurred to me that the conditions of the foregoing experiment were not altogether satisfactory, for this reason, that the wires were to some extent affected by the horizontal component of the earth's magnetic force. They hung vertically, and the vertical component of the terrestrial field was neutralised by a solenoid provided for that purpose. The only *longitudinal* magnetisation was therefore that given by the other or magnetising solenoid. But this arrangement left the horizontal component of the terrestrial field to act *transversely* on the wires, producing, of course, some small amount of transverse magnetisation, which must have been altered with every change of load. Now it was conceivable, though it appeared very unlikely, that the alterations of this *transverse* magnetism produced by changes of load should (even in the absence of all longitudinal magnetism) produce changes of susceptibility to subsequent longitudinal magnetisation sufficiently considerable to account for the phenomenon discussed in the preceding paragraphs (§§ 96–104). For this reason I judged it desirable to repeat some of the above experiments under conditions which would make any action of the kind impossible. Accordingly the wire to be magnetised was suspended along the line of dip, and the current in the neutralising solenoid was increased to the value necessary to make it neutralise the full terrestrial field. The wire was now free from all magnetising force, and, after being completely demagnetised, it was subjected to stress as before. It was then found that the peculiar residual effects of stress described in §§ 96–104 were as conspicuous as before. The susceptibility depended, just as much as when the wire was in the usual vertical position, on preceding as well as on actual states of stress. The experiment showed conclusively that the transverse magnetism which must have existed in the wire when set vertically did not materially affect the results, and could not have been the cause of the residual differences of susceptibility which were found when a wire, free from longitudinal magnetism, was brought to a given state of stress through different preceding states.

§ 106. *Magnetisation under constant Loads in a high Field.*—It appeared desirable to examine the form which curves of magnetisation under constant load took when

the field was raised to values considerably greater than those of preceding experiments of this type, chiefly to see whether with very strong magnetising forces the curves became convergent.

A specimen of the iron wire usually employed (0·79 mm. in diameter and 33 cm. long), was annealed and subjected to a magnetising force which was gradually increased to 97·9 units, first with a load of one kilo. on it, and afterwards (after demagnetisation by reversals) with a load of 6 kilos. The results are given in the table below, reduced to absolute measure. The same wire was then stretched (in a non-magnetic state, and in a neutral field) by applying a load of 15 kilos., giving an elongation of 2·2 cm., and was again magnetised, successively in two different conditions, namely, with no load and with a load of 8 kilos. The load of 8 kilos. was selected because previous experiments had shown that with moderate magnetising forces the susceptibility is greater under 8 kilos. than under any other load. At the same time the earlier experiments had shown that as the magnetising force was increased, the load giving maximum susceptibility became less and less, and therefore had made it likely that the 8-kilo. curve would be found to cross and run under the normal or no-load curve when the magnetising force was greatly heightened. The results of the present experiment, which are stated below, confirmed this expectation.

MAGNETISATION of Iron in a strong field under constant loads, Plate 66, fig. 55.

BEFORE stretching (soft annealed state).

$\mathfrak{H}$.	Load = 1 kilo., or 2·04 kilo. per sq. mm. $\mathfrak{B}$.	Load = 6 kilos., or 12·24 kilos. per sq mm. $\mathfrak{B}$.
0	0	0
2·3	346	
3·1	595	441
4·2	791	630
6·3	994	833
8·4	1074	945
12·7	1137	1050
20·3	1173	1120
33·6	1207	1169
48·9	1239	1204
60·9	1249	1221
74·7	1260	1232
88·6	1264	1242
97·9	1270	1246
0	1092 (residual)	847 (residual)

After stretching (hardened state).

Load = 0.		Load = 8 kilos., or 17·4 kilos. per sq. mm.	
$\mathfrak{H}$.	$\mathfrak{B}$.	$\mathfrak{H}$.	$\mathfrak{B}$.
0	0	0	0
3·1	56	6·3	600
4·2	97	8·4	735
6·3	142	21·0	1018
8·6	261	42·0	1138
12·9	455	63·0	1190
20·5	724	84·0	1231
33·8	1041	110	1261
50·4	1190		
62·6	1232	0	652
77·1	1268		(residual)
90·7	1292		
100·6	1309		
0	455		
	(residual)		

The same two pairs of observations are plotted in Plate 66, fig. 55. The two curves with 1 kilo. and 6 kilos. on the unstretched wire are approaching each other distinctly enough at high values of $\mathfrak{H}$, but this is not so clearly the case in the other pair, and so far as the question of ultimate coincidence is concerned, the experiment is inconclusive. It merely shows that considerable differences are still to be found under magnetising forces of the highest easily attainable value.

At the end of each magnetisation $\mathfrak{H}$ was gradually reduced to zero and the residual magnetism observed. Its value is entered at the foot of each column in the table above. It is curious to notice how much more the residual magnetism is affected by the presence of stress than the induced magnetism is when both have been produced by a strong magnetising force. In the soft wire the final induced magnetism under 1 and under 6 kilos. differs by less than 2 per cent., but the residual magnetism has the widely different values 1092 or 86 per cent. of the induced, and 847 or 68 per cent. The same thing is still more striking in the stretched wire, where indeed the condition of no load gives slightly *more* induced but much *less* residual magnetism than the condition of stress gives. The retentiveness of the soft wire is reduced, and that of the stretched wire greatly increased by the presence of stress during the application and removal of magnetising force.

§ 107. *The Villari Critical Point.*—Allusion has been made (§ 87) to the fact, discovered by Villari, and afterwards by Thomson, that if we take a wire under the influence of a longitudinal magnetising force, and alternately apply and remove a given load, "on" gives augmentation and "off" gives diminution of magnetism if the magnetisation is less than a certain value; but "on" gives diminution, and "off"

gives augmentation of magnetism if this value is exceeded. Sir WILLIAM THOMSON calls the value of the magnetising force at which this reversal of effect occurs, the "VILLARI critical value." It is clear (as he has also pointed out) that the "VILLARI critical value" depends on the particular amount of load alternately applied and removed, and also upon the mean value of the load.

An examination of my results shows that the effects of stress on magnetism are, on the whole, not greatly different whether the magnetism in question is residual or induced, and I am therefore disposed to regard the VILLARI reversal as depending rather on the value of the magnetisation than on the value of the magnetising force.

This view receives strong confirmation from experiments which I have described in a paper, "On the production of transient electric currents in iron and steel conductors by twisting them when magnetised, &c."* I have shown there that the inclined pull and push which make up torsional stress give effects on a very strongly magnetised wire which have the same sign as the effects of direct pull and push on a feebly magnetised wire; which is to be explained by the fact that these act on $\mathfrak{I}\cos^2\frac{\pi}{4}$ or only one-half of the whole magnetism, an amount which does not exceed the "VILLARI critical value" of $\mathfrak{I}$, although one-half of $\mathfrak{H}$ may be made to exceed very largely the value of the magnetising force sufficient to cause the VILLARI reversal when pull is applied in the direction of magnetisation.

I shall therefore apply the term "VILLARI critical point" to name that value of the magnetisation $\mathfrak{I}$ at which reversal occurs in the sign of the magnetic difference produced by two (assigned) states of longitudinal stress.

This point may be determined in two ways, which (on account of hysteresis) will lead to different results. We may (with THOMSON) cause a repeated alternation to take place from one to the other of the two assigned states of stress, and find the value of $\mathfrak{I}$ at which that alternation causes no magnetic change, or we may take two separate curves of magnetisation (that is, curves of the relation to $\mathfrak{I}$ to $\mathfrak{H}$) in the two assigned states of stress, and see where these curves cross each other. The value of $\mathfrak{I}$ at the crossing point is the critical value for the two given states of stress.

Several of the figures which have been given show values of the "VILLARI critical point" according to this last mode of definition.

We may, however, conveniently limit the term "VILLARI critical point" to the case where one of the two assigned loads is zero; that is to say, we may use the term only when we are comparing the magnetic condition reached under a given load with that reached under no load. With this restriction the VILLARI critical value of $\mathfrak{I}$ is (for any load) that value at which the curve of magnetisation under that load crosses the normal or no-load curve. Thus, in fig. 47 (§ 94) the curves for 12, 14, 16·2, and 18·5 kilos. all cross the normal curve within the range of the observations, and so determine the VILLARI critical points (as just defined) for these particular loads.

* Proc. Roy. Soc., Vol. 36 (1883), p. 117.

They show that the VILLARI reversal occurs earlier in stretched iron the greater the stress is, but that even with a stress approaching the breaking strength of the material it does not come till $\mathfrak{I}$ is more than 1000. For loads less than 12 kilos. it would, no doubt, occur at values of $\mathfrak{I}$ greater than those reached in this experiment.

§ 108. In soft annealed iron, however, the VILLARI critical point occurs much earlier, as the experiment of § 91, fig. 44, serves to show. There the curves for 1, 2, 4, and 6 kilos. all cross the normal or no-load curve, at values of $\mathfrak{I}$ which (as in the case of stretched wire) are higher the smaller the load, but which are very much lower than the values of the critical point in stretched wire. The following are the values of $\mathfrak{I}$ and $\mathfrak{H}$ at which the curves with load cross the normal curve in fig. 44 (each kilo. of load gives a stress of 2·15 kilos. per sq. mm.) :—

Load.	$\mathfrak{H}$.	$\mathfrak{H}$.
kilos.		
1	1220	7·3
2	1040	4·3
4	840	3·4
6	690	3·05

Effects of Stress on Retentiveness.

§ 109. In a previous part of this paper (§§ 37–47) I have described experiments by which the retentiveness of iron and steel was examined at various degrees of magnetisation, by applying a given magnetising force, reducing that to zero and observing the residual magnetism, then applying a stronger force, reducing that to zero, and so on. Those experiments showed that the ratio of residual to induced magnetism passes a maximum as the magnetisation is increased.

It seemed desirable to extend the same method of experiment to the case of iron or steel *under longitudinal stress*. Accordingly corresponding observations have been made on pieces of iron (in the soft annealed and also in the stretched condition) and steel, in several different states of stress.

(June 13, 1882.)—A piece of pianoforte steel wire, 31 cm. long, after being annealed, was stretched about 1 cm. by a load of 22 kilos. The diameter after stretching was 0·78 mm. Its retentiveness was then examined (by applying and removing progressively increased values of $\mathfrak{H}$) under four different states of load, namely, 0, 3, 10, and 20 kilos. The process of demagnetising by reversals served to reduce the wire to a magnetically neutral state after magnetisation under each load, and before the next value of the load was applied.

The table below shows the results of this experiment, by giving the observed values of the induced and residual magnetism, and their ratio, for various values of $\mathfrak{H}$:—

PIANOFORTE-STEEL Wire, stretched after annealing, Plate 67, fig. 56.

ϕ.	Load = 0.			Load = 3 kilos. = 6·27 kilos. per sq. mm.			Load = 10 kilos. = 20·9 kilos. per sq. mm.			Load = 20 kilos. = 41·8 kilos. per sq. mm.		
	$\mathfrak{Z}$ induced.	$\mathfrak{Z}$ residual.	Ratio.	$\mathfrak{Z}$ induced.	$\mathfrak{Z}$ residual.	Ratio.	$\mathfrak{Z}$ induced.	$\mathfrak{Z}$ residual.	Ratio.	$\mathfrak{Z}$ induced.	$\mathfrak{Z}$ residual.	Ratio.
0	0	0	..	0	0	..	0	0	..	0	0	..
0·52	5	0	0	..	..	..	..	..	..	..	..	..
1·04	11	0	0	12	0	0	12	3	0·25	18	7	0·39
1·56	17	0	0	..	..	..	..	..	..	..	..	..
2·08	23	1·4	0·06	25	4	0·16	32	11	0·34	39	17	0·44
3·12	38	5	0·13	42	8·4	0·20	57	25	0·44	64	31	0·48
4·16	57	11	0·19	67	17	0·25	90	43	0·48	94	48	0·51
5·20	82	23	0·28	90	31	0·34	129	70	0·54	129	70	0·54
6·24	115	42	0·37	126	56	0·44	176	105	0·60	168	98	0·58
7·28	161	76	0·47	174	89	0·51	231	146	0·63	213	126	0·59
8·32	216	120	0·56	232	136	0·59	297	197	0·66	263	162	0·62
9·36	277	170	0·61	300	190	0·63	367	255	0·69	314	199	0·63
10·40	346	229	0·66	371	252	0·68	435	315	0·72	365	235	0·64
11·44	417	291	0·70	..	..	..	..	..	..	..	..	..
12·48	484	353	0·73	524	389	0·74	566	431	0·76	470	314	0·67
13·52	546	409	0·75	..	..	..	619	473	0·76	518	350	0·68
14·56	599	459	0·77	..	..	..	666	517	0·78	554	384	0·69
15·60	648	501	0·77	..	..	..	..	..	..	598	413	0·69
16·64	692	540	0·78	733	582	0·79	747	588	0·79	633	440	0·70
17·68	727	574	0·79	..	..	..	776	622	0·80	664	468	0·70
18·72	760	605	0·796	..	..	..	805	644	0·80	..	..	..
19·76	792	633	0·799	826	666	0·806	829	669	0·81	720	507	0·70
20·80	815	655	0·803	..	..	..	851	690	0·81	739	529	0·71
23·4	875	703	0·804	897	734	0·817	897	728	0·81	788	563	0·71
28·9	955	773	0·809	..	..	..	..	..	..	..	..	..
29·5	..	..	..	987	815	0·826	981	804	0·819	877	636	0·725

The same results are shown in Plate 67, fig. 56, where three groups of curves are given. The lower group shows the induced magnetism, and the middle group the residual magnetism, in terms of $\mathfrak{H}$. The upper group shows the values of the ratio of residual to induced magnetism, also in terms of $\mathfrak{H}$.

The crossing of the curves of $\mathfrak{I}$ (induced) and $\mathfrak{H}$ should be noted. It corresponds, in kind, to the behaviour of iron, and shows that in steel, as in iron, any increase of stress gives first increase and then decrease of susceptibility, according as the magnetisation falls short of or exceeds a certain critical value. Each curve lies at first higher and afterwards lower than curves referring to smaller values of the load.

The middle group of curves shows that the same remark applies to residual magnetism.

Further, the curves which represent the ratio of residual to temporary magnetism in relation to $\mathfrak{H}$ cross each other in the same way. There is thus, for each load, a critical value of the magnetisation, below which the ratio of residual to temporary magnetism is greater, and above which it is less, when the wire is loaded than when it is unloaded (or loaded with a smaller load). For example, under a load of 20 kilos. the retentiveness is much greater than under a small load or no load, so long as the magnetisation is feeble, but less when the magnetisation is strong.

The observations taken with no load confirm what has been already said (§ 44) as to the absence of retentiveness in steel, under weak magnetising forces. But it is very remarkable how the presence of stress increases the retentiveness under weak magnetising forces. A force $\mathfrak{H}$ of 1 unit, for instance, leaves sensibly no residual magnetism in the wire when there is no load, but leaves a residue amounting to nearly 0·4 of the induced magnetism when there is a load of 20 kilos.

§ 110. A piece of iron wire, which, like the last, had (after annealing) been stretched beyond its limit of elasticity, was next examined in the same way (June 14, 1882), the successive states of load being 0, 5, 10, and 14·8 kilos. The diameter of the wire (after being stretched) was 0·72 mm. and its length was 30·5 cm. The load employed to stretch it was 20 kilos.

The induced and residual magnetism of this wire, found under these various states of stress by applying and removing a large number of progressively increased values of $\mathfrak{H}$, are given in the table below, and the same results are shown in Plate 67, fig. 57, where the curves give the relation to $\mathfrak{H}$ of $\mathfrak{I}$ induced, $\mathfrak{I}$ residual, and their ratio:—

IRON Wire, stretched after annealing, Plate 67, fig. 57.

LOAD = 0.

$\mathfrak{H}$.	$\mathfrak{B}$ induced.	$\mathfrak{B}$ residual.	Ratio.
0	0	0	..
0·54	5	..	..
1·35	19	4	0·21
1·62	23	6	0·26
2·16	35	10	0·28
2·70	50	16	0·32
3·24	73	28	0·39
3·78	101	43	0·43
4·32	132	65	0·49
4·86	167	88	0·53
5·40	205	114	0·56
5·94	242	140	0·58
6·48	280	166	0·59
7·02	312	189	0·60
7·56	353	211	0·60
8·10	390	232	0·60
8·64	429	252	0·60
9·18	457	270	0·59
9·72	488	286	0·59
10·26	520	301	0·58
10·80	550	315	0·57
11·88	611	336	0·55
12·96	670	354	0·53
14·04	725	372	0·51
15·17	777	387	0·50
16·20	821	397	0·48
17·55	874	406	0·46
18·90	930	416	0·45
20·25	975	423	0·43
21·60	1014	429	0·42
22·95	1053	432	0·41
24·30	1087	435	0·40
25·92	1123	437	0·39
29·16	1190	439	0·37

LOAD =5 kilos., or 12·2 kilos. per sq. mm.

Φ.	ℑ induced.	ℑ residual.	Ratio.
0	0	0	—
0·54	15	8	0·53
1·08	53	34	0·64
1·62	114	81	0·71
2·16	211	159	0·75
2·62	297	237	0·80
3·24	410	331	0·81
3·78	494	405	0·82
4·32	561	464	0·83
4·86	621	513	0·83
5·40	673	556	0·83
6·48	761	624	0·82
7·56	827	672	0·81
8·64	881	709	0·80
9·72	923	736	0·80
10·80	965	757	0·79
12·15	1004	780	0·78
13·50	1040	795	0·76
14·85	1069	806	0·75
16·31	1095	815	0·74
17·55	1112	822	0·74
18·90	1131	827	0·73
21·60	1164	832	0·72
24·30	1186	835	0·71
26·46	1202	838	0·70
29·16	1225	841	0·69

LOAD =10 kilos., or 24·4 kilos. per sq. mm.

Φ.	ℑ induced.	ℑ residual.	Ratio.
0	0	0	—
0·54	21	12	0·57
1·08	63	39	0·62
1·62	137	94	0·69
2·16	241	176	0·73
2·70	351	267	0·76
3·24	442	351	0·78
3·78	530	416	0·79
4·32	595	468	0·79
4·86	650	513	0·79
5·40	696	547	0·79
6·48	770	601	0·78
7·56	830	637	0·77
8·64	884	664	0·76
9·72	917	685	0·75
10·80	946	696	0·74
12·26	978	712	0·73
13·50	1004	725	0·72
14·85	1028	731	0·71
16·20	1048	739	0·70
18·90	1082	748	0·69
21·60	1105	754	0·68
24·30	1125	757	0·67
27·54	1144	760	0·66
29·16	1154	761	0·66

LOAD = 14·8 kilos., or 36·1 kilos. per sq. mm.

$\mathfrak{H}$.	$\mathfrak{I}$ induced.	$\mathfrak{I}$ residual.	Ratio.
0	0	0	—
0·54	19	9	0·47
1·08	53	29	0·55
1·62	104	62	0·60
2·16	176	110	0·63
2·70	258	172	0·67
3·78	419	289	0·69
4·86	530	371	0·70
5·99	619	432	0·70
7·02	679	471	0·69
8·10	731	489	0·68
9·45	790	524	0·66
10·80	835	546	0·65
12·26	874	559	0·64
13·50	910	570	0·62
16·20	966	581	0·60
18·90	1011	590	0·58
21·60	1040	595	0·57
24·30	1067	598	0·56
28·08	1093	598	0·55
29·16	1108	598	0·54

Here the results are somewhat different from those of the last paragraph. A distinct maximum occurs in the value of the ratio of residual to induced magnetism in each of the curves with load, as well as in the curve taken without load, and it occurs at earlier values of $\mathfrak{H}$ in the loaded curves than in the unloaded. The effect of stress on retentiveness is again very great, especially with weak magnetisations: thus for $\mathfrak{H}=1$ the ratio of residual to induced magnetism is nearly zero when there is no load, but ranges from ·55 to ·64 in the curves taken with load. It is to be noticed, however, that the curves of ratio and $\mathfrak{H}$, corresponding to 5, 10, and 14·8 kilos. of load, perhaps cross each other at a value of $\mathfrak{H}$ lower than that at which the observations can be said to begin. On the other hand, the curve of ratio and $\mathfrak{H}$ corresponding to no load may perhaps cross the others at a value of $\mathfrak{H}$ much greater than that to which the experiment extends.

The greatest ratio of residual to induced magnetism, throughout the whole group, occurs at the maximum point of the 5-kilo. curve, when its value is 0·83. This is a much greater value than is reached at any point under no load, where the maximum is only 0·6. Moreover, the presence of a moderate load increases, in a still greater proportion, the retentiveness of stretched iron under strong values of $\mathfrak{H}$. At the highest value of $\mathfrak{H}$ which was reached in this experiment the 5 and 10-kilo. curves of residual magnetism, and also of the ratio, lie much higher than the no-load curve.

§ 111. To illustrate this last point more fully the following additional observations were made. The same piece of wire was successively tested under loads of 0, 2, 4, 6, 8, 10, 12, and 14·8 kilos., by first demagnetising it in each case, then applying a force

$\mathfrak{H}$ of 29·16, and then removing that force. The following values of $\mathfrak{I}$ induced and $\mathfrak{I}$ residual were determined in this way :—

Load.	$\mathfrak{I}$ induced.	$\mathfrak{I}$ residual.	Ratio.
0	1157	423	0·37
2	1183	588	0·50
4	1193	757	0·63
6	1189	845	0·71
8	1180	809	0·69
10	1157	757	0·65
12	1138	683	0·60
14·8	1105	598	0·54

From this it will be seen that (for $\mathfrak{H}$=29·16) a load of 6 kilos. gives maximum retentiveness, and that the retentiveness under that load is nearly twice as great as when the same wire is not loaded.

§ 112. The same piece of iron was now re-annealed, and the effect of stress on its retentiveness when in the soft state was examined in the same way (June 16, 1882). The only states of load tested were 0 and 4 kilos. (a load so well within the limit of elasticity as to produce no important permanent mechanical change in the specimen). The results are given below, and also in Plate 68, fig. 58, where, in the lower group of curves, the two full lines show the induced, and the two dotted lines the residual magnetism, while the lines above show the ratio in the same way as before :—

ANNEALED Iron Wire, Plate 68, fig. 58.

Load = 0.				Load = 4 kilos., or 9·76 kilos. per sq. mm.			
$\mathfrak{H}$.	$\mathfrak{I}$ induced.	$\mathfrak{I}$ residual.	Ratio.	$\mathfrak{H}$.	$\mathfrak{I}$ induced.	$\mathfrak{I}$ residual.	Ratio.
0	0	0	—	0	0	0	—
1·08	66	32·5	0·49	0·54	38	21	0·53
1·62	202	141	0·70	1·08	141	94	0·69
2·16	460	381	0·83	1·62	325	242	0·745
2·70	684	601	0·879	2·16	532	419	0·788
3·24	846	767	0·907	2·70	677	543	0·802
3·78	939	860	0·916	3·24	796	640	0·804
4·32	999	920	0·921	3·78	876	705	0·805
5·40	1071	994	0·928	4·37	937	754	0·805
6·48	1109	1024	0·923	4·86	978	787	0·805
7·56	1139	1046	0·919	5·51	1022	816	0·800
8·64	1157	1063	0·919	6·48	1067	856	0·800
9·72	1168	1074	0·919	8·64	1121	891	0·795
10·8	1178	1082	0·918	10·8	1162	913	0·786
13·5	1196	1095	0·916	13·5	1186	926	0·781
16·2	1210	1105	0·913	16·2	1204	933	0·775
18·9	1219	1111	0·911	18·9	1211	939	0·775
21·6	1226	1116	0·910	21·6	1219	942	0·773
25·6	1236	1119	0·905	26·2	1232	946	0·768

The susceptibility is at first greater with 4 kilos. than with no load, but afterwards becomes less with 4 kilos. than with no load (*cf.* § 89). Moreover, the residual magnetism, and also the ratio of residual to induced magnetism, are with low magnetising forces greater, and with high magnetising forces less when there is load than when there is not load.

The ratio of residual to induced magnetism passes a distinct maximum in both cases; but the value of that maximum is greatest in the condition of no load, where it is as great as 0·93. My experiments have presented only one other instance in which the ratio reached so high a figure (*cf.* § 37).

§ 113. The manner in which curves showing the relation of $\mathfrak{I}$ to $\mathfrak{H}$ cross each other, when magnetisation is performed under various constant loads, has been very fully illustrated in a number of the experiments described above. As these, however, with the exception of the test of steel, referred almost exclusively to specimens of iron wire whose mechanical quality was pretty much the same, I judged it desirable to repeat some of the observations, using specimens of iron of very different quality. The results so obtained were in complete agreement with those which have been described above; and for that reason it is needless to repeat them. The general rule, apparently true for any piece of iron is, that the susceptibility, under any one load, is for low values of $\mathfrak{I}$ *greater*, and for high values of $\mathfrak{I}$ *less*, than under a slightly less load, the crossing point occurring at higher values of $\mathfrak{I}$ the smaller the load in question is.

Effects of Temperature on Magnetism.

§ 114. If a piece of magnetised iron or steel be subjected to a cyclic series of changes of temperature, it is well known, from the experiments of WIEDEMANN * and others that the changes of magnetism which it undergoes are, in general, not cyclic. If the magnetism which is being dealt with is the residual magnetism left after withdrawal of any magnetising force, then any cyclic process of heating and cooling, or of cooling and heating, results in a fall of magnetism. If, on the other hand, the magnetism dealt with is that which has been reached by the application of a magnetising force that is kept in action during the experiment, then any cyclic change of temperature results in a rise of magnetism. In both cases it is only after any cyclic change of temperature has been many times repeated that the accompanying changes of magnetism become even approximately cyclic.

In this respect the effects of temperature are closely analogous to those of stress (*cf.* § 77). Superposed upon the differential effects of heating and cooling there are progressive permanent changes of the nature of a shaking in of magnetism, when that is induced, or of a shaking out of magnetism, when that is residual.

By repeating any cycle of temperature changes often enough, however, we get rid

* "Galvanismus," II., § 522, *et seq.*

of these progressive changes, and may then study the differential effects of heating and cooling, after the magnetic changes have also become sensibly cyclic.

I have made a number of experiments of this kind with the view of seeing whether, after the magnetic changes become sensibly cyclic, they exhibit static *hysteresis* with respect to the changes of temperature. Since there is clear evidence of such hysteresis in changes of magnetism produced by other means, namely by changes of magnetic field, and also by changes of stress, I expected somewhat confidently to find hysteresis in the relation of magnetism to temperature. But in this expectation I have been altogether disappointed.

§ 115. A long iron wire was fixed inside a vertical glass tube, on which was wound a magnetising solenoid. The tube was connected by india-rubber piping at one end to one or other of three small boilers, capable of supplying a steady current of steam, alcohol vapour, and sulphuric-ether vapour respectively, or to a water-cistern filled with cold water. The other end of the tube led to a water vessel which served as a condenser. An adjacent mirror magnetometer, level with the top of the iron wire, measured the changes of magnetism in the usual way. The method of procedure was this:—

Steam and cold water were alternately passed through the tube many times, until the magnetic state of the wire was observed to change from one to the other of two nearly steady values. Then readings of the magnetometer were taken during the passage through the tube of (1) cold water; (2) ether vapour; (3) alcohol vapour; (4) steam; (5) alcohol vapour; (6) ether vapour; (7) cold water. This completed a cycle of temperature changes in which two intermediate points were fixed during each of the processes of heating and cooling.

This method was adopted in order that the magnetised iron might be exposed sufficiently long to an atmosphere of one definite temperature to give it time to assume that temperature throughout, and so avoid any possibility of error due to the sluggishness with which changes of temperature took place. The stream of vapour was in every case kept up until the magnetometer reading became perfectly steady.

In some experiments, in place of a magnetised wire, a magnetised iron pipe was used, surrounded by a non-conducting jacket, and steam and other vapours were passed through the iron tube itself. This had the advantage that the position of the magnetised iron, with respect to the magnetometer, could be maintained more absolutely constant and unaffected by temperature changes than when the magnet consisted of a wire inside a glass tube. The difficulty of fixing the magnet so that the process of heating and cooling it might not produce displacements whose effects on the magnetometer were comparable with the true change of magnetism, was somewhat formidable on account of the smallness of the whole range of magnetic change caused by alternations between atmospheric temperature and 100° C.

The small range of magnetic change also made it needful to raise the magnetometer to a condition of great sensibility. For this purpose a directing magnet

was applied, which neutralised the greater part of the earth's field at the magnetometer needle, leaving only a small directive force, whose intensity was estimated by comparing the period of the needle with its period when this directing magnet was removed. Further, the magnetism of the wire or iron pipe under examination was sufficiently balanced by a second deflecting magnet (arranged to deflect the magnetometer without altering its sensibility) to bring the spot of light on the scale. To determine what ratio the observed magnetic changes bore to the whole magnetism, it was only necessary to observe the whole deflection due to the magnetised wire or pipe, with the magnetometer in an insensitive state, and measure (by comparing periods) the relative sensibility of the two states of the magnetometer.

Much time was spent in tentative experiments before satisfactory results were obtained. These, however, when they were arrived at showed so conclusively that within the limits of temperature experimented on there was no sensible hysteresis in the relation of magnetic change to temperature, that the subject was not pursued at any great length; and for the same reason it will suffice to quote a single set of observations.

§ 116. (November 30, 1882.) In this experiment the magnetising field was the earth's vertical force, in which the iron under test was shaken beforehand. Then water and steam were passed alternately, five times each, after which the following readings were taken. The numbers given are the actual magnetometer readings: by adding 17,000 to them they can be made approximately proportional to the total magnetisation of the piece. The arrows show the sequence of the changes.

Water. (14° C.)	Ether vapour. (35° C.)	Alcohol vapour. (78½° C.)	Steam. (100° C.)
413 →	380 →	302 ↘	
			263
	383 ←	304 ↙	
415 ↙			
↘	381 →	301 ↘	
			261
	381 ←	304 ↙	
415 ↙			
↘	383 →	305 ↘	
			262
	382 ←	303 ↙	
416 ↙			
↘	382 →	304 ↘	
			262
	382 ←	304 ↙	
418 ↙			
↘	383 →	305	

It is clear from these figures that the changes of magnetism between 14° C. and

100° C. are nearly proportional to the changes of temperature,* and exhibit no hysteresis with respect to them. The whole range of change, from 17,415 at 14° C. to 17,262 at 100° C., is only 0·9 per cent. of the total magnetism.

§ 117. The actual amount of magnetic change, and even its sign, depends on the degree of magnetisation. This follows from the known fact that there is a critical value of the magnetising force, or (more probably) of the magnetisation, below which heating increases susceptibility, and above which heating reduces susceptibility. (Baur, Wied. Ann., xi., 1880.) My own experiments have illustrated this in two ways. In some of them the effects have been studied of repeated heatings (up to 100° C.) and coolings, on various degrees of induced or residual magnetism in an iron wire: in others, curves of the relation of $\mathfrak{I}$ to $\mathfrak{H}$ have been drawn when the same iron wire is magnetised, (1) at ordinary temperature, (2) at 100° C. Both methods of experiment have been applied to samples of iron wire both in the soft annealed and hard-drawn states. One or two representative experiments may be quoted.

§ 118. (Jan. 28, 1883.) An iron wire, 1·2 mm. in diameter and 35·7 centims. long, was annealed, and then stretched until its length was 40·5 centims. and its diameter 1·16 mm. In this state it was demagnetised by reversals, and then hung (without mechanical disturbance) under the influence of the earth's vertical force, and very near a magnetometer whose sensibility was raised to a high degree. The wire was surrounded by a tube, round which a magnetising solenoid was wound, and steam at 100° C. and cold water at 6° C. were alternately passed through the tube while the magnetic changes were noted. From time to time, at points which will be indicated below, the total magnetisation of the wire was raised a step, by momentarily setting up in the solenoid a suitable current, which was then interrupted, so that the magnetism dealt with was that part which was retained after the inducing field was reduced to the value 0·34, namely, the earth's vertical force. The following are the values of $\mathfrak{I}$ (expressed as usual in absolute c.g.s. units) which were observed throughout the process.

	$\mathfrak{I}$.	
Original condition of the wire	1·52	
While steam was passing	2·05	Here, in addition to a progressive shaking out and then shaking in of magnetism, there is augmentation of magnetism with steam, and diminution with water. $\mathfrak{I}$ is below the critical point.
,, water ,, ...	1·98	
,, steam ,, ...	2·11	
,, water ,, ...	2·05	
,, steam ,, ...	2·16	
,, water ,, ...	2·09	
,, steam ,, ...	2·20	
,, water ,, ...	2·12	
,, steam ,, ...	2·23	
,, water ,, ...	2·16	

* The ratio of change of magnetism to change of temperature increases slightly as the temperature rises.

	$\mathfrak{Z}$.	
Weak current made and broken raising $\mathfrak{Z}$ to	2·51	
Then, while *water* was passing	2·51	
,, steam ,,	2·42	
,, water ,,	2·34	
,, steam ,,	2·44	Still below the critical point.
,, water ,,	2·38	
,, steam ,,	2·46	
,, water ,,	2·41	
,, steam ,,	2·51	
,, water ,,	2·44	
,, steam ,,	2·55	
,, water ,,	2·46	
,, steam ,,	2·57	
Current made and broken	3·34	
Then, while steam was passing	3·07	
,, water ,,	2·99	
,, steam ,,	3·07	
,, water ,,	2·99	Ditto.
,, steam ,,	3·07	
,, water ,,	3·01	
,, steam ,,	3·10	
,, water ,,	3·02	
Current made and broken		
Then, while water was passing	4·03	
,, steam ,,	3·63	
,, water ,,	3·54	Ditto.
,, steam ,,	3·59	
,, water ,,	3·52	
,, steam ,,	3·56	
,, water ,,	3·51	
Current made and broken		
Then, while water was passing	8·38	
,, steam ,,	8·36	Just over the critical point.
,, water ,,	8·37	
,, steam ,,	8·37	
,, water ,,	8·38	
Current made and broken		
Then, while water was passing	9·09	
,, steam ,,	8·71	

	$\mathfrak{I}$.	
While water was passing	8·71	
„ steam „	8·67	
„ water „	8·69	Distinctly over the critical point.
„ steam „	8·67	
„ water „	8·69	Steam now gives *fall* and water *rise* of magnetism.
Current made and broken		
Then, while water was passing	9·72	
„ steam „	9·13	
„ water „	9·12	
„ steam „	9·05	Ditto, still more distinctly.
„ water „	9·09	
„ steam „	9·04	
„ water „	9·09	

This, as well as other experiments of the same class, shows that when tested in this way the critical point, at which the sign of the effects of heating and cooling (from 100° to, say, 6° C.) changes, occurs at a very low value of the magnetisation. In the above case it was reached when $\mathfrak{I}$ was about 8, or about the $\frac{1}{130}$ part of the so-called saturation value of $\mathfrak{I}$.

§ 119. (Feb. 5, 1883.) A similar experiment was made with an *annealed* specimen of wire cut from the same bundle as the last, and also 40·5 centims. long.

	$\mathfrak{I}$.	
Original condition	3·94	
While steam was passing	4·80	
„ water „	4·71	
„ steam „	4·95	
„ water „	4·77	Below the critical point.
„ steam „	4·95	
„ water „	4·77	
Current made and broken	9 90	
While steam was passing	9·27	
„ water „	8·88	
„ steam „	9·03	
„ water „	8·82	
„ steam „	9·00	Ditto.
„ water „	8·79	
Current made and broken	10·89	
While steam was passing	10·08	
„ water „	9·78	

	$\mathfrak{Z}$.	
While steam was passing	9·81	
„ water „	9·63	
Current made and broken	11·94	
While steam was passing	11·28	
„ water „	10·83	
„ steam „	10·92	
„ water „	10·74	
Current made and broken	20·04	
While steam was passing	18·63	
„ water „	18·03	
„ steam „	17·88	
„ water „	17·76	
„ steam „	17·73	
„ water „	17·61	
„ steam „	17·58	
„ water „	17·49	
„ steam „	17·43	
„ water „	17·28	Still below the critical
„ steam „	17·31	point.
Current made and broken	20·55	
While steam was passing	19·26	
„ water „	18·81	
„ steam „	18·69	
„ water „	18·51	
„ steam „	18·54	
„ water „	18·45	
„ steam „	18·42	Ditto.
„ water „	18·36	
Current made and broken	27·63	
While steam was passing	25·77	
„ water „	25·20	
„ steam „	24·90	
„ water „	24·84	
„ steam „	24·66	
„ water „	24·63	Now over the critical
„ steam „	24·45	point.
„ water „	24·48	
Current made and broken	41·49	
While steam was passing	41·19	
„ water „	39·24	

	$\mathfrak{I}$.	
While steam was passing	39·00	
„ water „	38·16	
„ steam „	38·16	
„ water „	37·74	Decidedly over the
„ steam „	37·89	critical point.
„ water „	37·53	
„ steam „	37·65	

Here the critical point appears to be reached when $\mathfrak{I}$ is about 20, but its position is not very sharply defined.

§ 120. The same two wires were also tested by finding, in the ordinary way by the direct magnetometric method, the relation of $\mathfrak{I}$ to $\mathfrak{H}$ during magnetisation: (1) when the temperature was that of the atmosphere, or about 7° C.; (2) when the temperature was maintained at 100° C. by the continuous passage of a current of steam through the tube on which the magnetising solenoid was wound. The following table gives the results, reduced to absolute measure. In each state several successive determinations of the curve of $\mathfrak{I}$ and $\mathfrak{H}$ were made, but as these agreed very exactly with each other, only one set of readings for each state has been reduced:—

EFFECT of Temperature on the Magnetic Susceptibility of Iron Wire, Plate 68, fig. 59.

Soft annealed wire.				Wire hardened by stretching.			
At 7° C.		At 100° C.		At 8° C.		At 100° C.	
$\mathfrak{H}$.	$\mathfrak{I}$.	$\mathfrak{H}$.	$\mathfrak{I}$.	$\mathfrak{H}$.	$\mathfrak{I}$.	$\mathfrak{H}$.	$\mathfrak{I}$.
0	0	0	0	0	0	0	0
1·36	120	1·24	125	2·16	23	2·16	28
1·70	184	1·70	193	3·60	51	3·60	60
2·27	298	2·27	306	7·21	162	7·21	186
2·84	389	2·84	405	10·88	305	10·81	341
3·69	539	3·97	612	14·42	449	14·49	490
4·54	673	5·67	850	18·02	589	18·02	623
5·67	827	7·09	960	21·63	711	21·63	741
7·09	942	8·50	1021	25·37	825	25·23	848
8·50	1012	11·34	1088	29·20	918	28·91	936
11·34	1088	14·18	1119	32·45	983	33·27	1017
14·80	1122	17·01	1140	36·19	1038	36·77	1065
17·01	1145	19·84	1151	39·66	1079	46·36	1150
22·79	1169	22·96	1158	53·21	1206		
30·33	1183	28·01	1160				
				0	363 residual.	0	341 residual.
0	911 residual.	0	894 residual.				

The same results are also shown graphically in Plate 68, fig. 59, where the full lines are the curves for ordinary temperature, and the dotted lines are for 100° C. Both with the annealed and hardened specimen the 100° C. curves lie at first higher and afterwards lower than the normal curves. The crossing points, at which the effect of this difference of temperature on magnetic susceptibility becomes reversed, occur at values of $\mathfrak{Z}$ far higher than the critical value in the former method of experiment. This difference is partly, perhaps, to be ascribed to the fact that here we are dealing with induced magnetism, whereas in the former case the magnetism dealt with was chiefly residual; but the principal reason for the very much higher values of $\mathfrak{Z}$ required to be reached here before the effects become reversed is no doubt the hysteresis which exists in the relation of magnetisation to magnetising force. The difference in the results of the two methods is analogous to that which is found when we determine the VILLARI critical point for stress (1) by loading and unloading, (2) by magnetising under constant loads (*cf.* § 107).

§ 121. The range through which magnetism was altered by heating and cooling in the experiment of § 116, and others like it, was so small that I considered it desirable to see whether an increased range of temperature variation would not afford evidence of hysteresis in the relation of magnetism to temperature. With this view, the effect of heating and cooling a steel bar magnet (23 centims. × 2·10 centims. × 0·94 centim.) in a vessel full of oil was investigated as follows:—The bar was hung, from a fixed support above the oil, perpendicular to and with its centre due south of the magnetometer. The temperature of the oil was then altered by means of a lamp, by steps and very slowly, to allow the bar's temperature to be sensibly uniform—and the deflection of the magnetometer was observed. The temperature of the oil was taken by two thermometers, the means of whose readings are given below.

After heating and cooling the magnet until its magnetic changes became nearly cyclic, the following continuous set of observations, extending over 17 hours, was made (Feb. 26–27, 1883). The magnetometer readings are approximately proportional to the total magnetic moment of the bar.

Temperature, deg. C.	Magnetometer.	Hours of observation.
		h. m.
8·6*	692	1 15 p.m.
34·6*	656	2 0
48·1	636	3 0
48·4*	635	3 15
67·7	607	3 40
73·3	598	4 10
74·8	596	4 18
75·8	594	4 27
76·1*	593	4 50
91·0	566	5 25
95·2	559	5 45
96·6	556	6 0
98·2	554	6 15
98·8*	552	6 33
113·1	523	7 4
119·8	511	7 20
122·7	505	7 43
123·8	502	8 0
123·8*	502	8 12
151·2	441	8 42
155·3	430	8 56
157·1	425	9 5
158·3	422	9 20
158·4*	422	9 30
Now began to cool.		
131·2	487	10 20
129·8	491	10 40
129·1	495	11 0
129·1*	495	11 10
108·8	538	12 0
105·5	542	12 21 a.m.
104·3*	545	12 45
104·7	546	1 0
103·4	546·5	1 13
79·6	589	2 15
78·4	591·5	2 28
77·4	593	2 35
76·4	593·5	2 45
76·0*	594	2 52
28·2	667	4 10
23·4*	675	4 25
17·8	684	4 45
16·8*	685	4 54
10·8	691	5 35
10·3*	691·5	6 0

The points at which the temperature of the bar was most nearly uniform (marked with an asterisk in the table) have been selected from the above readings and plotted in Plate 68, fig. 60, where the lower line shows the process of heating and the upper line the process of cooling. There is no appearance whatever of hysteresis, in spite of the very considerable amount of magnetic change which the bar underwent.

§ 122. In dealing with a subject such as the magnetisation of iron, to which so much attention has been given, it is difficult to master or to acknowledge adequately the labours of former observers. The comprehensive summary of G. WIEDEMANN,

who, both by his own work and by his analysis of the work of others, has contributed enormously to our knowledge of magnetism, and the paper of CHRYSTAL, to which allusion has been made, render some parts of this task at once comparatively easy and almost unnecessary. But contributions continue to be added at a rate which soon makes the best summary incomplete, and, to a greater extent than I am aware, some of the results of this paper may be repetitions of work already published. Even then, however, I hope they may not be without value, on account of the facts (1) that, with the single exception of the experiment of § 121, the magnetisation dealt with has been as nearly uniform as it is practicable to have it, and (2) that very nearly all the observed values of magnetism and magnetising force have been reduced to absolute measure. It may be added that the subject with which this paper deals has, in addition to its scientific interest, a special practical importance at the present time, on account of its very obvious bearing on many points relating to the industrial applications of electricity, and particularly to the design of dynamo-electric machines.

§ 123. The work described in this paper was done during 1881–1883 in the Physical Laboratory of the University of Tokio, Japan, with the help of four Japanese students, without whose assistance, both in the taking of observations and in the arithmetical labour of reducing them, it would have been impossible for me to have covered so much ground. To Mr. R. FUJISAWA and Mr. S. TANAKA I am particularly indebted for help in the first part of these experiments—the examination of the relation of $\mathfrak{I}$ to $\mathfrak{H}$ during the application and cyclic change of $\mathfrak{H}$, by the ballistic and magnetometric methods,—and to Mr. A. TANAKADATE for help in the second part—the investigation of the effects of stress on magnetism. These gentlemen threw themselves into the work with a thoroughly intelligent interest and with the utmost zeal. In the last part of the paper—on the effects of temperature on magnetism—the observations which are quoted were for the most part taken under my direction by Mr. S. SAKAI, who prosecuted this difficult portion of the subject with much perseverance, and to whom I am also indebted for valuable help in another investigation —of the effects of stress and magnetisation on the thermo-electric quality of iron—the results of which remain to be described. Those results, which are closely related to some parts of the foregoing inquiry, will form the subject of another paper.

XI. *Observations on the Chromatology of Actiniæ.*

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[Plates 69, 70.]

Very few observations have been made on the Chromatology of the Actiniæ, the reason probably being that observers have been led to believe that since their colouring matters cannot be got into solution with the ordinary solvents, the study of their spectra could hardly be attended with fruitful results.

Krukenberg,* who has added much to our knowledge of animal Chromatology generally, has attempted to enlarge the knowledge of this subject by his study of the pigments of *Anthea viridis* and *A. cereus*.

Professor Moseley† published a short paper in 1873 on the colouring matter found by him in *Bunodes crassicornis*, which he called Actiniochrome, and in a subsequent paper,‡ in which an immense number of new animal pigments were brought to light, he describes a dark red colouring matter in *Anthea* which absorbed the blue end of the spectrum from before E onwards, and another in *Adamsia* which was pink, and in the fresh state gave a band between D and E. The filaments emitted from the pores of the body wall also gave two bands in green.

Heider§ examined *Cerianthus membranaceus*, var. *fusca* and var. *violacea*, and the colouring matter obtained, which Krukenberg calls *Purpuridin*, was extracted by ammoniacal water. It appears to resemble in some respects (*e.g.*, colour-changes) one of the pigments which I have found in *Sagartia parasitica*, to be referred to again.

O. and R. Hertwig‖ have tried to show that the so-called "yellow cells" or "pigment

* 'Vergleichend-physiologische Studien,' 1ste Reihe, 5te Abth., 1881, pp. 38–42, and, ditto, 2te Reihe, 3te Abth., 1882, pp. 72–87.

† "On Actiniochrome, a Colouring Matter of Actiniæ." Quart. Journ. Micro. Soc., vol. xii., N.S., 1873, p. 143.

‡ "On the Colouring Matters of various Animals, and especially of Deep Sea Forms dredged by H.M.S. Challenger." Quart. Journ. Micro. Soc., vol. xvii., N.S., 1877, pp. 1–23. Polyperythrin was also found in some Actiniæ by Prof. Moseley.

§ "*Cerianthus membranaceus*." (Haime.) "Ein Beitrag zur Anatomie der Actinien." Sonderabdr. aus den Sitzb. d. kk. Acad. der Wiss. zu Wien. Bd. lxxix., I. Abth., 1879, S. 7; also "Ueber Sagartia troglodytes." Sitzungsb. der Wien. Akad., 1877.

‖ "Die Actinien." 'Jenaische Zeitschrift für Naturwiss.' Bd. xiii., 1879, S. 495–500.

bodies" of certain Actiniæ are parasitic or rather symbiotic algæ, a view which P. GEDDES* by his remarkable experiments has endeavoured to support.

I had made some preliminary observations† on the occurrence of a band in *Actinia mesembryanthemum*, which I believed to be due to MOSELEY's Actiniochrome, but since I have been able to extract the pigment by means of glycerin I find that the band of Actiniochrome is not coincident with that seen in *Actinia mesembryanthemum*, as the former is nearer the red end of the spectrum, and the latter belongs to a colouring matter which, as I will shortly endeavour to show, yields certain decomposition products having a most remarkable resemblance to those obtainable from hæmoglobin. No purpuridin is found in the last-mentioned Actinia (although supposed to be present by KRUKENBERG).

Method of Examination.—The solid portions of Actiniæ are first examined by means of the microspectroscope, and they are then treated with various solvents. The solid portions are examined in the "compressorium," which enables any desired thickness to be obtained; they have to be well illuminated,‡ and for this purpose a substage condenser is used. The solutions are examined first with the microspectroscope, and then their bands measured with the large spectroscope, obtained by means of the grant allowed me by the Royal Society. Wave-lengths calculated by means of the microspectroscope cannot be sufficiently relied upon owing to the shortness of the spectrum, but owing to its superior definition the microspectroscope is indispensable, without it faint bands would be missed, and a comparison of a chlorophyll solution in both instruments shows the presence of bands with the microspectroscope which are almost invisible with the chemical spectroscope.

Actiniæ examined.—I have examined the following Actiniæ: *Actinia mesembryanthemum*, *Bunodes crassicornis*, *B. ballii*, *Sagartia bellis*, *S. dianthus*, *S. parasitica*, *S. viduata*, *S. troglodytes*, and *Anthea cereus*. I have not been able to obtain other species, and my results would have been more valuable if I had had a greater abundance of material; this dearth of material has been the cause of my not having been able to attempt the complete isolation of some of the pigments to be described, and of my not being able to generalise as fully as I might otherwise have done. I trust on these grounds too much will not be expected from these preliminary observations.

* "Further Researches on Animals containing Chlorophyll." 'Nature,' Jan. 26, 1882, pp. 303–5. And "On the Nature and Functions of the Yellow Cells of Radiolarians and Cœlenterates." Proc. Roy. Soc. Edinburgh, vol. xi., 1881–82, pp. 377–96. *Cf.* also Dr. BRANDT, 'Sitzungsbericht der Gesellschaft naturforsch. Freunde zu Berlin,' No. 9, 1881; and Professor LANKESTER, Quart. Journ. Micro. Soc., vol. xxii., pp. 229 *et seq.*; *vide* also 'Nature' for 1882, for letters of Professors MOSELEY and E. P. WRIGHT.

† Proc. Birm. Philos. Soc., vol. iii., p. 374.

‡ The source of light is an Argand gas-burner, or sometimes a "9-candle-power" SWAN lamp. The fanciful curves, recently published in Germany, obtained by means of a heliostat and chemical spectroscope, cannot be accurate, and show more imagination on the part of the observer than most people possess. The flat and other shaped summits of the bands are very remarkable.

*Actinia mesembryanthemum.**—A tolerably uniform result is obtained by the examination of all red-coloured specimens of this species. When the solid portions from the ectoderm, endoderm, and tentacles (in most cases) are examined in the manner described, a band which closely resembles that of reduced hæmoglobin is seen, accompanied generally by two other bands nearer the violet end of the spectrum, Chart I., spectrum 1. The extreme edges of the shading of the band extend from λ 600 to λ 560, while its darkest part is from λ 580 to λ 563. These measurements vary, however, according to the colour of the specimen, for in brown specimens the dominant band is nearer the violet, and in some a band is also present before D. As subsequent observations showed, the latter spectrum belongs to modifications of the same colouring matter, for the same decomposition products are obtained in both cases. The best way to show this variation of the band according to the colour of the specimen is by drawing a number of spectra from different cases, which accordingly I have done in the accompanying Chart I., spectrum 1 to 8. The blue chromatophores (="eye-spots") show always a spectrum in which the band is nearer the red than in the ectoderm and endoderm, and has a likeness to that of indigo-blue. This spectrum is shown in Chart I., spectrum 9. In purely brown specimens the spectrum is tolerably constant, as they all showed a band between D and E, and generally one at D, spectra 5 and 6, Chart I. They also—as I have stated—give the same decomposition products as the red specimens, and this remark applies to greenish specimens. The spectrum of brown specimens has a close resemblance to that of the pigments to which I have given the name histohæmatins, and a spectrum even more closely related to these is seen in *Sagartia troglodytes*.

I tried by the use of alcohol, ether, chloroform, bisulphide of carbon, and other solvents, to get this colouring matter out of the different parts of *Actina mesembryanthemum*, but failed. I succeeded in getting it out changed by boiling with rectified spirit and caustic potash or caustic soda, also by digesting in the cold with the same solutions, but at last I found that it could be extracted with glycerin.

When boiled with caustic potash and rectified spirit, or slowly extracted in the cold, a reddish solution was always obtained, which showed a band at D, spectrum 10, Chart I., generally extending from λ 625 to λ 589, which recalls to mind the spectrum of alkaline hæmatin; when sulphide of ammonium was added to this, the band at D disappeared, to be replaced in every case by two well-defined bands, which are undistinguishable from hæmochromogen, spectrum 11, Chart I. The first extended from λ 564·5 to λ 553, the second from λ 537 to λ 521·5. Their peculiarities of shading and their position are those of hæmochromogen. I then observed that all the red colouring-matter in the Actiniæ gave after this treatment in the solid state the spectrum of hæmochromogen. Now Hoppe-Seyler† found that if solutions of hæmoglobin are treated with caustic alkalies

* In every case the finely divided portions of the Actiniæ were well washed in distilled water before examination.

† "Weitere Mittheilungen über die Eigenschaften des Blutfarbstoffs." Zeitschrift f. physiol. Chem., vol. i., p. 138, and Physiol. Chemie, also Professor Gamgee's 'Physiological Chemistry,' vol. i., 1880.

in an apparatus from which the air is carefully excluded, the hæmoglobin becomes changed into hæmochromogen. In the solid tissues of the Actinia, a similar reaction occurs, but in the solution used to extract the pigment the hæmatin becomes oxidised as it comes out of the tissue and shows the alkaline hæmatin spectrum, which, however, can be reconverted into hæmochromogen by the addition of ammonium sulphide.

I could not, with certainty, obtain acid hæmatin, but I did succeed in converting the colouring matter into hæmatoporphyrin. By digesting portions of an Actinia in sulphuric acid, and filtering through asbestos, a purple-red solution was obtained which showed bands like those of acid hæmatoporphyrin, spectrum 12, Chart I., a little rectified spirit being added to the acid solution, but the band nearer the violet is not placed exactly in the same position as the corresponding band of hæmatoporphyrin obtained from hæmoglobin. The first band (in one experiment) extended from λ 605 to λ 595, and second from λ 563 to λ 551, but owing to the presence of biliverdin and proteids these measurements may not be quite reliable, still they possess a certain value when the results are compared with other cases. If this spectrum be that of a kind of hæmatoporphyrin, it ought to be changeable into alkaline hæmatoporphyrin, and such is the case. The solution was largely diluted with water, and ammonia added to cause the precipitation of the pigment. A flocculent precipitate fell; on filtering, an ochry-coloured precipitate was left, which, on being dissolved in alcohol and ammonia, gave a red solution, showing the spectrum of alkaline hæmatoporphyrin, as shown in spectrum 13, Chart I. The solution was too dilute to enable me to take the wave-lengths of its bands in the large spectroscope, but in the microspectroscope every band could be measured easily.

Hence there can no longer be a doubt that in *Actinia mesembryanthemum* a colouring matter is present which can be changed into hæmochromogen and hæmatoporphyrin.* But a more remarkable likeness to the higher animals as regards its pigments is shown by this Actinia, as I find that it contains a pigment which cannot in any way be distinguished from biliverdin. I could not believe this at first, but there is now no doubt that such is the case, my conclusions being based on a great number of experiments.

Beneath the ectoderm of many specimens, and also in the base (of attachment), a green coloration is perceptible. I found that if such portions of an Actinia were put into strong sulphuric acid they immediately assumed a vivid green colour, but on examining such portions spectroscopically they were found free from absorption bands, but absorbed some of the red end of the spectrum and transmitted the green intensified. I then found that if portions having the green colour were put into a mixture of alcohol and sulphuric acid, and even in alcohol alone, the solution soon

* The hæmatin of the bile of pulmonate mollusks and of the crayfish is not changeable into alkaline hæmatin, nor into hæmatoporphyrin, so far as one can judge; it is an immature kind of hæmatin, as Sorby has shown. Krukenberg's name, helicorubin, is not appropriate, as this pigment is met with in other animals. *Enterohæmatin* is the name I now propose for it for obvious reasons.

acquired a green colour. If red portions of Actiniæ were removed with the green ones, then I could sometimes see hæmatoporphyrin-like bands in the solution obtained by the action of alcohol and sulphuric acid upon them, but if the green coloured parts alone were treated in this manner the resulting solution showed no bands. On treating the solution so prepared on a white dish with nitric acid the blue, violet, red, and yellow colours of GMELIN'S reaction were successively and distinctly seen; and if the solution was placed in a glass-vessel and its spectrum observed, the very characteristic bands which accompany the colour-changes of this reaction were distinctly visible. But a purer solution of biliverdin is obtained by extracting the integument after the above treatment with alcohol; this purely green solution now absorbs more of the red end and transmits green intensified, as shown in the map, spectrum 14, Chart I., which is quite free from absorption bands, except a very feeble shading before F. Treated on a white dish with nitric acid the green, blue, violet, red, and yellow stages of GMELIN'S reaction are clearly seen. If placed in a test tube and examined before the slit of the spectroscope, and nitric acid added, as the colour changes a band appears before D and one at F, then a band also appears after D, the last fades away, then that before D fades away, and finally that at F. No other pigment but biliverdin behaves in this manner, as I have previously shown.*

It is no easy matter to read the bands in the case of GMELIN'S reaction, they disappear so soon, but I believe the following are fairly accurate, for the above solution:—the 1st band is about λ 623 to λ 593, the second faded too quickly to measure it, and the third extended from λ 511·5 to λ 488,† spectrum 15, Chart I. *Compared with an alcohol solution of biliverdin obtained from bile, these bands corresponded exactly.* These experiments were repeated several times, and the result was always the same. Hence *Actinia mesembryanthemum* contains in its mesoderm and elsewhere a colouring matter undistinguishable from biliverdin. From the fact that dull green parts of *Actinia mesembryanthemum* assume a vivid green under the influence of the acidulated alcohol, it is probable that a portion at least of the biliverdin is present in the condition of a chromogen.

For a long time I could not exactly determine whether the band in red specimens of *Actinia mesembryanthemum*, occupying the position nearly of that of reduced hæmoglobin, was not the same as that of Actiniochrome, because Professor MOSELEY'S drawing of the spectrum was lost, but on examining *Bunodes crassicornis*, in which Actiniochrome occurs, I was able to decide this point. The band of Actiniochrome is nearer the red, and is represented in Chart I., spectrum 16.

The next point to be determined was whether the same decomposition products could be obtained from the respective colouring matters, but as long as I dealt with solid tissues I got conflicting results; at length, however, I found that I could get Actinio-

* 'Spectroscope in Medicine,' 1880, and Proc. Roy. Soc., No. 208, 1880, No. 226, 1883, and Journ. Physiol., vol. vi., pp. 22–39.

† The reading of the violet edge of the last band may not be quite accurate.

chrome into solution to a slight extent by extracting parts in which it was seen to be present—by preliminary spectroscopic examination—with glycerin, which also extracts —to a slight extent—the hæmatin-yielding pigment. But from the glycerin it could not be obtained in sufficient quantity in the solid state to be thoroughly studied. Knowing that various ferments can be precipitated out of glycerin extracts of organs by means of absolute alcohol I tried to precipitate these pigments in the same way, but failed, owing to dearth of material and the presence of mucus, &c. I shall, however, attempt this again when I have an opportunity. Examination of glycerin extracts, however, enabled me to decide the question referred to above, namely: whether MOSELEY's Actiniochrome yields hæmochromogen or not.

Every specimen of *Actinia mesembryanthemum*, whether its colour was red, reddish-brown, brown, or greenish-brown, gave to the glycerin, after some days' extraction, a certain amount of colouring matter, *which in every case could be made to change into hæmochromogen, while Actiniochrome never could be changed into it; hence the respective pigments are very different. One is a respiratory colouring matter, the other is an ornamental one.*

If a glycerin extract is made of Actiniochrome, a band may be seen from about λ 593 to λ 566 (centre at λ 579). On adding to this caustic soda the band is moved slightly nearer violet, and then on adding sulphide of ammonium it is only *darkened*.

If a glycerin extract be made of the ectoderm of an anemone yielding the broad band of actiniohæmatin, referred to before, it gives a band at D and one between D and E, spectrum 17, Chart I. If now sulphide of ammonium be added, no change, or only darkening, of the second band takes place; but if caustic soda or caustic potash be added previously, and afterwards sulphide of ammonium, the spectrum is changed into that of hæmochromogen, spectrum 18, Chart I.

The caustic soda in this case does not, however, cause the spectrum of alkaline hæmatin to appear, but removes the band at D altogether.

In such glycerin solutions of *Actinia mesembryanthemum* the first band reads from λ 613 to λ 576, the second from λ 566 to λ 545·5, and generally a third from λ 479 to λ 458·5. After caustic soda the first band disappears and the second is made very faint; on adding ammonium sulphide the hæmochromogen bands read: first from λ 564·5 to λ 554·5, and the second from λ 537 to λ 524·5. I reserve for the end of this paper the conclusions which one may draw as to the functions of the hæmatin-yielding pigment. This Actinia, without doubt, contains a pigment yielding hæmatin and hæmatoporphyrin, and perhaps it may save confusion to name the mother-substance provisionally *Actiniohæmatin*. In none of the specimens of this Actinia examined could I find any "yellow cells." It appears that this hæmatin-yielding pigment does not give the same spectrum in brown specimens as in red; but the spectrum of the glycerin extract of red Actiniæ has a close resemblance to that of the spectrum of the solid ectoderm and other parts of brown specimens. This does not

show that the pigment has been altered by extraction with glycerin, but its molecular condition may be altered. It is well known that the spectrum of a pigment may differ in the solid and liquid state without any necessary change in its composition (Vogel and Kundt).

Bunodes crassicornis.—As already stated, this species has been examined by Professor Moseley,* who found that the parts scraped off from the general body wall and tentacles gave no bands, while those from the circumoral disc did give a band, nearly coinciding with the less refrangible band of oxy-hæmoglobin; he also found that most of the western species were entirely green, and in about one in ten the tips of the gonidial tubercles retained a bright red colouring. In two specimens the tentacles were a beautiful rose colour, and this colour was found to be due to Actiniochrome, though it was absent from the entire remainder of the body. I have found that the colour and spectra of this species differ considerably in different cases. In some obtained from Weymouth the specimens were mostly a dull brownish-green, mottled irregularly with red, and the base was also mottled with red. In some a green layer was present beneath the ectoderm. (The colour of other specimens will be referred to further on.) In the ectoderm of these a band, occupying the same position as that in *Actinia mesembryanthemum*, was present in the red parts. The tentacles were colourless at the apices, and those parts of the tentacles which had a yellowish-red colour gave spectrum 1, Chart II, which belonged to the ectodermal layer. No Actiniochrome could be detected. The red parts of the ectoderm gave spectrum 2, Chart II. In none of the tentacles of this variety could "yellow cells" be detected. On cutting out the red portions of the ectoderm and digesting them in rectified spirit and caustic potash a reddish-yellow solution was obtained, giving an ill-defined band before D, and absorption of the violet end of the spectrum, spectrum 3, Chart II. On adding ammonium sulphide the hæmochromogen bands appeared, the solution becoming redder; the first band extended from λ 564·5 to λ 554·5, and the second from λ 537 to λ 524·5. An extraction of the tentacles with alcohol was without result.

The colours of other specimens from the same locality were a dirty green in the ectoderm which was mottled with light green spots and with a bright red circumoral ring, the tentacles being yellowish with a red zone belonging to the ectodermal layer.† The red ectoderm and the brown-red endodermal parts gave the same spectrum as the former specimens. The ectodermal and endodermal parts, and also the tentacles, contained actiniohæmatin, as proved by the action of rectified spirit and caustic potash, sulphide of ammonium, &c., the same decomposition products having been obtained as in former experiments. No Actiniochrome could be detected with certainty.

* *Loc. cit.*

† The spectrum of these tentacles in the yellowish parts is shown in Chart II., spectrum 1. They had a bluish-white base, a red median zone, and almost colourless points. The red zone, as said *supra*, gave the same band as the ectoderm, &c., and yielded hæmochromogen.

In other specimens of *Bunodes crassicornis*, from Llandudno, the ectoderm had a dark purple-red colour, and the tentacles were purple-red, with more lightly tinted apices; here the tentacles gave MOSELEY'S Actiniochrome spectrum with great distinctness, spectrum 16, Chart I. and 5, II., and on squeezing them in the compressorium two other bands nearer the violet end of the spectrum. The red colouring matter (Actiniochrome) was confined to the ectodermal layer of the tentacles. Its band read from λ 600 to λ 560, including the feeble shadings on each side of it. The ectoderm of the body wall gave a band somewhat like the last specimens, but nearer violet, which evidently resembles the band of actiniohæmatin in former cases. In the endodermal parts the same band could be seen, but less distinctly. Beneath the ectoderm (mesoderm) a greenish layer was seen showing no bands.

On digesting the tentacles for three days in glycerin a violet-red solution was obtained, and this gave spectrum 5, Chart II. This band of Actiniochrome in glycerin extended from λ 596·5 to λ 563,* centre at λ 579. The band in violet of the same spectrum from λ 477 to λ 458·5 (?). On treatment with caustic soda the former band remained *unchanged*, and no change—except very slight shifting towards violet—was produced by adding ammonium sulphide. On treatment with acetic acid (of a glycerin extract), the colour and spectrum became less distinct; sulphuric acid also made the band fainter. On treating some tentacles with caustic potash and spirit, and subsequently ammonium sulphide, no evidence of the presence of hæmochromogen could be obtained. Now the red colouring matter of the ectoderm, when treated in the same way, yielded a solution which, with caustic soda and subsequently ammonium sulphide, gave the hæmochromogen bands. Other portions of ectoderms were extracted with rectified spirit and caustic potash, and yielded a red solution, showing an ill-defined band before D (as in other cases), and on treatment with ammonium sulphide the bands of hæmochromogen appeared, the first, from λ 563 to λ 554·5, and the second from λ 540 to λ 524·5. The solid parts of such ectoderms after this treatment with caustic potash and alcohol—as in all cases when actiniohæmatin is present—showed two well-marked hæmochromogen bands, which on measurement were found to have the following wave-lengths, the first, from λ 567·5 to λ 556, and the second, from λ 537 to λ 521·5. On comparison with the spectrum of an alcoholic or glycerin solution of hæmochromogen, a discrepancy in the measurements is noticeable, the reason is simply this: that in one case the pigment is in the solid state, in the other in solution (spectrum 6, Chart II.). The endodermal parts also contained actiniohæmatin, as proved by the same treatment.

Just as in *Actinia mesembryanthemum* so also in *Bunodes crassicornis*, I succeeded in getting hæmatoporphyrin from the actiniohæmatin. Portions of ectoderm, in which the pigments had been converted into hæmochromogen by treatment with rectified spirit and caustic potash, were acted upon with sulphuric acid, and the resulting purple-

* Its darkest part was from λ 589 to λ 569.

red solution filtered through asbestos; in this solution the acid hæmatoporphyrin spectrum was plainly seen, much more plainly than in the case of other Actiniæ, so that readings could be easily taken, the first band extended from λ 613 to λ 596·5, and the second from λ 566 to λ 551; here a third feeble shading from λ 524·5 to λ 501 was also seen. On comparing these readings with those got in former cases the agreement is tolerably close, especially when we take into consideration the fact that in the case of *Actinia mesembryanthemum* the bands of a solution diluted with rectified spirit were measured, while here the pigment was dissolved in pure sulphuric acid. In the latter case the pigment could also be converted into alkaline hæmatoporphyrin.

If a *Bunodes crassicornis* be taken whose tentacles show the actiniohæmatin band, and if the tentacles be extracted with glycerin, the solution shows a much broader band between D and E than that of Actiniochrome; then on adding caustic soda the band is made fainter, and on adding sulphide of ammonium the hæmochromogen bands are seen with distinctness, but if those tentacles which show Professor MOSELEY'S Actiniochrome are treated in the same way a negative result is obtained.

Biliverdin appears to be present in the mesoderm and in other green parts.

In some specimens I found actiniohæmatin and actiniochrome mixed together, and in the glycerin extract of some tentacles two bands nearer the violet, as shown in spectrum 7, Chart II. I do not think these bands belong to a lipochrome (= lutein) as they are not found in solutions which dissolve the lipochromes.* Similar bands were seen in the case of other anemones, as will be seen by referring to the spectra figured.

Hence in *Bunodes crassicornis* we find actiniohæmatin with tolerable constancy, occasionally actiniochrome and also biliverdin, besides the lutein-like pigments. In the ectoderm as well as in the endoderm, and sometimes in the tentacles, actiniohæmatin is present. In none of the specimens were "yellow cells" present, and by no other solvents except glycerin, and alkaline and acid alcoholic solutions, could any pigments be got into solution.

Bunodes ballii (large variety).—The specimens which I have examined came from Weymouth. The body wall was dotted with red, the tentacles and body at their base were green, and the tentacles presented a contrast in their opacity to the Actiniæ, examined *supra*. In other specimens the body wall (ectodermal layer) was red or brownish-red, the upper fifth or so greenish, with rows of white dots running from base to disc.

The tentacles gave a number of bands, (see spectrum 10, Chart II.,) which showed

* "Lipochromes" is a name proposed by KRUKENBERG for a class of pigments some of which were formerly known as luteins. See his 'Vergleichend-physiologische Studien,' 1880–1882, and 'Grundzüge einer vergleichenden Physiologie der Farbstoffe und der Farben,' 1884. I hope to have more to say on this point in a future paper.

that a chlorophyll-like* pigment was present in them. The interior parts—mesenteries, &c., gave the same spectrum. The ectoderm in the redder parts gave a faint band in green, as shown in spectrum 11, Chart II., while in its green parts—with few exceptions—the same spectrum as that of the tentacles could be detected. On extracting the tentacles with absolute alcohol for a couple of days and filtering, an orange-coloured solution, with a distinct red fluorescence, was obtained, and this gave spectrum 12, Chart II. These bands gave the following readings: 1st band, λ 675, to λ 657, 2nd band, λ 642·5 to λ 629, and 3rd band, λ 595 to λ 579, the light being completely absorbed at λ 543. On treatment with one drop of nitric acid the spectrum changed completely, a darker band appeared placed over a lighter one, the whole compound band extended from λ 675 to λ 647, and its dark part from λ 675 to λ 665; another also before D from λ 623 to λ 596·5, light being completely absorbed at λ 521·5. Spectrum 13, Chart II. A feeble band also was seen after D, not shown in the map. The first alcohol extract left the tentacles a vivid green, but on repeated extraction with alcohol the colour became much diminished, and the second alcohol extract did not differ from the first.

On extracting the interior parts of this anemone with absolute alcohol and filtering, an orange-coloured solution was obtained, and this gave a very faint red fluorescence and a faint spectrum, not differing from that of the alcohol solution of the tentacles, after allowing for the smaller amount of pigment present in the solution. In a thin layer of the solution a feeble band was seen extending approximately from λ 509 to λ 484·5.

On extracting the ectoderm with absolute alcohol and filtering, an orange-coloured solution was also obtained, giving the same spectrum as the parts referred to, spectrum 14, Chart II., and having a red fluorescence. These bands read as follows: 1st from λ 675 to λ 657, 2nd from λ 645 to λ 629, and 3rd from λ 595 to λ 580·5, the spectrum being extinguished at λ 551.

On treating this last alcoholic solution† with caustic potash *the spectrum changed completely*, as shown in spectrum 15, Chart II. The result obtained by extracting the ectoderm with rectified spirit and caustic potash and filtering was no less striking; the resulting solution was distinctly green, and gave the spectrum shown in spectrum 16, Chart II. The first and second band read: 1st from λ 649 to λ 627, 2nd λ 609 to λ 585. In a thin layer a third became detached at λ 492·5 to λ 475. No hæmochromogen bands could be obtained. At the same time I believe the red portions contain a small amount of actiniohæmatin, its presence being masked by

* In the list of chlorophyll-containing animals, enumerated by Professor Lankester, in the second English edition of Sachs's 'Botany,' the word *chlorofucin* is placed after *Anthea cereus*. I have accordingly introduced the spectrum of chlorofucin from *Fucus serratus* (Linn.), and there is seen a remarkable likeness, if not an identity, between it and the pigments of the above, *S. bellis* and *A. cereus*. Spectra 8 and 9, Chart II. Sorby was the first who showed the presence of *chlorofucin* in *A. cereus*. Proc. Royal Soc., No. 146, vol. xxi., 1873, p. 454.

† Or that of the tentacles.

the chlorophyll-like pigment. On microscopic examination the tentacles were found packed with "yellow cells," lodged in part in their endodermal lining. They differed in no respect from the "yellow cells" of *Sagartia bellis* and *Anthea cereus*, to be described further on.* SCHULZE's fluid developed a distinct cellulose reaction in their outer wall, and evidence of starch was obtained by treatment with iodine in iodide of potassium, especially after maceration for some time in a weak solution of caustic potash and neutralizing with acetic acid (GEDDES). Further, it would seem that these "yellow cells" had taken the place of the red colouring matter of other species, to a great extent, since the latter was present in mere traces.

On examining the *small variety* of *Bunodes ballii* from the same locality no "yellow cells" could be detected in the tentacles or elsewhere. Their tentacles were mostly coloured pink in the inner row, and colourless in the outer row, and the former gave a well-marked actiniochrome-like spectrum, spectrum 17, Chart II. The ectoderm was in places lake-red (and the base dotted with red), and gave spectrum 18, Chart II. This spectrum, on comparison with the tentacles, shows the presence of a band near the violet, while the other band is well marked. In some specimens the lining of the body cavity (endoderm) was pink, and in such cases gave spectrum 1, Chart III.; the second band of which is remarkably dark, in other cases it was brownish-yellow, and gave no well-marked band. I failed to procure hæmochromogen, however, from these anemones, which is very remarkable. Digestion of the different parts in rectified spirit and caustic potash gave a negative result after repeated trials. Still the fact is interesting that the colouring matter of the ectoderm resembles, with regard to the first band of its spectrum, that of *Actinia mesembryanthemum*. It may have been a pigment which is intermediate between actiniochrome and actiniohæmatin. But this I hope to decide when I have an opportunity. The replacement of this pigment by the colouring matter of the "yellow cells" in the large variety is of great interest, and teaches that the presence of the colouring matter has something to do with the absence of "yellow cells" in the small variety. It would be premature to say that this pigment is peculiar to the small variety, but it may possibly be a special one like that of *Sagartia parasitica*.

Sagartia dianthus.—Specimens of this Actinia were procured from Weymouth. Some were yellow-brown externally, others white. The tentacles generally flesh-coloured; the lining of the body cavity generally reddish. The ectoderm of the yellow-brown specimens showed a very badly marked band before D, and a feeble shading in green, which recalled to mind the spectrum of brown specimens of *Actinia mesembryanthemum;* the ectoderm of white specimens showed no band. The endodermal parts showed only a faint shading in green; and all these spectra were too indistinct to be mapped.

In a small orange coloured *Sagartia dianthus* a bit of ectoderm and the orange brown-red endodermal parts gave spectrum 2, Chart III.

* These yellow cells measured $\frac{1}{1000}$th inch in diameter, others $\frac{1}{3000}$th; they were mostly $\frac{1}{1500}$th.

But although the examination of the solid parts of this species is so unsatisfactory, I got distinct evidence of the presence of a hæmatin-yielding pigment. On digesting portion of the ectoderm of brown specimens in rectified spirit and caustic potash a reddish-yellow solution was obtained which gave a band at D, and a shading at the blue end of green, the former from λ 611 to λ 579. And then, on adding ammonium sulphide, this band *did not disappear*, but a faint band resembling the first reduced hæmatin band was distinctly seen, spectrum 3, Chart III. On digesting the lining of the body cavity of the same specimens in the same solution, a very feeble band at D was detected, and on adding ammonium sulphide the result was the same as in the case of the ectoderm. The tentacles were digested in absolute alcohol, and the almost colourless solution gave a feeble band from λ 505 to λ 484·5, unchanged by caustic potash. The examination of the endodermal parts of white specimens gave the same result as above.

In a large brown *Sagartia dianthus* I also got distinct evidence of the presence of actiniohæmatin. The reddish-yellow filtered extract, obtained by digesting the ectoderm in rectified spirit and caustic potash, showed a band from λ 623 to λ 585, and on adding sulphide of ammonium two hæmochromogen bands were seen, of which the first (approximately) measured from λ 564·5 to λ 554·5. The same ectoderm yielded nothing to absolute alcohol, nor did the same part of an orange *S. dianthus* to the same solvent.

Sagartia viduata.—The few specimens which I had an opportunity of examining were striped with brown and white, and no well-marked band could be detected either in the ectoderm or endoderm. A faint shading in green was the only noticeable appearance.

On extracting the ectoderm for twenty-four hours with rectified spirit and caustic potash a yellowish solution was obtained, and this gave spectrum 4, Chart III. In this spectrum the first band read from λ 657 to λ 631, and the second from λ 611 to λ 582 (?). If this result is compared with the result of the examination of *Bunodes ballii* a resemblance is noticed. Faint traces of hæmochromogen were detected on adding ammonium sulphide.*

Sargatia parasitica.—In some specimens of this species—of a brown colour, externally striped with yellow and being internally a reddish-yellow colour,—no distinct band could be seen. In the brownish-red base no band could be detected, but both by the ectoderm and by the latter the violet end of the spectrum was strongly absorbed. The tentacles were white, but here and there dotted with a purplish pigment which showed no band. In some specimens the circumoral part had a purple-red colour, which gave an ill-defined band between D and E. Microscopically two kinds of pigment could be seen in them—brown and purple-red. Corresponding to these appearances was the result of extracting the various parts with solvents, as by their

* I believe the above chlorophyll-like spectrum was due to presence of "yellow cells," but I failed to detect them in the specimens examined.

use the presence of *two* pigments was proved. One of them was the familiar actinio-hæmatin, the other a pigment different from any I had examined before.

On digesting the interior parts in rectified spirit and caustic potash an orange-coloured solution was obtained, and on treatment with ammonium sulphide two bands like those of reduced hæmatin appeared, as shown in spectrum 5, Chart III.,* of which the first read from λ 563 to λ 554·5, the second was not distinct enough to be read.

A different result was obtained on examining the solutions obtained from the ectoderm.

As usual, I digested the ectoderm in rectified spirit and caustic potash, and after filtering obtained a red solution, and at the end of six weeks a similar solution retained a fine carmine-red colour. A deep layer transmitted red and a little green, and in a thinner one a broad band with ill-defined shading occupied the middle of the spectrum; both are shown in spectra 6 and 7, Chart III. Measurement showed that a deep layer absorbed the spectrum from λ 557 onwards to the violet, and the broad band extended from λ 540 to λ 467. On treatment with ammonium sulphide the red colour was replaced by a brownish-yellow, and now a broad shading covered D and extended towards E (spectrum 8, Chart III.). But it is not necessary to use an alkali for extracting this colouring matter, as it goes into solution when the ectoderm is digested in absolute alcohol. Such a solution is red in deep layers and yellow in thin, and in the former gives a band at D, broad and ill-defined, and absorbs strongly the whole of the spectrum from close before E onwards towards violet,† spectrum 9, Chart III., while in a thin layer a double band became detached, as shown in the spectrum 10, Chart III. The broad band at D extended from λ 640 to λ 576, and the second about λ 535 to λ 511·5, while the third was from λ 505 to λ 484·5 (?). On adding sulphide of ammonium the solution became a dark purple colour, changing rapidly to yellowish-red, which appeared a fine red by gaslight. A curious appearance was here noted, showing that this colouring matter is capable of existing in two states of oxidation; a dark purplish zone was noticed on the surface of the liquid in contact with the air, which momentarily disappeared on inverting the test tube, but at once reappeared when left to rest. The reduced solution gave spectrum 11, Chart III. The purple colour is due to the action of alkali, and the yellowish-red to the action of the reducing agent; and the change from the alkaline oxidised state to the alkaline reduced state, could be brought about as often as one wished. The band of the reduced alkaline solution extended from λ 623 to λ 572. The band of the purple solution (oxidised state) is broader and darker than the latter. This solution, after being kept for four months, preserved a blue colour, the H_2S having gradually escaped.

* They are slightly nearer violet than in other cases, as can be seen by comparison with former spectra.

† The spectrum of this body is not unlike KÜHNE's "rhodophan," also tetronerythrin, but it does not belong to one or the other, as its colour-changes show.

On treating an alcoholic solution with a little nitric acid the band at D disappeared, the solution changed to gamboge-yellow (gaslight), and a band from λ 535 to λ 509 was seen. The abrupt absorption of the violet end of the spectrum was not removed by the acid.

On treating some alcoholic solutions with caustic potash the colour became redder (gaslight) and the band at D was no longer seen, and in a thin layer a band could be seen from about λ 548·5 to λ 509, but in a deeper layer it extended up to λ 458·5. On treatment of the latter solutions with ammonium sulphide a faint bluish tinge appeared and quickly disappeared, and now some shading at D appeared.

If an alcoholic solution be treated with ammonia alone a splendid purple-blue solution is obtained, giving a dark band from λ 660 up to λ 516, and another from λ 498·5 to λ 475. The first broad band was strongly shaded at first on the redward side, but it eventually changed to that shown in spectrum 12, Chart III. On diluting this solution with more alcohol it became a deep blue, and in the more dilute solution a band was seen from λ 645 to λ 548·5 with its darkest part from λ 627 to λ 582. On neutralising this solution with acetic acid it became yellow, and the whole of the blue and violet part of the spectrum was now strongly absorbed, and in a thin layer the same kind of double band as that of the original alcoholic solution was seen.

The same colouring matter occurs in the tentacles, as I have proved, and they contain no hæmatin-yielding substance, this being confined to the endodermal parts of the body from which I could not extract any of the above-mentioned pigments. I do not think, in the present state of knowledge of the Chromatology of Actiniæ, that one is justified in calling this pigment by a new name, but so far as my experience goes it is peculiar to this species. In its colour-changes with acids it has a very remote resemblance to the purple pentacrinin of Professor MOSELEY,* also to the colouring matter of *Aplysia*,† but differs in spectrum and in some colour-changes. It has also a slight *resemblance* in character of spectrum and colour-changes to the colouring matter obtainable from the petals of some red flowers, *e.g.*, scarlet geranium, red rose, &c., but the respective spectra are not the same.

Out of *Cerianthus membranaceus*, according to HEIDER,‡ by means of ammoniacal water, a colouring matter can be extracted which KRUKENBERG§ calls *purpuridin*, but he says it gives no absorption bands, and as the above pigment does give bands, they cannot be the same.

I failed to find any yellow cells in *Sagartia parasitica*; its colouring matter appears to me to be capable of uniting with oxygen and of giving it up again, and is, therefore, probably of respiratory use.

* *Loc. cit.*

† Professor MOSELEY, *loc. cit.*, also myself in Proc. Birm. Phil. Soc., vol. iii., 1883, p. 392.

‡ *Loc. cit.*

§ "Vergleichend-physiologische Studien," 2te Reihe, 3te Abth., 1882, p. 72.

Sagartia troglodytes.—The solid tissues of this species—at least in the specimens examined by me—are poor in colouring matters. They were mostly a pale flesh colour, or yellow-white, or almost white, externally. Internally almost colourless or yellowish-white, the tentacles slightly green or almost colourless. A dirty brown tint was sometimes present in the circumoral part (externally), and in the same place some greenish spots, which on microscopic examination appeared to be due to the presence of some foreign substance, but not "yellow cells."

In the solid ectoderm a spectrum is detectable which is of great interest, as it indicates the presence of a colouring matter which is related to hæmochromogen, and in some parts the second band of that pigment can be seen. In the stomach wall of some invertebrate animals, *e.g.*, *Uraster rubens*, *Asterias*, &c., a similar band or bands can be seen. These belong to a *histo-hæmatin*, and I have no doubt that such is present in *S. troglodytes*. I have only shown the single band in spectrum 13, Chart III. In the interior portion a feeble band before D is also present. I could not see a band in the tentacles or in the brown part of the ectoderm referred to.

Although the lining of the body cavity was so pale, a solution obtained by digesting it in rectified spirit and caustic potash, on the addition of ammonium sulphide, gave the bands of reduced hæmatin. Traces of the same were found in a similar solution of the ectoderm.

The tentacles on extraction with absolute alcohol yielded only an almost colourless solution, showing no bands.

Sagartia bellis.—Some were yellowish externally, with a greenish zone around the mouth (externally); internally, brownish; the tentacles were quite opaque, and when squeezed out between glasses brownish in colour, and under the microscope they were found packed with "yellow cells." Other specimens were of a dirty yellow, with white stripes running from the tentacles to the base, their interior was orange and the tentacles dirty green; round the bases of the latter a ring of greenish-brown colouring matter was seen, and this was also present in the others. In some, however, a pale reddish-yellow zone encircled the *Sagartia*.

The first specimens referred to on spectroscopic examination gave in the ectoderm some uncertain shading in green, while in a few I saw a narrow band in green like that of *S. troglodytes*, and perhaps a second one. In the lining of the body cavity no distinct bands were seen.

In others the results were the same. In striking contrast to these results was the examination of the tentacles, as in every instance they showed a banded spectrum reminding of chlorophyll.* This spectrum belongs to the mass of "yellow cells" which were confined to the interior of the tentacles, and probably embedded in their endodermal linings. This spectrum is shown in spectrum 1, Chart IV. On digesting the ectoderm in rectified spirit and caustic potash, a pale yellow solution was obtained, giving spectrum 2, Chart IV. On treatment with sulphide of ammonium no reduced

* Or rather *chlorofucin*.

hæmatin bands appeared. The solution evidently contained a trace of the pigments belonging to the yellow cells which are present in the ectoderm. A similar solution of the interior parts was faintly greenish-yellow, and its spectrum was the same as that of the last solution; it contained no hæmatin; the band in red was rather doubtful.

The examination of solutions of the tentacles, however, furnished more interesting results.

An absolute alcohol solution of tentacles after filtering was deep yellow in colour, with a tinge of green, and had a red fluorescence; a deep layer, showed spectrum 3, Chart IV., while in a thinner layer two other bands were present. The whole series of bands read as follows:—1st, λ 675 to λ 660; 2nd, λ 642·5 to λ 629; 3rd, λ 593 to λ 577·5; 4th, λ 505 to λ 481; and 5th (about) λ 458·5 to λ 445. On adding a drop of nitric acid the colour did not appear much changed, but the bands were changed, as shown in spectrum 4, Chart IV., and the first two bands read from λ 669 to λ 649 and λ 613 to λ 593 (?).

Although very little, if any, effect was produced by ammonia, *the spectrum was completely altered by caustic potash.*

The result of treatment with this reagent is shown in spectrum 5, Chart IV., and the wave-lengths of these bands are as follows:—The shading on the red side came up to λ 636, next band λ 613 to λ 589, next λ 574 to λ 553, and a shading from λ 532 to λ 513, and another λ 496·5 to λ 473·5 (?). *This reaction distinguishes the colouring matter of the "yellow cells" from enterochlorophyll, other animal chlorophylls and plant chlorophyll, and teaches that it is quite useless to saponify this pigment, for if decomposed by caustic alkali in the cold it would become further decomposed by boiling with it.* (The change is the same as in *Bunodes ballii* and *Anthea cereus.*)

By the tests adopted in other cases the yellow cells were found to have a cellulose wall and to contain starch, but they had to be treated with a weak solution of caustic potash, then a little acetic acid, before the reactions with iodine in iodide of potassium and "SCHULZE's fluid" could be distinctly seen.

The small amount of other pigments present is very noticeable, especially when we compare these results with those obtained in the case of *Bunodes ballii* and *Anthea cereus.* It would appear that the presence of the "yellow cells" has something to do with the absence or suppression of respiratory pigments.

Anthea cereus.—In some specimens the ectoderm was a pale red, also the base, and the tentacles a pale green, tipped with violet. In the violet apices of the tentacles actiniochrome was detected (see spectrum 6, Chart IV.). The rest of the tentacles gave a spectrum resembling that of chlorophyll,* and those of the tentacles of *Bunodes ballii* and *Sagartia bellis*, spectrum 8, Chart IV. If the contents of a tentacle be squeezed out, an entirely different spectrum is obtained from the empty tentacle, spectrum 7, Chart IV. It is probably the pigment to which these bands belong that is found in

* Chlorofucin.

glycerin extracts of some tentacles to be referred to further on. The base in some specimens had a pale reddish colour which showed a faint band like that of actinio-hæmatin. *The "yellow cells" of Anthea are not confined to the tentacles*, for they could be detected in several places in the ectoderm by the aid of the microscope, and they seemed disposed in rows. They were also found in the lining of the body cavity. In other specimens examined the ectoderm was brownish, with lighter-coloured stripes running from above downwards, the base tinged orange-red, and the tentacles bluish-white, without any violet coloration. In both tentacles and ectoderm there was a great abundance of "yellow cells." On extracting the tentacles with absolute alcohol, and filtering, an orange coloured solution having a red fluorescence was obtained, which showed spectrum 9, Chart IV., the bands of which read: 1st, from λ 675 to λ 657; 2nd, from λ 645 to λ 629; 3rd, λ 595 to λ 579. On adding nitric acid in small quantity a violet and a blue ring formed in contact with the acid, and on mixing acid and solution a decided change took place, the solution giving spectrum 10, Chart IV., and then acquired a pale yellow colour. On adding a little more nitric acid a curious change ensued, as spectrum 11, Chart IV., now appeared. After some time the band in red could not be seen; this reaction distinguishes the present colouring matter from enterochlorophyll, other animal chlorophylls, and plant chlorophyll, but a more striking difference is apparent when this alcoholic solution is treated with caustic soda.* On treatment with this reagent the fluid assumed a redder colour (gaslight), and the spectrum changed to that shown in spectrum 12, Chart IV.; these bands gave the following wave-lengths: 1st, λ 609 to λ 589; 2nd, λ 569 to λ 554·5, and also a shading between green and blue which were too faint to be measured.

On digesting some of the reddish parts of *Anthea cereus* in rectified spirit and caustic potash, a greenish solution was obtained which had a red fluorescence, this solution gave spectrum 13, Chart IV., the more prominent bands of which read: λ 649 to λ 633·5, and λ 607 to λ 587. And there was also a band between green and blue visible in a thin layer from λ 498·5 to λ 477. With ammonium sulphide alone a change was brought about in an alcoholic solution as shown in Chart IV., spectrum 14.

The "yellow cells," on treatment with SCHULZE'S fluid, gave a distinct cellulose reaction, and in their interior the presence of starch was detected with iodine in iodide of potassium, but, as in former cases, the best results were obtained by previously extracting the tentacles with alcohol, and then macerating them in a weak solution of caustic potash, and neutralising with acetic acid. In young specimens of *Anthea cereus* I found much fewer cells than in the larger ones, and in these the starch and cellulose reactions could be more easily obtained.

Since caustic potash and caustic soda change this colouring matter completely, it is useless to attempt to saponify it, and this very fact distinguishes, as I said before, the colouring matter of the "yellow cells" from enterochlorophyll, other animal chloro-

* KRUKENBERG noticed this change with caustic soda, *loc. cit.*, but failed in seeing that this test distinguished the pigment from what he called hepatochromates (= enterochlorophyll).

phylls, and plant chlorophyll. I cannot, on these grounds, agree with KRUKENBERG* that it resembles the hepatochromates, which is his name for the pigment named by me enterochlorophyll,† nor does he by any means succeed in proving that it is a purely animal pigment.

The other violet pigment referred to by KRUKENBERG is probably the actiniochrome of Professor MOSELEY. In some specimens the apices of the tentacles are tinged of a violet colour, which showed the actiniochrome spectrum well marked, spectrum 6, Chart IV. The glycerin extract of the tentacles of some specimens of *Anthea cereus*, after some days' extraction, gives a solution which, although only yellowish-red, possesses a magnificent emerald-green fluorescence. I have no doubt that this is the pigment whose bands are seen in the almost colourless tentacle after removal of the yellow cells, as it gives a band in blue and in the violet, spectrum 15, Chart IV., besides the first. They read: 1st, λ 582 to λ 560; 2nd, λ 520·5 to λ 506; 3rd, λ 494·5 to λ 475. The first band probably belongs to a trace of actiniohæmatin. That this fluorescent pigment was not a so-called "lipochrome" is shown by the fact that, on adding caustic soda, one of the bands near the violet end disappeared, while with acetic acid the second and third bands appeared merged into one.

Summary and Remarks,

The above observations show that a respiratory colouring matter is present in *Actinia mesembryanthemum*, *Bunodes crassicornis*, and other Actiniæ. That it must be respiratory is shown by the fact that one of its decomposition products is capable of existing in a state of oxidation and reduction. That it is closely related to hæmoglobin is shown by the results attending my attempts to convert it into hæmochromogen and hæmatoporphyrin, as the pigments corresponding to these, obtained from the above Actiniæ, are undistinguishable from those obtained from hæmoglobin.

The occurrence of biliverdin in such lowly-organised animals is of great interest; it may probably be looked upon as excretory, and, as I said above, a part of it seems to be present in the state of a chromogen, as proved by the action of acids upon it.

The discovery of biliverdin in the shells of certain mollusks by KRUKENBERG‡ acquires an additional interest from these observations; but it is premature to conclude, as KRUKENBERG does, that here it is independent of the presence of hæmoglobin, because it is possible that the pharyngeal muscles of the animals in which he found biliverdin may contain hæmoglobin as Professor LANKESTER has shown to be the case in other mollusks; or their tissues may contain the colouring matters which I find widely

* *Loc. cit.* KRUKENBERG did *not* show that enterochlorophyll and plant chlorophyll give the same spectrum; he figures spectra of liver-extracts, but in no case is the fully-developed spectrum shown. The position of the FRAUNHOFER lines in most of his spectra is not quite correct.

† Proc. Roy. Soc., No. 226, 1883

‡ "Zur Kenntniss der Genese der Gallenfarbstoffe und der Melanine." Sep. Abdr. a. d. Centralblatt für die med. Wiss., 1883, No. 44.

distributed in the animal kingdom, and which I have called *histohæmatins* or tissue hæmatins, so that the biliverdin may in that case also be looked upon as excretory, and at the same time useful for decorative purposes.

A similar instance of a used-up pigment being got rid of in the integument is the occurrence of hæmatoporphyrin in the integument of starfishes and slugs, as I have proved.* This colouring matter can be extracted by digesting the integument in the cold for some hours in alcohol acidulated with sulphuric acid, and it can be easily shown that it is present in the state of hæmatoporphyrin *as such*. The presence again of enterohæmatin in the "bile" of pulmonate mollusks, in that of the crayfish, and in that of *Patella vulgaris*, as I have lately found, and of histohæmatins in various parts of their bodies, is a parallel instance of the presence of an immature respiratory pigment or pigments taking the place of the hæmatin-yielding pigment in the Actiniæ. The latter cannot, however, be looked upon as a colouring matter intended to *carry* oxygen, but rather to *keep* it in combination until it is wanted by the cells for purposes of metabolism. As it is distributed all over the surface of some Actiniæ, the whole body of such an animal may, in a physiological as well as in a morphological sense, be considered comparable to a single organ of a higher animal, so far, at least, as *internal*† respiration is concerned.

In *Sagartia parasitica* the hæmatin-yielding pigment is replaced by a special one,‡ as already referred to, and in every species of Actiniæ, even in those almost destitute of colour, the presence of respiratory pigments has been detected.

In *Anthea cereus*, *Sagartia bellis*, and *Bunodes ballii* the same colouring matter is present not only in the tentacles but in other parts, and from the observations recorded in this paper, it is quite clear that all the chlorophylloid colouring matter in these three species is entirely due to the presence of "yellow cells." It is not within the scope of this paper to enter on a discussion of the nature of these "yellow cells," it will be sufficient to call attention to the facts that whenever present they have been found to possess a cellulose wall and to contain starch. This is in favour of the view held by GEDDES,§ BRANDT, the HERTWIGS, and others, that they are of a vegetable nature, and are symbiotic algæ. Another point in favour of this view is the behaviour of their solutions with caustic potash and soda, which, as I have already stated, distinguishes them from animal chlorophyll and ordinary chlorophyll (of green land plants). The colouring matter itself, as Dr. SORBY and Professor LANKESTER‖ show

* Proc. Birm. Philos. Soc., vol. iii., pp. 378 *et seq.* Another very suggestive connexion between biliverdin and hæmatoporphyrin is furnished by the fact discovered by SORBY, namely, that these pigments occur in birds' eggs. I have also found hæmatoporphyrin in the integument of the earth-worm.

† *I.e.*, tissue respiration. ‡ In the ectoderm, see above.

§ See 'Nature,' January, 1882, for GEDDES's paper and the subsequent letters of Professors MOSELEY and E. P. WRIGHT as to BRANDT's priority, also P. GEDDES's reply.

‖ See list of chlorophyll containing animals drawn up by Professor LANKESTER, for 2nd English edition of SACHS's 'Botany;' also his paper "On Chlorophyll Corpuscles and Amyloid Deposits of Spongilla and Hydra," Quart. Journ. Micro. Soc., vol. xxii., p. 229, &c.; also note to page 650, *supra*.

(in the case of *Anthea cereus*), is probably identical with chlorofucin. To see if this is the case I have figured the spectrum of chlorofucin in the natural condition in *Fucus serratus* and in alcoholic solution, and on comparison of these spectra with those of the above-mentioned Actiniæ a remarkable likeness is apparent. (See spectra 8 and 9, Chart II.)

As I have already shown, the conclusion of KRUKENBERG* that the colouring matter of the yellow cells of *Anthea cereus* is identical with that of the hepatochromates (= enterochlorophyll) is not borne out by this fact; viz., that in the case of enterochlorophyll no such change is produced by caustic alkalies as is produced in solutions of the colouring matter of the "yellow cells" with these reagents.

Another very remarkable fact noticed should go to support the view of the "yellow cells" being symbiotic algæ, namely, that they appear to cause a suppression of those pigments which in other Actiniæ appear to discharge a respiratory function. In most, if not in all cases, this fact impressed itself strongly on my attention, and I believe the observation is correct. At the same time we must remember that the "yellow cells"—if they are symbiotic algæ—only give oxygen up to the tissues of the animal, which would still require to be *fixed* in the tissues by a combination with something else, such as actiniohæmatin or other pigments; so that too much importance ought not to be attached to the apparent absence or presence of such pigments in the ectoderm, endoderm, or other parts. Their absence too ought not to be concluded from the mere fact that they cannot be detected in certain solutions of these parts, as there is no doubt that they cannot always be got into such solutions even when present.

Besides these pigments of a direct respiratory use, there are others which appear to be of use for decorative purposes, and to this class Professor MOSELEY'S actiniochrome belongs. I always found (with one or two doubtful exceptions) that this is a pigment confined to the tentacles, and, as already stated, it cannot be changed into anything capable of being oxidised and reduced; but whether it is intended for a protective purpose or as a means of attracting prey, further research may decide.

Another kind of pigment is that found in the "eye-spot" of *Actinia mesembryanthemum* (spectrum 9, Chart I.). It is a noteworthy fact that in the eye of *Musca domestica* † a red colouring matter occurs which gives a band covering D. It is also found in other insects' eyes. Possibly, the band in *Actinia mesembryanthemum* denotes that the pigment is capable of absorbing certain rays of light, so as to enable the animal to distinguish light from darkness. The presence of other light-absorbing pigments, and their possible use in some obscure photo-chemical processes in the bodies of Actiniæ, ought not to be overlooked.

The preceding observations have brought to light the following facts among others:—

(1.) That *Actinia mesembryanthemum* contains a colouring matter which can be

* *Loc. cit.*

† *Cf.* KRUKENBERG, *loc. cit.*, he did not notice the band referred to.

changed into hæmochromogen and hæmatoporphyrin, and that it is present in other Actiniæ. It is named *actiniohæmatin*.

(2.) It is not actiniochrome, which is a colouring matter found widely distributed in Actiniæ, which is not changeable into other pigments capable of oxidation and reduction, and is mostly confined to the tentacles.

(3.) A special colouring matter not identical with either is found in *Sagartia parasitica*, which is capable of existing in the oxidised and reduced state.

(4.) In the mesoderm of *Actinia mesembryanthemum*, in other parts of the same Actinia, and in other Actiniæ a green pigment occurs which gives all the reactions of biliverdin.

(5.) *Anthea cereus* (as already known), *Bunodes ballii*, and *Sagartia bellis* yield to solvents a colouring matter resembling chlorofucin, and all the colouring matter which shows this spectrum is derived from the "yellow cells" which are abundantly present in their tentacles and elsewhere. By its behaviour with reagents this pigment can be shown to be quite different from enterochlorophyll, plant chlorophyll, and the chlorophyll of *Spongilla*, or other animal chlorophyll.

(6.) When "yellow cells" are present there is probably a suppression of those colouring matters, which in other Actiniæ appear to be of respiratory use.

(7.) Other colouring matters which give bands in the blue and violet, and spectroscopically resemble lutein, are found especially in the tentacles, but they are not generally soluble in those solvents which dissolve the luteins.

(8.) Pigments whose nature is yet uncertain are also met with, which have been described in this paper.

Explanation of the Spectrum Charts.

CHART I. Plate 69.

Sp. 1. Spectrum of the ectoderm of a brownish-red *Actinia mesembryanthemum*.
Sp. 2. Spectrum of a red *Actinia mesembryanthemum* (ectoderm).
Sp. 3. Spectrum of a reddish-brown *Actinia mesembryanthemum* (ectoderm).
Sp. 4. Spectrum of a red *Actinia mesembryanthemum* (ectoderm).
Sp. 5. Spectrum of a brown *Actinia mesembryanthemum* (ectoderm).
Sp. 6. Spectrum of a brown *Actinia mesembryanthemum* (ectoderm).
Sp. 7. Spectrum of a brownish-red *Actinia mesembryanthemum* (ectoderm).
Sp. 8. Spectrum of a reddish-brown *Actinia mesembryanthemum* (ectoderm).
Sp. 9. Blue "eye-spot" of *Actinia mesembryanthemum*.
Sp. 10. Spectrum like that of alkaline hæmatin, from a rectified spirit and caustic potash extract of the ectoderm of *Actinia mesembryanthemum*.

Sp. 11. The same with ammonium sulphide (hæmochromogen).
Sp. 12. Spectrum of acid hæmatoporphyrin from a solution obtained as described, from *Actinia mesembryanthemum*.
Sp. 13. Spectrum of alkaline hæmatoporphyrin, from the same.
Sp. 14. Biliverdin in alcohol solution from the mesoderm of *Actinia mesembryanthemum*.
Sp. 15. The same solution with nitric acid, early stage of reaction.
Sp. 16. Spectrum of Actiniochrome and another pigment, from tentacles of *Bunodes crassicornis*, &c.
Sp. 17. Glycerin extract of the ectoderm of *Actinia mesembryanthemum*.
Sp. 18. The same after treatment with caustic soda or caustic potash and afterwards ammonium sulphide (hæmochromogen).

CHART II. Plate 69.

Sp. 1. Yellowish-red tentacles of *Bunodes crassicornis*.
Sp. 2. Red ectoderm of *Bunodes crassicornis*, showing the spectrum of the hæmatin-yielding pigment.
Sp. 3. Rectified spirit and caustic potash solution of the same.
Sp. 4. The same on addition of ammonium sulphide (hæmochromogen).
Sp. 5. Glycerin extract of tentacles of *Bunodes crassicornis*, showing spectrum of actiniochrome.
Sp. 6. Solid ectoderm of *Bunodes crassicornis* after digestion in rectified spirit and caustic potash.
Sp. 7. Glycerin extract of the tentacles of some specimens of *Bunodes crassicornis*, showing spectrum of actiniochrome and another pigment.
Sp. 8. Spectrum of *Fucus serratus* in solid state.
Sp. 9. Alcohol solution of the colouring matter of same, showing chlorofucin bands for comparison with those of the pigment of "yellow cells."
Sp. 10. Spectrum of "yellow cells," *in situ*, in tentacles of *Bunodes ballii*. *Cf.* 8.
Sp. 11. Ectoderm of *Bunodes ballii*, showing presence of a trace of the hæmatin-yielding pigment.
Sp. 12. Absolute alcohol extract of tentacles of *Bunodes ballii*. *Cf.* 9.
Sp. 13. The same with nitric acid.
Sp. 14. Absolute alcohol extract of ectoderm of *Bunodes ballii*.
Sp. 15. Absolute alcohol extract of tentacles of *Bunodes ballii*, with caustic potash.
Sp. 16. Rectified spirit and caustic potash extract of the ectoderm of *Bunodes ballii*, showing that the spectrum of the pigment of the "yellow cells" is changed by KHO.
Sp. 17. Tentacles of small variety of *Bunodes ballii*, showing actiniochrome.
Sp. 18. Ectoderm of the same.

CHART III. Plate 70.

Sp. 1. Spectrum of endoderm (of body cavity) of a small *Bunodes ballii*.
Sp. 2. Ectoderm of small orange *Sagartia dianthus*.
Sp. 3. Rectified spirit and caustic potash extract of ectoderm of a brown *Sagartia dianthus* (trace of hæmochromogen ?).
Sp. 4. Rectified spirit and caustic potash extract of ectoderm of *Sagartia viduata*.
Sp. 5. Endodermal parts of *Sagartia parasitica* in rectified spirit and caustic potash after addition of ammonium sulphide.
Sp. 6. Rectified spirit and caustic potash extract of ectoderm of *Sagartia parasitica*.
Sp. 7. Thinner layer of the same.
Sp. 8. The same solution with ammonium sulphide.
Sp. 9. Absolute alcohol extract of ectoderm of *Sagartia parasitica*.
Sp. 10. Thinner layer of same.
Sp. 11. The same with sulphide of ammonium.
Sp. 12. Alcoholic solution of this colouring matter with ammonia.
Sp. 13. Spectrum obtained from ectoderm of *Sagartia troglodytes*.
In other parts a second band nearer violet is seen.

CHART IV. Plate 70.

Sp. 1. Spectrum due to "yellow cells" in tentacles of *Sagartia bellis*. *Cf.* Chart II., spectra 8 and 10, and also 8, *infra*.
Sp. 2. Rectified spirit and caustic potash extract of ectoderm of same.
Sp. 3. Absolute alcohol solution of the tentacles of same. *Cf.* spectra 9 and 12, also 9, *infra*.
Sp. 4. The same with nitric acid, early stage of reaction.
Sp. 5. Change of spectrum of an alcoholic solution of tentacles with caustic potash.
Sp. 6. Actiniochrome in the tentacles of *Anthea cereus*.
Sp. 7. An empty tentacle of *Anthea cereus* (*i.e.*, free from yellow cells).
Sp. 8. A tentacle full of "yellow cells." *Cf.* 1, *supra*, &c.
Sp. 9. Absolute alcohol extract of tentacles of *Anthea cereus*.
Sp. 10. The same with nitric acid, early stage of reaction.
Sp. 11. The same with nitric acid, later stage of reaction.
Sp. 12. An alcoholic solution of the tentacles of *Anthea cereus* with *caustic soda*. *Cf.* also spectrum 5, *supra*, also Chart II., 15.
Sp. 13. Rectified spirit and caustic potash extract of ectoderm, &c.
Sp. 14. An alcoholic solution of the same pigment with ammonium sulphide.
Sp. 15. Glycerin extract of the tentacles of a specimen of *Anthea cereus*; it showed a splendid green fluorescence, but this pigment does not occur in every specimen of *Anthea cereus*.

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XII. *On the Development and Morphology of* Phylloglossum Drummondii.

By Professor F. O. Bower.

Communicated by W. T. Thiselton Dyer, *F.R.S.*

Received and Read June 18, 1885.

[Plates 71–73.]

Part I.—The Vegetative Organs.

Within the last twelve months the interest of Botanists has been centred afresh in the investigation of the *Lycopodinæ*. It is sufficient to refer to the recent writings of Bruchmann,* Treub,† and Solms-Laubach,‡ to indicate how actively the investigation of these plants is being pursued. It had long been the wish of the author of this paper to obtain material for the careful investigation of the less known members of the *Lycopodinæ*, and especially of *Phylloglossum;* the authorities of the Royal Gardens at Kew made this known to Baron Ferdinand von Mueller, and from him there were received during the latter half of 1884 two consignments of living tubers of *Phylloglossum Drummondii*, as well as a supply of mature plants preserved in alcohol. The tubers germinated successfully, in the propagating pits at Kew, and the material was thus at hand for the investigation of the development and histology of this little known plant. As circumstances will for the present prevent the author from continuing his work, and as the *Lycopodinæ* are now one of the focal points of botanical interest, it appears desirable that the chief facts thus far acquired should at once be published, while a discussion of these results, and their bearing upon recent observations of other forms, may for the present be postponed. The structure and development of the sporophore generation only will be described at present. Many of the plants now growing at Kew already bear immature sporangia, but the description of their development will be deferred till the sporangia have arrived at maturity, and until attempts have been made to germinate the spores: the general treatment of the subject will be better in place when it is seen what results, if any, are acquired from those cultures of the spores. For similar reasons it would be both inconvenient and inopportune now to review the writings already published by various

* 'Botanisches Centralblatt.'

† "Études sur les Lycopodiacées." 'Ann. du jardin botanique de Buitenzorg,' vol. iv., p. 107.

‡ "Der Aufbau des Stockes von Psilotum Triquetrum." 'Ann. du jard. Buitenzorg,' vol. iv., p. 130.

authors on *Phylloglossum*. This memoir will therefore deal merely with the observations recently made by the author; in some points it will be seen that they confirm the observations of previous writers, in others they are not in accord with them: they will be placed in relation with those of other authors when the investigations, which are still in progress, have been as far as possible completed.

Structure of the Tuber during the resting stage.

The tubers as sent over from South Australia were in the mature condition, and undergoing a period of rest. The upper parts of the plants which produced them had already died off. In no case was there evidence of more than one tuber being produced by one parent plant.

In this condition the structure of the tuber is as follows: it consists of an ovoid solid mass of tissue, connected at its upper end with the stalk by which it was attached to the parent plant. This stalk widens out near the apical end of the tuber (*i.e.*, that nearer the parent plant), and is there hollowed, covering in the broad organic apex of the tuber, which thus has the appearance of being of endogenous origin. A transverse section of the stalk which bears the tuber shows a nearly circular outline. The chief bulk of the tissue consists of thin walled parenchyma, with intercellular spaces: in this may be recognised a point where the cells are of smaller size, and for the most part without intercellular spaces (fig. 1, *c*): in many cases a transverse section will show a cavity between these cells (*c*, fig. 2), and this is especially marked in sections taken near the tuber, or near the attachment of the stalk to the parent plant. At the point in question lies the original channel of communication of the apex of the tuber with the outer air; as will shortly be seen the apex of the tuber is of exogenous origin, but the channel of communication becomes closed during development. Further, a vascular strand of small size may be seen cut through in the transverse section (*xy*, fig. 1); it consists of a few elements with lignified walls, which are, as shown by longitudinal sections, spirally or reticulately thickened. At the periphery is a layer of epidermis, with peculiarly thickened walls: it is continuous with a similarly developed layer covering the tuber externally (see below). Passing downwards to the tuber itself, the stalk widens out, and becomes hollowed; the vascular strand spreads laterally, so as to assume a funnel-like form, covering the apex of the tuber; but it stops short about the level of the apical cone, and is not continuous into the fleshy tissue of the tuber.

The mature tuber itself consists of a central bulky mass of thin walled parenchyma, which is entirely undifferentiated: there are intercellular spaces. The cells are filled with store materials, viz., protoplasm, starch and oil. Towards the periphery of the tuber the cells have little or no contents, and the last two or three layers below the epidermis thus form a more transparent band, the elements being often squeezed out of shape. The superficial layer is developed as a clearly defined epidermis (figs. 3, 4); the cells themselves are of oblong form, their radial walls are strongly thickened and

stratified, and are also pitted; irregular pits are found in the outer part of them, which has accordingly a reticulate appearance when seen in surface view; clearly marked pits are also found occasionally near the internal limit of the radial walls (fig. 4). These thickened walls give a mauve colour with SCHULZE's solution, and are not stained by an acid solution of aniline sulphate: it is thus shown that they are not lignified. Some superficial cells grow out as root-hairs, which are sometimes cut off by a wall from the parent cell (as shown in fig. 3, *h*). As might have been expected, stomata have not been seen in the epidermis of the tuber. An investigation of the base of the tuber shows that in the mature state this epidermis is continuous over it: a longitudinal section of a tuber in course of development shows no trace of those characters which are peculiar to the root, a conclusion which is equally borne out by the mature structure.

Passing now to the apex of the tuber, median longitudinal sections show that the mode of division of the cells up to the period of maturity is that known as the ⊥ mode of division.* I have not in any case observed any wedge-shaped cell at the apex in this stage of development of the axis (fig. 6). The arrangement of the cells as seen in superficial view from without is represented in fig. 7; it is suggested that the four cells marked (*x*) may be initial cells in this particular example, but it is not insisted that this is universally the case. As will be seen later, there is some inconstancy in structure of the apical meristem of this plant, and as the number of tubers investigated has been necessarily small, I am not prepared to assert that the structure of the apex above described is uniform for all mature tubers. Before germination there is no sign of formation of appendicular organs, the apex of the tuber is broad, slightly conical, and quite smooth.

Germination of the Tuber.

The living tubers, which were sent over in the dry state, were germinated with success in the propagating pits at Kew; the first plants made their appearance above ground about one month after they were sown. The tubers were of very variable size, and the character and number of the organs formed depends, in great measure, upon the size of the tuber. Thus it has been found that where the tuber is small, foliage leaves are formed, together with roots (one or more), and one new tuber; the number of the leaves also depends upon the size of the tuber, being sometimes only one, where the tuber is very small. Where the tuber is relatively large, sporangia are produced, their position and arrangement being as already described by other writers; in this case the tuber will also produce foliage leaves, roots (one or more), and one new tuber, as in the former case. As there is a difference in the mode of development of the plant, according as sporangia are formed or not, it will be necessary to deal with the two cases separately; it is however to be borne in mind

* See PRANTL, "Unters. zur Morph. der Gefässkryptogamen." Heft i., p. 4.

that, as far as observation extends, the starting-point is in either case the same. If we leave the size of the tuber out of account, there is in the structure of the apex of the mature tuber no indication by which it has been possible to decide beforehand whether the tuber will produce sporangia or merely vegetative organs.

Before entering on the details of the process of germination, it will be well to state what the results of that process are, as shown by mature plants. Of the twenty-two mature plants which were preserved in spirit, seventeen bore sporangia, five had only vegetative organs. From a careful comparison of them the following conclusions have been drawn, viz.:—

(A.) Where sporangia are produced, the number of foliage leaves is 3–6; the number of roots 1–3, usually 3. Two tubers are found in all cases, one of these being the spent tuber of the previous year, the other the new tuber.

(B.) Where sporangia are absent, the number of foliage leaves is 4–7; the roots 1–2, usually 1. As before two tubers are present, and it may be stated that in no case have more tubers than two been observed: if it is constantly the case that only one new tuber is formed during each period of activity, it is obvious that there can be no increase in number of individuals by means of the tubers.

Taking, first, the more simple case, where the germinating tuber produces only vegetative organs, the conical apex elongates, and a massive rounded outgrowth appears upon it: this is the first leaf. The position of this first leaf is lateral, though in some cases more clearly so than in others: its position relative to the parent plant which produced the tuber has not been found to be constant in those cases where it was possible to judge on this point; thus, it has been found in some cases to be inserted on the side of the axis more remote from the parent plant; again, it has been observed in other cases to be lateral as regards the parent plant. As will be pointed out below, the other organs developed later, have more or less clearly defined positions, relative to the first formed leaf, and consequently we may conclude generally, that as regards the position of its organs, the germinating tuber develops independently of the plant which produced it: this conclusion might well be anticipated since the parent plant is in a state of advanced decomposition before the tuber begins to germinate. Where the tuber is a small one, and the supply of reserve materials is therefore small, the leaf which appears first may be the only one developed: this has been the case in the plants represented in figs. 8 and 9: in both of these, the solitary leaf is the most prominent member of the plant. I am not in a position to state definitely that in these cases the leaf is clearly of lateral origin on the apex of the tuber, though the natural conclusion would be that it is lateral, and this view is supported by a comparison with the other types of development of the plant, to be described below.

In one irregular example represented in fig. 10, there are apparently two leaves of equal size. This case may be compared with that shown in fig. 20 at a more advanced stage of development.

Where a single leaf is formed first (and this is the case in the large majority of specimens) it is usually followed by two of approximately equal size, which are formed laterally on the apex of the tuber, and are disposed one on either side of the first formed leaf. This is clearly seen to be the case in the specimens represented in figs. 11 and 12 (lii.). This pair of leaves may be again followed by a second pair (fig. 13, liii.); but here again irregularities of arrangement may, and often do occur: thus though in fig. 14 their general arrangement is the same as that above described, the leaves do not in this case appear as successive pairs. From the examples cited it is evident that there is no strict constancy in number, or arrangement of the leaves; their number is seen to vary from one to five, and the number as seen in the mature plant may be occasionally higher still, as many as seven have been seen in one mature specimen. It may be again noted that the larger number of leaves is usually associated with the absence of sporangia. The arrangement of the leaves also shows some inconstancy, there being in the majority of cases one leaf formed first, and followed by successive pairs; in other cases the order of succession and position being less regular. In those examples in which the arrangement of the leaves, and their order of appearance on the apex are most regular, it is clear that we have to deal with a *successive whorl* of leaves. Such successive development of members of a whorl is well known to exist in certain flowers, as for example in *Reseda*, the flowers of the *Papilionaceæ*, &c. The successive formation of the whorl of leaves derived from a node of *Chara* may also be cited. Further, the irregularities of arrangement and succession of the leaves in *Phylloglossum* acquire additional interest when it is remembered that in the genus *Lycopodium* irregularities are also prevalent, and have always presented difficulties to those who attempt to reduce morphological description to strict formulæ. The leaves originate on the apex of the tuber as rounded masses of meristematic tissue, in which no single apical cell is to be distinguished; divisions of the superficial cells by periclinal walls are frequent, and it is only at a comparatively late period that a superficial layer of epidermis becomes apparent.

Returning now to the central point around which the leaves are disposed, it is at no time a prominent cone in those plants which do not form sporangia. According to the strength of development of the leaves it is liable to be more or less displaced; this is most clearly seen in those plants in which only one leaf is produced (figs. 8, 9), here the apex of the tuber assumes a lateral position at the base of the solitary leaf. Even in those cases where the successive whorl of leaves is most complete, the apex becomes displaced, owing to the more rapid development of the earlier formed leaves (fig. 13). At an early period the apex thus displaced becomes depressed; it grows slowly in length itself, while the tissues immediately surrounding it advance more rapidly, the result is a conformation of the surface as seen at (*a*) in figs. 8–14. A longitudinal section shows the depression still more clearly (fig. 15). There can be little doubt that, in those plants which do not form sporangia, it is actually the apex of the plant which becomes thus depressed, since the depression appears constantly in

an approximately central position, notwithstanding differences in number and arrangement of the leaves. Following still the changes of external conformation, it is found that the young tuber begins to project, since the growth of tissue is continued actively round the apical depression (fig. 16, *a*); it gradually becomes inverted and elongated, as the result of a special localisation of intercalary growth (figs. 17–20). Meanwhile the swelling of the tuber begins by increase of the tissues lying below the depressed apex (fig. 20). Further details of the development of the tuber, as illustrated by sections, will be given below.

In some cases a leaf smaller than the rest and of later development is to be seen inserted above the tuber (fig. 18); its position is, as a rule, not directly above the tuber (fig. 17). A leaf holding this position has been observed by other writers, but it is by no means of constant occurrence, and its occasional appearance illustrates once more how great is the irregularity of number and position of the leaves in *Phylloglossum*. The description above given of the development of those simpler plants of *Phylloglossum* which do not bear sporangia coincides with the structure of those plants when mature, and it may especially be noted that in the mature plant there is no trace of an apical cone or bud to be found in a central position at the base of the leaves, an observation which completely bears out the view that in these plants it is actually the apex of the plant which gives rise to the new tuber.

While the development of the tuber is progressing, an outgrowth makes its appearance on the opposite side of the plant to that on which the tuber is formed; it is situated at a point below the insertion of the oldest leaf. It arises as a smooth rounded cone; there is no sign of any rupture of the tissues, indicative of endogenous origin, its outer surface being unbroken and glossy; as it grows older a slight excrescence appears at its apex, the convexity of its surface being there greater; this outgrowth ultimately develops as the first root (*r*, in figs. 9, 16*b*, 18, 19, 20). In one plant this outgrowth did not appear as a simple cone, but, as shown in fig. 21, two cones appeared, joined laterally, a mode of development which may well be compared with that exceptional development of the leaf shown in fig. 20. As far as external observation is concerned, it may be concluded from the above description that the root is of exogenous origin. This is borne out by the results derived from the study of sections, which will be detailed below.

The second alternative mode of development remains to be described, viz., that in which the plant forms sporangia. As above stated, this appears to occur only on the germination of large and well-developed tubers. The formation of leaves and their order of succession appear to proceed in the same way as above described, but it is to be noted that the number of ordinary vegetative leaves is usually smaller in sporangium-bearing plants than is the case in plants of purely vegetative development. When sporangia are to be formed, the apex of the plant is convex, and grows on in an erect position; on it are formed successive leaves, which originate like the ordinary foliage leaves, but remain of relatively small size, while the axis which bears them

becomes elongated below the insertion of the lowest of them (figs. 22–25). As to the arrangement of the leaves on the cone I have nothing to add to the description of previous writers. It will be seen from figs. 22–25 that the first two leaves of the cone correspond roughly in position to the second pair of leaves in fig. 13.

Since in sporangium-bearing plants the apex of the main axis continues its growth, and gives rise to the sporangium-bearing cone, it is clear that there must in this case be a different origin for the tuber than that described for the purely vegetative plant. Up to a stage of development, such as is shown in fig. 23, no trace of a new tuber has been observed; but subsequently a depression is seen at a point near the base of the pedicel, which is surrounded by a circular weal (fig. 24); this is the young tuber, which is again shown in a more mature condition in figs. 25, 26. It would thus appear that in those plants in which the main axis develops as a sporangium-bearing cone, the new tuber is of adventitious origin, and exogenous. It does not bear any constant relation to the leaves, though, as has been observed in mature plants, a leaf of smaller size than the rest is occasionally inserted immediately above the attachment of the new tuber (fig. 27). A supernumerary leaf of this sort has been observed in four out of eleven specimens of mature sporangium-bearing plants. In plants of this type the roots originate in the manner above described.

The chief points in the development, as seen on germination of the tuber, having been described, as far as their external characters are concerned, the description of the development of the tuber and of the root, as illustrated by sections, may now be proceeded with. If a median longitudinal section be cut from a plant, such as that represented in fig. 13, it will present an outline as shown in fig. 15. At the point marked (a) there is a slight depression, which indicates the first origin of the new tuber. Seen under a higher power the arrangement of the cells appears as in fig. 28. Here it is evident that growth and cell-divisions, especially periclinal divisions, are less frequent at the point (a) than in the tissues immediately surrounding that point: the result is that the point marked (a) becomes depressed, while the surrounding tissues gradually overarch it (fig. 29), leaving only a narrow channel open above the apex. Finally the channel which is seen still open in fig. 29 is closed up, and the apex of the new tuber is thus completely covered in by the more rapidly growing tissue which surrounds it (fig. 30). It is to be observed that the apex thus covered in is in the first instance of very small area, and in the case of fig. 29, it appears to be only a single cell in width: whether this is always the case I am not in a position to state. Subsequently divisions appear in the cells at the base of the depression (fig. 30), which are repeated, and as the tuber increases in bulk the apical cone also increases in width, until finally in the mature tuber it is a massive structure of convex form, such as is seen in fig. 6. Even before the apex of the new tuber is thus covered in by the increasing bulk of the tissues surrounding it, growth is localised in a special way in the more deeply seated tissues: this growth results in the whole body of the tuber being made to project beyond the general surface of the axis, while, since the

intercalary growth is more strong on the lower than on the upper side of the new tuber, the whole body of it becomes inverted, in a manner not unlike that already well known in the tubers of certain orchids.* Fig. 31 represents in general outline the form and position which are thus assumed by a developing tuber: the apex (*a*) still maintains an oblique position; above it is a small cavity, more clearly seen in the specimen drawn under a higher power in fig. 32; from this cavity the now closed channel, which originally communicated with the open air, may be traced upwards, as seen in figs. 31, 32, till on approaching the outer surface it again widens out as a funnel-like depression (shown in fig. 33), which may often be recognised even in the mature plant (fig. 26). It may further be noted that a vascular strand, already described as existing in the stalk of the mature tuber, runs down from the main body of the plant till it approaches the apex of the new tuber; but, as above stated, it stops short before reaching the new apex. It may here be again remarked that among the plants grown at Kew no case has been observed of the formation of more than a single tuber: if this be uniformly the case with *Phylloglossum*, there can be no increase in number of individuals by means of the tubers, and at present no other form of vegetative reproduction is known in this plant: this would lead us to expect that the plant depends for its multiplication upon the germination of its spores, and this will enhance the interest which attempts to raise the plant from spores would naturally arouse.

If a median longitudinal section be cut through the conical outgrowth which ultimately develops as the root, while it is still young, its structure is seen to be such as is shown in fig. 34. It is clear from such preparations that there is no question of endogenous origin of the outgrowth, since the outer margin of the section is continuous, smooth, and unbroken from the surface of the axis, over the whole of the swelling. There can be no doubt that the whole outgrowth is of exogenous origin. The superficial cells are subject to both periclinal and anticlinal division, and consequently there is no clearly marked dermatogen; this is the case for all the other organs of the plant while young. As the outgrowth increases in age and size, periclinal divisions, which may already be observed at the point of greatest convexity in fig. 34, become more frequent, and being repeated in the same cells, result ultimately in the formation of a root-cap, which may then be recognised externally as the slight excrescence at the apex, as above described; the root-cap is thus derived in this case from superficial cells, and the whole root is of *exogenous origin*. In *Phylloglossum* another plant is added to the list of those in which the root is described as being of exogenous origin†: it is interesting to note that among them is *Isoetes*, in which the first root of the embryo is stated to originate from superficial cells.‡

* Irmisch. "Biologie und Morphologie der Orchideen." Leipzig. 1853.

† Compare Goebel. "Vergleichende Entwickelungsgeschichte," p. 350.

‡ Sadebeck. "Gefässkryptogamen," p. 229.

It has been above stated that the mature leaf is of simple form, and almost cylindrical: a transverse section of it shows an almost circular outline, limited peripherally by a more or less regular epidermis, in which are numerous stomata of simple structure, and distributed uniformly on all sides (fig. 36). Below the epidermis is a massive, thin-walled mesophyll, arranged almost uniformly all round: there are large intercellular spaces, and respiratory cavities. The more peripheral cells have their longer axis disposed obliquely to the outer surface (fig. 37), while in those which are nearer the centre of the leaf the longer axis is directed longitudinally: thus in the transverse section the cells nearer the centre of the section appear almost circular. At the centre of the transverse section there is a small and simple vascular bundle, consisting only of four or five xylem elements, surrounded by tissues with cellulose walls, and small cell-cavities with protoplasmic contents. Longitudinal sections show that the xylem elements are elongated, with annular or irregular spiral thickening: there is no clearly marked phloem, and no bundle-sheath. The more central portion of the mesophyll consists of elongated and pointed elements (fig. 37); the oblique terminal walls where two such elements adjoin one another are of irregular thickness, and as seen in surface view are marked with shallow pits: there is no special character of their cell-contents, and a nucleus is present. It seems probable that this tissue replaces the phloem not only anatomically, but also physiologically, in fact that it is a rudimentary type of phloem. It is thus seen that the vascular bundle of the leaf is of a very simple type.

A peculiarity of structure is to be noted in the parenchyma at the base of the leaf. There the intercellular spaces are large, and into them there project numerous peg-like outgrowths from the walls of the adjoining cells (fig. 40); these are similar to those already well known as existing in the parenchyma at the base of the leaf in many ferns. The substance of which they are composed gives the following reactions: (1) They swell slightly with strong potash solution, and become less highly refractive, but do not lose their clearly defined outline; they do not stain with Schulze's solution, even after previous treatment with potash, though the adjoining walls give a deep blue stain; (3) in sulphuric acid they behave similarly to the middle lamella, that is, they resist its action longer than the layers adjoining the protoplasm; (4) with methylene blue they stain very slightly or not at all. From these reactions it is clear that the outgrowths are not composed of ordinary cellulose; but it is difficult to define what their substance really is; it would appear, however, that it is not slightly cuticularised cellulose, such as described by Luerssen for similar outgrowths in ferns.*

A transverse section of the peduncle, or sporangium-bearing axis, shows an epidermis with stomata; its outer wall is thick, and is covered with cuticle, and a narrow cuticularised layer. Beneath the epidermis is parenchyma, with large inter-

* "Ueber Intercellularverdickungen im parenchymatischen Grundgewebe der Farne." Sitzungsber. der naturforsch. Gesellschaft zu Leipzig, 1875, p. 76.

cellular spaces, the cells being rounded, with thin walls, and containing protoplasm with chlorophyll granules. Here again there is no clear limit between the outer tissue and the vascular tissue; on passing from the periphery inwards the intercellular spaces become smaller, and in the tissues immediately surrounding the xylem they are almost or entirely absent (fig. 39); there are, however, intercellular spaces between the elements of the xylem themselves. The thin-walled tissues surrounding the xylem show characters similar to those of the tissue in a similar position in the leaf, which suggest for them a similar function. The elements of the xylem are tracheides with annular or irregular spiral and reticulated thickening of the walls.

A transverse section of a mature root shows a structure which coincides with that of the simpler forms of root in the *Lycopodineæ*. There is no clearly defined epidermis; cells of the superficial layer grow out as simple root hairs. There is a broad band of cortex with large intercellular spaces; it is limited internally by the bundle-sheath, which is not a definite layer of cells, but is, as in *Lycopodium*, a somewhat irregular band of cells with corky walls (fig. 41); between this and the vascular tissues an irregular band of cells with cellulose walls intervenes, which may represent the pericambium. The vascular tissues consist of a single group of xylem elements, and a single group of elements of the phloem. No example of branching of the root has been observed.

As regards the distribution of the vascular bundles in the whole plant, and their mutual relations, there is considerable irregularity; this might be anticipated from the irregularity of number and arrangement of the leaves, &c., as above described. From each leaf one bundle of the leaf-trace passes into the axis; one strand passes from the axis into each root, and, as above described, one vascular strand passes into the stalk of the tuber. These are the fundamental points on which the arrangement of the vascular tissues is based. Taking first the simple case where only vegetative organs are formed, the vascular system was found, in two specimens, each with four leaves and one root, to be disposed as shown in fig. 42A. The bundles from three of the leaves united in the axis to form one trunk, which was continuous downwards directly into the single root. The bundle from the fourth leaf, probably the latest formed or supernumerary leaf, was traced directly downwards into the stalk of the tuber, without its being connected with the other bundles. This arrangement is a very simple one, and is obviously less efficient than those to be described below, since the vascular system here consists of two disconnected portions. In another specimen having five leaves, two roots and a new tuber, the vascular system was found to be disposed as shown in fig. 42B. The bundle passing down from one leaf, obviously from its position the first formed leaf of the successive whorl, divided in the axis into three strands; of these one passed to the stalk of the tuber (t), the other two curved to right and left, and finally coalesced with those descending from the other leaves, to form two trunks which passed directly into the two roots (r).

In those plants which bear sporangia, seated on an elongated stalk or peduncle, a

composite bundle passes down that stalk, the structure of which is shown in fig. 39. At the base of the stalk, about the level of the insertion of the foliage leaves, this composite bundle separates into strands of varying number, according to the number of the other organs of the plant; two examples from such plants are shown in figs. 43A, B. In these it may be observed that the general plan is in both cases the same, though there is a difference in the number of the roots; also in (A) the strand which passes to the tuber (t), is derived directly and solely from the axis, while in B the bundle of the tuber (t) is derived partly from the axis and partly from the leaf (l_1), which is situated above the insertion of the tuber, and appears to be the supernumerary and latest formed leaf, described above as being of inconstant occurrence. These few examples will suffice to show that there may be considerable variety in the arrangement of the vascular system of *Phylloglossum*, and that the irregularities are closely connected with the variations of arrangement of the organs.

Concluding Remarks.

The above description applies only to the vegetative organs of *Phylloglossum Drummondii*. The description of the structure of the sporangia, their development, and the possible results of their germination must be deferred for the present. In the light of TREUB'S recent researches on the *Lycopodiaceæ*, the development of the vegetative organs of *Phylloglossum* acquires additional interest, and a comparison immediately suggests itself between the early stages of development of the sporophore of *Lycopodium*, and the germination of the tuber of *Phylloglossum*. It can hardly have escaped the notice of those acquainted with TREUB'S memoir, that in certain points the correspondence is very marked, and it may be well to point out those characters where the similarity is most striking. As regards the general conformation of the plant of *Phylloglossum* while young, a comparison may well be drawn with the young plant of *Lycopodium cernuum* shown in fig. 1, Plate XVI. of TREUB'S memoir. At the base is a parenchymatous tuber ("tubercle embryonnaire" of TREUB), which corresponds in position and structure, though not in size, to the tuber of *Phylloglossum*; in both cases it is composed uniformly of parenchymatous cells, and in both plants the superficial cells may develop as root hairs. In that same figure three leaves are represented similar in their form and position to those of the simpler plants of *Phylloglossum*. As yet no root is visible in the young plant of *Lycopodium*, and, similarly in *Phylloglossum*, the formation of the first root is subsequent to the appearance of the leaves. The foot, which is not shown in the figure of *Lycopodium*, is absent in *Phylloglossum*, but it is to be remembered that in this plant the place of the foot is taken physiologically, if not morphologically, by the stalk of the young tuber. Thus the correspondence of parts, as regards their external form, between the young dlant of *Lycopodium cernuum* and *Phylloglossum* is very close. The chief difference is in the relative size of the tuber; in *Phylloglossum* the development of the tuber, and storing of nutritive materials in it, precede the development

of the new organs at the apex, an arrangement which is well adapted to the growth of the plant under circumstances which alternately favour and prevent vegetative activity. In the young *Lycopodium* the supply of nutritive materials is drawn by the foot from the prothallus as it is wanted by the young sporophore. It is further to be observed that the same difficulty has been experienced by Dr. TREUB in distinguishing the morphological character of some of the young organs (*l.c.*, p. 130) as meets us also in the case of *Phylloglossum:* shall an outgrowth which appears not clearly lateral, such as that described as the first leaf in *Phylloglossum*, be designated a leaf or not? It is only to be expected that such difficulties should arise as we investigate plants which are low in the scale of development, and especially when we observe the development of their embryos. As regards the origin of the root, there is no doubt that in *Phylloglossum* it is exogenous, and that the first root is adventitious. In *Lycopodium cernuum* also the origin of the first root is comparatively late, and it is adventitious. I cannot think, however, that the one example shown by TREUB (Plate XVII., fig. 1) excludes the possibility of the first root being of exogenous origin, and it is to be observed that other figures (*e.g.*, figs. 2, 3, 4 of Plate XVII.) seem rather to point to an exogenous than an endogenous origin. It may also be observed that the connexion of the vascular bundles of the leaves and root, and their absence in the tuber ("tubercle embryonnaire") in *Lycopodium cernuum* correspond with the distribution of the bundles in *Phylloglossum*. Taken as a whole, and discounting the absence of the foot, and the storage of nutritive materials in the tuber before the development of the apex to form leaves and other organs, it may be said that the whole development of *Phylloglossum* from the tuber is strikingly similar to the embryonic stages of *Lycopodium* as described by TREUB. A comparison of the two leads clearly to the conclusion that, provided the sporophore generation of *Phylloglossum* corresponds in its more important points to that of *Lycopodium*, we may regard *Phylloglossum* as a form which retains, and repeats in its sporophore generation, the more prominent characteristics of the embryo as seen in *Lycopodium cernuum;* it is a permanently embryonic form of Lycopod.

DESCRIPTION OF THE FIGURES.

PLATE 71.

Fig. 1. Part of transverse section of the stalk of a mature tuber: xy=the strand of xylem. *c* indicates the position of the channel, which is in this case completely closed. (×175.)

Fig. 2. Part of a similar section: here the channel (*c*) is still open. (×175.)

Fig. 3. Peripheral part of a transverse section of a mature tuber, showing the superficial epidermis, with peculiarly thickened walls, and one root-hair (*h*). (×130.)

Fig. 4. A small piece of epidermis of the tuber, showing under higher power the thickened, stratified walls, with pits as described in text. (×325.)

Fig. 5. Median longitudinal section of the base of a young tuber. (×325.)

Fig. 6. Median longitudinal section of the apex of a mature tuber. (×130.)

Fig. 7. Apex of mature tuber as seen from above. (×325.)

Fig. 8. Apex of a very small tuber, after germination for fifteen days. l_i=the first, and in this case the only leaf. a=apex of new tuber. (×20.)

Fig. 9. A similar specimen more advanced. l_i=first leaf, t=new tuber, a=apex of new tuber, r=root. (×20.)

Fig. 10. Apex of tuber germinated for thirteen days, showing a rather unusual disposition of parts, viz., two leaves of equal size (l), apex of new tuber (a), and a small outgrowth (l ?), which may be a young leaf. (×20.) A, seen from above; B, lateral view.

Fig. 11. Apex of tuber germinated thirteen days: l_i=first leaf, l_{ii}=pair of subsequent leaves, a=apex of new tuber. (×20.) A, seen from above; B, lateral view.

Fig. 12. Apex of a similar tuber germinated ten days. (×20.)

Fig. 13. Apex of a strong tuber which has, on germination, formed a first leaf (l_i), and two subsequent pairs (l_{ii}), (l_{iii}), a=apex of new tuber. (×20.)

Fig. 14. Apex of a strong tuber after germination thirteen days: l=the four leaves, a=apex of new tuber. (×20.) A, lateral view; B, seen from above.

Fig. 15. Outline of a median longitudinal section of a germinating plant similar to that represented in fig. 13.

Fig. 16. A, frontal aspect of a plant germinated for twenty days; B, lateral view of the same. r=root. (×20.)

Fig. 17. A more advanced plant with three leaves (l), new tuber (t_2) with the apex at (a), sh=sheath.

Fig. 18. A plant after germination for three months. l=ordinary leaves, l^*= last formed smaller leaf, t_1=original tuber, t_2=new tuber, r=root, sh=sheath. (×about 5.)

Fig. 19. A similar plant. (×5.)

Fig. 20. A plant germinated three-and-a-half months. l=two leaves, in this case connate. (×about 5.)

N.B.—All the figures 8–20 represent plants of purely vegetative development which would not produce sporangia.

Fig. 21. A young plant seen from above, and showing two connate roots; ap= apex of sporangium-bearing axis, which has formed two leaves (l_1). (×20.)

Fig. 22. Young plant germinated thirteen days, and developing a sporangium-bearing axis. A, as seen from above; B, lateral aspect. (×20.)

Fig. 23. Young plant rather more advanced, showing leaves (l) and sporangium-bearing axis, but no sign of a new tuber. (×20.)

PLATE 72.

Fig. 24. A more advanced plant, showing, in addition to the parts seen in fig. 23, a new tuber (t_2), and two roots (r). (×20.)

Fig. 25. A plant germinated about five weeks, showing the same parts as fig. 24. c=sporangium-bearing cone. (×5.)

Fig. 26. A mature plant, showing the arrangement of parts. l=leaves, r=roots, t_1=old tuber, t_2=new tuber, a=sporangium-bearing axis. (×3.)

Fig. 27. A mature plant with similar parts, with the addition of the supernumerary leaf (l).

Fig. 28. Median longitudinal section of a new tuber in its first stage of development, compare fig. 15. The depression of the surface at (a) is already apparent. (×175.)

Fig. 29. A similar section through an older tuber; the depression at a is deeper, and the channel above it is already beginning to be closed up. (×175.)

Fig. 30. A similar section through a still older tuber; the channel is here completely closed. (×325.)

Fig. 31. An older tuber drawn under low power (20) to show its general outline.

Fig. 32. Apex of a similar tuber, with the completely closed channel. (×325.)

Fig. 33. Opening of the channel of same tuber as fig. 32, at the external surface. (×130.)

Fig. 34. Median longitudinal section of a root in early stage of development. Compare fig. 16. (×175.)

PLATE 73.

Fig. 35. A rather older root. The periclinal divisions in superficial cells are very regular. (×175.)

Fig. 36. Transverse section of a mature leaf. (×130.)

Fig. 37. Median longitudinal section of a mature leaf. (×130.)

Fig. 38. Two elements from a longitudinal section of the mature leaf. (×325.)

Fig. 39. Central part of a transverse section of a mature peduncle. ii=intercellular spaces in the xylem. (×175.)

Fig. 40. Part of a transverse section of the base of the leaf, showing peg-like projections into the intercellular spaces. (×325.)

Fig. 41. Part of a transverse section of a root, showing the vascular tissue, surrounded by an ill-defined bundle sheath (*b. sh.*).

Fig. 42A, B, vascular systems of two plants bearing no sporangia. l, l, bundles from leaves. t, bundle running to tuber. r, bundle running to root.

Fig. 43A, B, vascular systems of sporangium-bearing plants. ax=bundle running down the stem.

XIII. *Results deduced from the Measures of Terrestrial Magnetic Force in the Horizontal Plane, at the Royal Observatory, Greenwich, from* 1841 *to* 1876.

By Sir G. B. Airy, *K.C.B., F.R.S., late Astronomer Royal.*

Received June 24,—Read November 26, 1885.

[Plate 74.]

In offering to the Royal Society some results deduced from the systems of magnetic observation and magnetic self-registration established several years since at the Royal Observatory, Greenwich, during a portion of the time in which I presided over that institution, I think it desirable to premise a short statement on the origin of the Magnetic Department of the Royal Observatory, and on the successive steps in its constitution.

It appears to have been recognised many years ago, that magnetic determinations would form a proper part of the business of the Royal Observatory. When I commenced residence at the Royal Observatory, at the end of 1835, I found in the garden a small wooden building, evidently intended for the examination of compasses, perhaps of the size of those used in the Royal Navy. But the locality was inconvenient, and the structure was totally unfit for any delicate magnetic purpose; for instance, the balance-weights of the sliding windows were of iron. For some preliminary experiments a small observatory was borrowed from Captain Fitzroy, but no real progress was made in magnetism.

In the beginning of 1836, a scheme for the erection of a Magnetical Observatory was brought before the Board of Visitors. The Board approved the plan, and recommended it favourably to the Admiralty. The Government Department superintending the Park gave their consent to an extension of the grounds of the Observatory, and the ground was inclosed in 1837. The Magnetic Observatory was built, from my plans, in the spring of 1838. Since that time, no alteration has been made in

the building, except in 1864, when the ground below the east, west, and south arms, was excavated, in order to obtain positions for the three fundamental instruments in which the severity of the temperature-changes would be much diminished. Small accidental interruptions of observations occurred in 1847, January, and 1861, July.

The interest taken in the subject of terrestrial magnetism in the first half of this century was occasioned principally by the enterprise of Gauss and other German philosophers. Magnets were, therefore, established at the Royal Observatory, furnished with apparatus adapted to eye-observations corresponding to those of Gauss, and some observations were made in concert with the Germans. The observations to the end of 1847 with these instruments were made entirely by eye; the instruments (magnets 2 feet in length) being furnished with small plane reflectors, to which telescopes were directed, and by which fixed marks were observed. The observations were made at every two hours, day and night; proper precautions were taken for assurance of the general accuracy of the times of observation; and I do not doubt that the results interpreted from these observations are each as good as those derived from the succeeding system; though the intervals of two hours were longer than I could wish. But the labour was great, and (as measured by the interruption of assistants' work) was expensive.

The idea of self-registration by photography of the movements of the instruments (an idea little entertained before that time) then suggested itself; and, at the Cambridge Meeting of the British Association in 1845, it was proposed for consideration of the Council of that body, that the Government should be requested to promote, by offer of a pecuniary reward, the construction of a photographic self-registering instrument. This proposal was adopted by the Council; letters were addressed by Sir John Herschel, President of the Association, to Her Majesty's Treasury, and by myself to the Admiralty; and, finally, the assistance of Dr. Charles Brooke was secured, for forming an efficient apparatus, and making the necessary chemical arrangements adapted to our wants.

I do not propose here to describe the photographic recording apparatus. Allusions to the construction will be found in the Introductions to the Greenwich Observations for successive years, and especially, and in great detail, in the introduction to the volume for 1847. The only alteration that was made in it for several years is the following. Mr. Brooke had conceived that advantage would be gained by making the recording barrel to revolve in twelve hours. But this caused a doubling of the

curves traced on the photographic paper which is wrapped upon the barrel; and the inconvenience produced by this doubling was soon found to be so great that I thought it necessary to alter the clock-work so as to produce a revolution of the barrel in twenty-four hours. The records of the change of western declination from the north, and of the change of horizontal force, are made on the same barrel; and by alterations, first suggested by myself about 1881, and carried out by the present Astronomer Royal (then Chief Assistant), the two curves are now so traced that the simultaneous records of the two instruments at all times are in close juxtaposition.

While the observations were made by eye, at every two hours, the mean of the two-hourly readings was adopted as base for the day, and the excess of each two-hourly reading above the mean was adopted as "magnetic inequality" of that ordinate for that hour; producing twelve measures of "inequality" for each day. When the photographic system was introduced, the elevation of a pencil curve drawn by eye so as to smooth down the irregularities of the photographic trace above a photographic base was measured for every hour, producing twenty-four measures of "inequality."

In the instances of excessive and rapid disturbances of the magnets during magnetic storms, no measures of ordinates were taken for the present purposes.

Thus the daily measures at each hour or two hours were obtained.

The next step was to collect for each month all the daily measures on corresponding hours through each month, and to take their mean. These are the measures for the hours which are actually treated in the present memoir. By combining (for each month) the inequality of magnetic horizontal force at every two hours or each hour, as abscissa, with the inequality of magnetic declination (on the same scale of measure) at the same two hours or hour, as ordinate, points were defined in every monthly curve representing completely the mean diurnal changes of magnetism for each month. On the recommendation of the Board of Visitors of the Royal Observatory, reduced photographic copies of these curves were prepared by the Astronomer Royal for publication with the volume of Greenwich Observations for 1884.

The number, and the character, of the curves produced uninterruptedly on this plan, and the circumstance that they are intended for publication in the Greenwich Observations, appear to render them unfit for dissemination in the Royal Society's

Transactions. I have, therefore, decided on the following course. With the permission of the Astronomer Royal, I have adopted the three years 1863, 1864, and 1865, for partial exhibition of results. (Any other years would have answered equally well, for general exhibition.) For each of these years I have attached to this paper the curves for the months January, April, July, October, which suffice for showing generally the characteristic changes of magnetism for the several months. But some general account may be given, for which this is perhaps a suitable place.

The form of the curves, and the position of the points on them corresponding to hours of solar time, leave no doubt that the diurnal inequality is due mainly—and, as far as I can judge, entirely—to the radiant heat of the sun; and, it would seem, not to the sun's heat on the earth generally, but to its heat on parts of the earth not very distant from the magnets. In the hot months of the year, the curve, though far from circular, surrounds the central point in a form which, as viewed from that central point, never crosses itself; and is, roughly speaking, usually symmetrical with regard to E. and W. But in the cold months, the space included in the curve is much smaller; in many cases, probably not more than one-sixth of what it is in the summer months; and the curve often crosses itself in the most bizarre fashion with irregular loops stretching out, three crossings in one curve occurring very frequently. In the summer months there is a certain degree of symmetry; but here is, constantly, a preponderance on the west side, which leads me to imagine that the magnetic effect of the radiant heat upon the sea is considerably greater than the effect on the land.

To obtain some numerical basis for a report, which though exceedingly imperfect may convey some ideas on this wonderful subject, I have adopted the following course. I have confined myself to the months of June and July as probably the two hottest, and the months of December and January as probably the two coldest. For each of the curves applying to these months, I have laid down a system of rectangular co-ordinates, corresponding to the Greenwich meridian and to the line at right angles to the meridian (or the geographical E. and W.). The extreme north ordinate and the extreme south ordinate are measured, and their sum is taken, and interpreted by a scale of measure formed in accordance with the theory of the instruments; and this interpreted sum forms the "Range of Meridian Force" in terms of the whole Meridian Force. In the same manner, the "Range of Transversal Force" is measured. As the time of each of the two-hourly or hourly records

is marked on the curve, there is no difficulty in fixing approximately on the solar times corresponding to the extreme N. and S. values, and the extreme E. and W. values, mentioned above. These are all the elements of the magnetic record which are described in the subjoined table.

The changes in the monthly records are very remarkable. They leave no doubt in my mind that the diurnal magnetic changes are produced by the sun. But I cannot account for every change that takes place in the course of a day; nor can I undertake to say whether we can found, on these, the theory that general terrestrial magnetism is a part of solar radiation, perhaps sometimes acting through or sometimes impeded by the masses of land and sea on which that radiation acts.

Still I think that a considerable step is made by the establishment of a connexion between terrestrial magnetism (on one hand), and the radiation, or, at least, the visibility of the sun (on the other hand).

Monthly Means of Daily Range of Horizontal Magnetic Force at

Mean Daily Range of Horizontal Force in the Direction of Astronomical Meridian.

Year.	Month.	Range of Meridian Force.	In Göttingen Mean Solar Time. Approximate Hour of Minimum Meridian Force.	In Göttingen Mean Solar Time. Approximate Hour of Maximum Meridian Force.	Month.	Range of Meridian Force.	In Göttingen Mean Solar Time. Approximate Hour of Minimum Meridian Force.	In Göttingen Mean Solar Time. Approximate Hour of Maximum Meridian Force.
			h. m.	h. m.			h. m.	h. m.
1841	June . . .	·0032	23 50	8 0				
	July . . .	·0033	23 50	8 0				
					December . .	·0012	2 0	18 0
1842	June . . .	·0022	0 0	12 0	January . .	·0009	23 50	20 0
	July . . .	·0028	0 0	9 0				
					December . .	·0010	0 0	18 0
1843	June . . .	·0027	23 20	8 30	January . .	·0014	0 0	18 0
	July . . .	·0028	23 50	8 0				
					December . .	·0011	2 0	12 0
1844	June . . .	·0029	23 20	8 0	January . .	·0013	0 0	7 30
	July . . .	·0030	23 40	8 0				
					December . .	·0016	0 10	6 0
1845	June . . .	·0025	22 40	8 0	January . .	·0010	0 0	6 0
	July . . .	·0022	23 30	8 0				
					December . .	·0015	0 0	9 0
1846	June . . .	·0030	22 40	7 50	January . .	·0011	0 20	19 0
	July . . .	·0033	23 10	7 50				
					December . .	·0016	0 0	8 10
1847	June . . .	·0026	23 40	8 0	January . .	Observations interrupted.		
	July . . .	·0032	23 30	8 0				
					December . .	·0015	0 40	20 0
1848	June . . .	·0033	0 30	8 0	January . .	·0023	0 10	18 30
	July . . .	·0045	0 0	8 0				
					December . .	·0018	0 30	19 0
1849	June . . .	·0035	1 0	7 10	January . .	·0019	0 0	19 0
	July . . .	·0036	0 0	8 0				
					December . .	·0013	0 0	19 0
1850	June . . .	·0028	0 0	19 0	January . .	·0015	0 10	18 0
	July . . .	·0026	23 50	8 0				
					December . .	·0019	23 40	19 0
1851	June . . .	·0027	0 0	8 0	January . .	·0021	1 0	19 0
	July . . .	·0026	0 0	9 0				
					December . .	·0015	4 50	18 0
1852	June . . .	·0029	0 15	18 10	January . .	·0021	1 10	20 0
	July . . .	·0028	1 0	17 40				
					December . .	·0017	2 0	18 30

Göttingen Mean Solar Time is greater by 40^{m} than

Greenwich expressed as factors of the Mean Horizontal Magnetic Force.

Mean Daily Range of Horizontal Force in the Direction Transversal to the Astronomical Meridian.

Month.	Range of Transversal Force.	In Göttingen Mean Solar Time. Approximate Hour of		Year.
		Maximum Easterly Force.	Maximum Westerly Force.	
		h. m.	h. m.	
June . . .	·0036	20 0	2 0	1841
July . . .	·0027	20 0	3 0	
December . .	·0016	10 0	2 0	
January . .	·0019	12 0	1 50	1842
June . . .	·0026	20 10	3 0	
July . . .	·0029	20 10	2 10	
December . .	·0016	10 0	2 0	
January . .	·0018	12 0	2 0	1843
June . . .	·0035	20 30	2 10	
July . . .	·0036	20 20	2 30	
December . .	·0014	10 0	2 0	
January . .	·0016	14 0	1 50	1844
June . . .	·0033	20 0	2 15	
July . . .	·0029	20 0	2 0	
December . .	·0014	12 0	2 10	
January . .	·0020	14 0	1 50	1845
June . . .	·0032	20 0	2 20	
July . . .	·0036	20 0	2 40	
December . .	·0015	10 0	2 0	
January . .	·0018	10 20	1 50	1846
June . . .	·0033	20 0	2 10	
July . . .	·0038	20 20	3 0	
December . .	·0015	12 0	2 0	
January . .	Observations interrupted.			1847
June . . .	·0034	19 50	3 30	
July . . .	·0038	19 30	2 20	
December . .	·0026	14 0	2 0	
January . .	·0028	10 0	2 20	1848
June . . .	·0037	20 30	2 10	
July . . .	·0041	20 50	2 20	
December . .	·0019	13 20	2 0	
January . .	·0022	11 0	2 0	1849
June . . .	·0041	20 0	3 0	
July . . .	·0039	20 20	2 40	
December . .	·0015	12 0	1 30	
January . .	·0022	10 40	2 30	1850
June . . .	·0034	20 0	2 0	
July . . .	·0034	20 30	2 0	
December . .	·0012	9 0	1 0	
January . .	·0015	10 0	2	1851
June . . .	·0027	20 20	2 50	
July . . .	·0029	20 40	2 10	
December . .	·0017	9 40	1 0	
January . .	·0015	11 0	2 0	1852
June . . .	·0038	21 0	2 0	
July . . .	·0022	21 0	2 10	
December . .	·0015	9 40	2 10	

Greenwich Mean Solar Time for the same instant.

Monthly Means of Daily Range of Horizontal Magnetic Force at Greenwich

Mean Daily Range of Horizontal Force in the Direction of Astronomical Meridian.

Year.	Month.	Range of Meridian Force.	In Göttingen Mean Solar Time. Approximate Hour of Minimum Meridian Force.	In Göttingen Mean Solar Time. Approximate Hour of Maximum Meridian Force.	Month.	Range of Meridian Force.	In Göttingen Mean Solar Time. Approximate Hour of Minimum Meridian Force.	In Göttingen Mean Solar Time. Approximate Hour of Maximum Meridian Force.
			h. m.	h. m.			h. m.	h. m.
1853	June . . .	·0031	22 50	9 50	January . .	·0019	1 10	20 0
	July . . .	·0026	23 10	7 0	December . .	·0020	3 0	19 0
1854	June . . .	·0021	23 40	8 0	January . .	·0019	1 0	19 0
	July . . .	·0024	0 0	18 0	December . .	·0012	23 10	19 0
1855	June . . .	·0020	0 0	18 0	January . .	·0014	0 0	19 0
	July . . .	·0022	1 0	18 0	December . .	·0015	2 0	19 0
1856	June . . .	·0023	23 10	17 30	January . .	·0014	1 0	19 0
	July . . .	·0020	0 20	19 0	December . .	·0013	2 0	20 0
1857	June . . .	·0027	0 0	18 0	January . .	·0020	0 30	20 0
	July . . .	·0025	0 0	18 0	December . .	·0020	0 0	19 0
			In Greenwich Mean Solar Time.				In Greenwich Mean Solar Time.	
1858	June . . .	·0037	23 50	16 30	January . .	·0024	1 0	18 0
	July . . .	·0033	23 0	16 40	December . .	·0023	0 0	18 0
1859	June . . .	·0032	0 0	17 0	January . .	·0023	0 0	17 30
	July . . .	·0031	1 0	16 0	December . .	·0023	0 0	18 0
1860	June . . .	·0034	22 40	6 30	January . .	·0026	1 0	17 0
	July . . .	·0035	23 50	13 0	December . .	·0020	0 0	17 0
1861	June . . .	·0025	0 10	7 0	January . .	·0021	0 0	18 0
	July . . .	Observations interrupted.			December . .	·0021	0 0	18 0
1862	June . . .	·0028	22 50	8 0	January . .	·0014	0 0	17 0
	July . . .	·0024	20 0	6 0	December . .	·0014	0 0	18 0
1863	June . . .	·0033	23 0	6 0	January . .	·0014	23 0	17 0
	July . . .	·0028	22 40	7 30	December . .	·0013	0 0	10 30
1864	Observations interrupted during construction of the Magnetic Basement.							

Göttingen Mean Solar Time is greater by 40m than

expressed as factors of the Mean Horizontal Magnetic Force (continued).

Mean Daily Range of Horizontal Force in the Direction Transversal to the Astronomical Meridian.

Month.	Range of Transversal Force.	In Göttingen Mean Solar Time. Approximate Hour of Maximum Easterly Force.	Maximum Westerly Force.	Month.	Range of Transversal Force.	In Göttingen Mean Solar Time. Approximate Hour of Maximum Easterly Force.	Maximum Westerly Force.	Year.
		h. m.	h. m.			h. m.	h. m.	
June . . .	·0032	21 0	3 0	January . .	·0014	12 0	2 0	1853
July . . .	·0030	19 40	2 50	December . .	·0015	10 20	20 0	
June . . .	·0027	20 50	2 20	January . .	·0019	9 0	1 0	1854
July . . .	·0026	22 0	3 0	December . .	·0013	10 0	1 0	
June . . .	·0021	21 0	2 10	January . .	·0015	12 0	2 30	1855
July . . .	·0017	19 50	2 0	December . .	·0012	9 20	1 0	
June . . .	·0022	21 0	2 10	January . .	·0010	7 0	18 30	1856
July . . .	·0022	21 0	2 15	December . .	·0013	9 0	1 3	
June . . .	·0013	21 0	15 0	January . .	·0014	11 0	0 0	1857
July . . .	·0011	6 20	2 0	December . .	·0012	9 40	1 30	
		In Greenwich Mean Solar Time.				In Greenwich Mean Solar Time.		
June . . .	·0022	20 40	2 0	January . .	·0012	9 30	1 0	1858
July . . .	·0028	20 0	2 10	December . .	·0014	9 30	2 0	
June . . .	·0036	19 30	2 20	January . .	·0013	9 20	1 30	1859
July . . .	·0032	20 0	2 10	December . .	·0014	10 0	1 30	
June . . .	·0041	19 30	1 30	January . .	·0012	9 0	2 0	1860
July . . .	·0038	19 0	2 20	December . .	·0018	10 0	2 0	
June . . .	·0034	19 30	2 30	January . .	·0015	10 0	0 30	1861
July . . .	Observations interrupted.			December . .	·0019	10 0	1 0	
June . . .	·0037	19 40	2 0	January . .	·0020	12 0	1 50	1862
July . . .	·0030	20 0	3 30	December . .	·0011	13 0	1 30	
June . . .	·0033	19 50	2 20	January . .	·0026	10 0	1 0	1863
July . . .	·0030	20 0	2 0	December . .	·0020	10 0	1 0	
Observations interrupted during construction of the Magnetic Basement.								1864

Greenwich Mean Solar Time for the same instant.

MONTHLY Means of Daily Range of Horizontal Magnetic Force at Greenwich

Mean Daily Range of Horizontal Force in the Direction of Astronomical Meridian.

Year.	Month.	Range of Meridian Force.	In Greenwich Mean Solar Time. Approximate Hour of Minimum Meridian Force.	In Greenwich Mean Solar Time. Approximate Hour of Maximum Meridian Force.	Month.	Range of Meridian Force.	In Greenwich Mean Solar Time. Approximate Hour of Minimum Meridian Force.	In Greenwich Mean Solar Time. Approximate Hour of Maximum Meridian Force.
			h. m.	h. m.			h. m.	h. m.
1865	June . . .	·0025	23 30	11 0	January . .	Observations interrupted.		
	July . . .	·0024	22 10	7 0				
					December . .	·0009	23 10	18 0
1866	June . . .	·0020	0 0	7 30	Janaary . .	·0011	0 0	19 0
	July . . .	·0022	22 10	7 0				
					December . .	·0008	23 0	18 0
1867	June . . .	·0024	23 0	7 0	January . .	·0010	23 20	8 0
	July . . .	·0022	23 0	9 0				
					December . .	·0009	23 0	18 0
1868	June . . .	·0025	23 0	7 0	January . .	·0007	0 0	19 0
	July . . .	·0026	0 0	8 0				
					December . .	·0010	23 20	18 0
1869	June . . .	·0036	23 0	7 0	January . .	·0014	23 50	19 0
	July . . .	·0033	22 30	7 0				
					December . .	·0012	23 0	17 0
1870	June . . .	·0041	23 0	7 0	January . .	·0016	23 40	17 0
	July . . .	·0037	23 0	7 0				
					December . .	·0017	0 30	18 0
1871	June . . .	·0035	0 0	7 0	January . .	·0019	0 0	11 0
	July . . .	·0036	23 40	7 0				
					December . .	·0017	23 30	18 0
1872	June . . .	·0037	23 0	7 0	January . .	·0017	0 0	18 0
	July . . .	·0031	0 10	7 0				
					December . .	·0010	0 0	18 0
1873	June . . .	·0027	23 30	7 10	January . .	·0017	0 0	18 0
	July . . .	·0031	22 30	6 50				
					December . .	·0008	23 20	17 30
1874	June . . .	·0024	23 0	6 20	January . .	·0011	0 0	18 0
	July . . .	·0023	23 0	7 30				
					December . .	·0007	0 0	18 30
1875	June . . .	·0019	23 0	7 20	January . .	·0007	23 0	18 30
	July . . .	·0021	23 0	7 30				
					December . .	·0008	23 20	20 0
1876	June . . .	·0018	22 30	8 0	January . .	·0009	0 0	19 0
	July . . .	·0020	23 30	7 0				
					December . .	·0006	0 0	19 0

expressed as factors of the Mean Horizontal Magnetic Force (continued).

Mean Daily Range of Horizontal Force in the Direction Transversal to the Astronomical Meridian.

Month.	Range of Transversal Force.	In Greenwich Mean Solar Time. Approximate Hour of		Month.	Range of Transversal Force.	In Greenwich Mean Solar Time. Approximate Hour of		Year.
		Maximum Easterly Force.	Maximum Westerly Force.			Maximum Easterly Force.	Maximum Westerly Force.	
		h. m.	h. m.			h. m.	h. m.	
				January . .	Observations interrupted.			1865
June . . .	·0032	20 0	2 0					
July . . .	·0028	18 50	2 0					
				December . .	·0015	10 50	1 0	
				January . .	·0019	10 40	1 0	1866
June . . .	·0029	19 30	2 30					
July . . .	·0029	19 30	2 10					
				December . .	·0013	9 0	1 0	
				January . .	·0013	10 20	1 0	1867
June . . .	·0027	20 0	1 40					
July . . .	·0026	20 0	2 0					
				December . .	·0012	9 0	1 0	
				January . .	·0016	10 0	1 0	1868
June . . .	·0029	20 0	2 0					
July . . .	·0030	19 30	2 0					
				December . .	·0017	10 0	1 30	
				January . .	·0018	12 10	2 0	1869
June . . .	·0042	19 50	2 10					
July . . .	·0038	19 30	2 30					
				December . .	·0015	10 30	1 0	
				January . .	·0020	10 20	1 0	1870
June . . .	·0046	20 10	2 10					
July . . .	·0047	20 0	2 10					
				December . .	·0017	10 40	2 0	
				January . .	·0021	11 10	2 0	1871
June . . .	·0046	20 10	2 20					
July . . .	·0043	20 0	2 30					
				December . .	·0019	12 0	1 30	
				January . .	·0023	11 10	2 0	1872
June . . .	·0044	20 0	2 0					
July . . .	·0038	20 0	2 10					
				December . .	·0022	11 0	1 0	
				January . .	·0026	10 30	2 0	1873
June . . .	·0032	20 30	2 50					
July . . .	·0036	20 0	2 20					
				December . .	·0017	11 0	1 0	
				January . .	·0021	11 0	1 0	1874
June . . .	·0032	19 50	2 0					
July . . .	·0033	20 20	2 0					
				December . .	·0012	10 30	1 0	
				January . .	·0014	10 50	1 0	1875
June . . .	·0027	19 40	2 0					
July . . .	·0024	20 0	2 10					
				December . .	·0012	10 0	1 0	
				January . .	·0014	10 0	1 0	1876
June . . .	·0029	19 30	2 30					
July . . .	·0030	19 20	2 0					
				December . .	·0012	9 30	1 0	

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XIV. *On Radiant Matter Spectroscopy.*—Part II. *Samarium.*

By William Crookes, *F.R.S.*

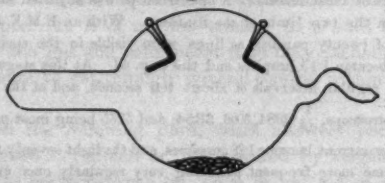
Received May 21,—Read June 18, 1885.

[Plate 75.]

Introduction.

90. In the concluding sentence of the Bakerian Lecture which I had the honour to deliver before the Royal Society, May 31, 1883, I said that the new method of Radiant Matter Spectroscopy there described had given me not only spectrum indications of the presence of yttrium as an almost invariable, though very minute, constituent of a large number of minerals, but had likewise revealed signs of another spectrum-yielding element. I stated that I had repeatedly seen indications of another very beautiful spectrum characterised by a strong red and a double orange band. That this second spectrum was not then new to me is shown by a paper sent to the Royal Society in 1881,* in which I described a double orange band occurring in the phosphorescent spectrum of an earth less frequently met with than the "pale yellowish coloured earth" (since identified as yttria) which gave me the "red, orange, citron, and green bands."

Fig. 1.



91. The method adopted to bring out the orange band is to treat the substance under examination with strong sulphuric acid, drive off excess of acid by heat, and finally to raise the temperature to dull redness. It is then put into a radiant matter tube, of the form shown in fig. 1, and the induction spark is passed through it after the exhaustion has been pushed to the required degree. The anhydrous sulphate thus

* Proc. Roy. Soc., vol. xxxii. (1881), p. 206.

left frequently shows the orange band in the radiant matter tube, though before this treatment the original substance shows nothing.

The Elimination of Mercury Vapour from Vacuum Tubes.

92. The presence of mercury vapour is liable to interfere with the spectrum indications, and when working in an entirely new field, where the mutual reaction of bodies one on the other is the subject of the research, it is always desirable to reduce the unavoidable constants to a minimum. In a tube exhausted in the ordinary manner by the SPRENGEL pump more mercury vapour is present than is generally supposed. Unless the tube is exhausted and sealed off without loss of time, mercury diffuses through the connecting tubes, and soon gets into the vessel under experiment. It is not easily detected by the spectroscope so long as the tube remains cold, but when the passage of the current has warmed the glass and the metal poles the mercury lines become distinctly visible, and the same result is obtained at once when the tube is heated by a lamp.*

The following experiment illustrates this behaviour of mercury:—A small radiant matter tube, having flat aluminium poles connected to platinum terminals passing

* In incandescent lamps exhausted by means of a mercury pump, a peculiar blue phosphorescence is observed to fill the globe when the filament is raised much above its normal incandescence. This phenomenon has been examined and discussed by Mr. EDISON in America and by Mr. PREECE in England (Proc. Roy. Soc., vol. xxxviii., No. 236, p. 219). During the course of my work on the manufacture of incandescent lamps I have frequently noticed the same effect. Examined in the spectroscope the phosphorescent light filling the globe shows the mercury lines strongly. In order to examine this phenomenon more closely the following experiments were tried:—An incandescent lamp with fine carbon filament which had been plated up in vapour of carbon tetrachloride was placed in one of the sides of a Wheatstone bridge and connected with a dynamo-electric machine, where by means of a suitable rheostat it could be raised to any desired degree of incandescence. A spectroscope was adjusted so that the slit was opposite the rarefied space between the two limbs of the filament. With an E.M.F. of 64·5 volts, a current of 0·97 ampère, and a light of twenty candles, no lines were visible in the spectrum; on increasing the volts to 73·6 the current became 1·15 ampères, and the c. p. 43. At this stage flashes of blue phosphorescence filled the bulb at regular intervals of about ten seconds, and at the same instant the mercury lines were seen in the spectroscope, $\frac{1}{\lambda^2}$ 2984, 3006, 3354, and 5265 being most prominent. On increasing the E.M.F. to 81·3 volts, the current became 1·28 ampères, and the light seventy-four candles, the flashes of phosphorescent light became more frequent, pulsating very regularly once every three seconds, and at each flash showing the mercury spectrum strongly. The E.M.F. was next increased to 99·7 volts, and the ampères to 1·83; at this point the light was equal to 172 candles. The flashes of light became more and more frequent, rising quickly from once in two seconds till they occurred too rapidly to count, and the blue phosphorescence became continuous and seemed to fill the globe. The mercury spectrum at this stage was permanent, and most of its lines could be detected. The flashes of light were probably owing to some such action as this:—the hot filament offering a considerable resistance to the passage of the current, the long length of insulated wire in the circuit (about half a mile, including the lamps used for house-lighting) charged up like a condenser and then at intervals discharged through the highly rarefied vapour in the bulb.

through the glass, was sealed to the pump and quickly exhausted. An induction current was immediately passed through, and the light examined in the spectroscope. No mercury lines were detected. The tube was then heated strongly, the spectroscopic examination being continued: no mercury lines were seen. The apparatus was then allowed to rest for a day, and the experiments were repeated. In the cold no mercury lines could be seen, but on heating the tube they became distinctly visible.

93. The tube was then exhausted to the highest possible point, and sealed off. In the cold it was impossible to get an induction-spark through the tube—it was absolutely non-conducting. However, on heating the tube with a BUNSEN flame, keeping the coil going all the time, suddenly the current passed, lighting up the inside of the tube with a greenish-blue light, in which the spectroscope showed strong mercury lines. On allowing the tube to cool it again became non-conducting, and the experiment could be repeated.

This shows that in a vacuum-tube which is so highly exhausted as to be non-conducting there is plenty of mercury present to carry an induction current, only it is not in the form of vapour, but is condensed on the metallic poles or on the sides of the glass.

94. It is much more difficult than is generally supposed to keep mercury vapour from diffusing itself into the experimental tubes. Gold-leaf was packed loosely in a tube about 2 feet long, which was then interposed between the pump and the radiant matter tube. This answered for a short time, but whilst it certainly kept back some of the mercury, it let much through.

For a long time I used a tube about 2 feet long, packed with coarsely powdered sulphur. This is effectual at first, but there is a liability of sulphur vapour getting into the tube, and this might be as disastrous as mercury. The sulphur was therefore kept out by interposing another tube containing finely divided copper. This, however, was not quite effectual: after a time mercury was found to have run the gauntlet of both sulphur and copper. It is of no use trusting to a solid reagent to absorb a gaseous body. Where the gas and solid meet there may be action, but some of the gaseous molecules are sure to get through without coming within the solid's sphere of attraction.

95. I finally adopted the following plan, which answers perfectly so far as my experiments have yet gone:—Sulphur is first prepared by keeping it fused at a high temperature till bubbles cease to come off, so as to get rid of water and hydrogen compounds. It is then allowed to cool, and is then pounded and sifted so as to get it in the form of granules averaging a millimètre in diameter. A glass tube, a centimètre in diameter and about 2 feet long, is lightly packed for half its length with this sulphur, and next about 2 inches of iodide of sulphur (I_2S_2) is added, and the rest of the tube is then filled up with sulphur. Ignited asbestos is packed in at each end to keep the sulphur from blowing out whilst the vacuum is being made, or from being sucked through when air is suddenly let in. This contrivance entirely

keeps mercury vapour from passing through, since the iodide of sulphur holds its iodine very loosely, and fixes the mercury in the form of non-volatile red iodide. A glass tube containing finely divided copper must follow in order to keep sulphur out. With this blockade interposed between the pump and experimental tubes I have been unable to detect mercury vapour in any of the tubes, whether in the cold or on heating them.

Mixed "Citron" and "Orange" Spectra.

96. Since the date of my last paper I have devoted myself to the task of solving the problem presented by the double orange band first observed in 1881. With the yttrium experience as a guide it might be thought this would not be a difficult task, but in truth it helped me little beyond increasing my confidence that the new like the old spectrum was characteristic of an element. The extreme sensitiveness of the test is a drawback rather than a help. To the inexperienced eye one part of "orange band" substance in ten thousand gives as good an indication as one part in ten, and by far the greater part of the chemical work undertaken in the hunt for the spectrum-forming element has been performed upon material which later knowledge shows does not contain sufficient to respond to any known chemical test. It is as if the element sodium were to occur in ponderable quantity only in a few rare minerals seldom seen out of the collector's cabinet. With only the yellow line to guide, and seeing the brilliancy with which an imponderable trace of sodium in a mineral declares its presence in the spectrum, I venture to think that a chemist would have about as stiff a hunt before he caught his yellow line as I had to bring my orange and citron bands to earth.*

Chemistry, except in few instances, as water-analysis and the detection of poisons, where necessity has stimulated minute research, takes little account of "traces;" and when an analysis adds up to 99·99, the odd 0·01 per cent. is conveniently put down to "impurities," "loss," or "errors of analysis." When, however, the 99·99 per cent. constitutes the impurity and this exiguous 0·01 is the precious material to be extracted, and when, moreover, its chemistry is absolutely unknown, the difficulties of the problem become enormously enhanced. Insolubility as ordinarily understood is a fiction, and separation by precipitants is nearly impossible. A new chemistry has to be slowly built up, taking for data uncertain and deceptive indications, marred by the interfering power of mass in withdrawing soluble salts from a solution, and the solubility of nearly all precipitates in water or in ammoniacal salts when present in traces only. What is here meant by "traces" will be better understood if I give an instance. After six months' work I obtained the earth didymia in a state which most chemists would call absolutely pure, for it contained not more than one part of

*So recently as the year 1880 a high authority on blowpipe chemistry seriously argued that the yellow coloured flame "which is ascribed to sodium" is not caused by that metal, but is due to "chemical water." ('Chemical News,' vol. xli., p. 159, April 2, 1880.)

impurity in five hundred thousand parts of didymia (131). But this one part in half a million profoundly altered the character of didymia from a radiant-matter-spectroscopic point of view, and the persistence of this very minute quantity of interfering impurity entailed another six months' extra labour to eliminate these final "traces" and to ascertain the real reaction of didymia pure and simple (131).

For a long time the "citron-band" and the "orange-band" spectra were confounded. That they were due to two different states or kinds of matter was not easily decided, since in all the early experiments I was dealing with a mixture; consequently the spectra obtained were not only mixed but differed considerably in the relative intensities and faintness of the different lines (146).

97. At last having separated yttria and obtained its spectrum pure (71), the characteristic lines in the other spectrum or spectra could be provisionally mapped out by difference, and a systematic hunt was instituted for the new "orange band" substance, which to avoid periphrasis was termed x. Naturally my thoughts turned to samarskite and the yttria earths. A wide, prolonged survey over every available substance had convinced me that the number of bodies giving a discontinuous phosphorescent spectrum is extremely limited, and to be counted on the fingers of one hand; and having satisfactorily mated one of these spectra to yttria it became in the highest degree probable that the twin spectrum should belong to one of the nearest chemical associates of yttria.

Chemistry of the "Orange-Band" forming Body.

98. At first it was necessary to take stock, as it were, of all the facts regarding x which had turned up during the search for the orange band. In the first place x is almost as widely distributed as yttria, generally occurring with the latter earth. Sometimes, however, the orange band was strong where the citron band was almost or quite absent. It is almost certainly one of the earthy metals, as it occurs in the insoluble oxalates, in the insoluble double sulphates, and in the precipitate with ammonia. It is not precipitated by sodic thiosulphate, and, moreover, it must be present in very minute quantities, since the ammonia precipitate is always extremely small, and as a rule x is not found in the filtrate from this precipitate.

99. At this stage of the enquiry the chemical reaction of x were much more puzzling than with yttria. At the outset an anomaly presented itself. The orange band was prone to vanish in a puzzling manner. Frequently an accumulation of precipitates tolerably rich in x was worked up for purposes of concentration, when the spectrum reaction suddenly disappeared, showing itself neither in precipitate nor filtrate (3, 101, 108, 115); whilst on other occasions, when following apparently the same procedure, the orange band became intensified and concentrated with no apparent loss. The behaviour of the sulphate to water was also very contradictory; on some occasions it appeared to be almost insoluble, whilst occasionally it dissolved in water readily (115).

100. For some time I debated whether the orange-band spectrum might not be a

mere modification of the yttrium spectrum induced by the presence of some extraneous body. We know that yttria *per se* has little or no phosphorescence (75), that this power chiefly resides in the ignited sulphate. Might it not happen that some other earth with molecules peculiarly sensitive to the longer vibrations would confer upon yttria some of its sensitiveness to the red end of the spectrum?

It would be too much like a repetition of my paper on the yttrium spectrum quest were I to detail the numerous experiments and false starts with samarskite, orangite, thorite, strontianite, cœlestine, perofskite, cerite, coral, &c.; but I may be permitted to extract from an enormous mass of chronicles which must remain unpublished, some few experiments which will usefully emphasize what I may call the nodal points in this research.

"X" from Samarskite.

101. It was to be expected that samarskite would contain x. It occurred, however, very little in the yttria group, but was found with the decipia residues (47, 49), or the earths forming, with potassium, insoluble double sulphates,—ceria, lanthana, didymia, decipia, samaria, together with a little thoria and zirconia. These residues were dissolved in hydrochloric acid, precipitated with ammonia, washed till free from potassic salts, re-dissolved, and precipitated as oxalates. The filtrate was set aside in Winchester quart bottles, and after standing for some weeks a further quantity of insoluble oxalates was found deposited at the bottom of the bottles. These were collected, and appeared to be very rich in x; but on attempting to work them up vexatious anomalies constantly started up: suddenly the orange band would disappear, and after being lost sight of for a week or two, would return in a manner equally unaccountable (3, 99, 108, 115).

Thorite and Orangite.

102. Early in my research thorite and orangite (26) had given a brilliant spectrum, afterwards identified with that of yttria (70). When hunting for x some of the insoluble double sulphates from these minerals (32) were treated like the samarskite double sulphates to remove potassium (101), and examined in the radiant matter tube. Here, also, was found the orange-band spectrum, quite different from the yttrium spectrum of the soluble sulphates; but, as usual, it behaved in a most capricious manner.

Perofskite.

103. An American friend, Mr. George F. Kunz, with great kindness sent me some pounds' weight of the rare mineral perofskite (calcic titanate), in fine crystals, from Magnet Cove, Arkansas, together with a large number of specimens of associated minerals from the same locality. The perofskite was found to be richer in x than any mineral yet examined. At great sacrifice of material a small portion of an earthy

body was obtained giving the orange-band spectrum more brilliantly than I ever had seen it before. Analysis failed to detect anything in it but lime (5,9), the flame spectrum showed lime, and the atomic weight came out RO = 55·3, CaO being 56.

Calcite.

104. Mr. LETTSOM, understanding I was engaged in quest of an unknown body supposed to be associated with calcium, most kindly sent me specimens of rare and curious minerals; and through his good offices Professor ALBIN WEISBACH presented me with an extensive set of calcites; these, prior to the invention of the spectroscope, had been measured by Professor A. BREITHAUPT, who, owing to the discordant measurements, held what is known as "calcium" to consist of two or more allied elements, which as yet chemists were unable to separate.

These calcites were treated as usual and examined most carefully in the radiant matter tube. In one of them only was a trace of yttria found, but the orange-band spectrum was very faintly seen in six of the thirteen specimens. The others shone with the usual greenish-blue phosphorescence of calcic sulphate, giving no lines or bands in the spectrum.

I am also indebted to Mr. LETTSOM for a specimen of calcite from Branchville, S. Carolina, which, when heated, has the curious property of glowing strongly with a golden-yellow light showing a faint continuous spectrum. In the radiant matter tube the phosphorescence was very brilliant, but there was no discontinuity in the spectrum, only a concentration of light in the red portion.

Dolomite.

105. Another curious mineral, for which I am also indebted to Mr. LETTSOM, is a granular dolomite from Utah. When scratched with a knife or struck with a pick it emits so strong a phosphorescent red light that the miners call it Hell-fire Rock. By itself in the radiant matter tube it brightly phosphoresces with a reddish light, showing no bands, but a concentration of light in the red. Treated with sulphuric acid in the usual manner, and then examined in a vacuum tube, a similar continuous spectrum was observed. Chemical analysis showed that it was a nearly pure double calcic and magnesic carbonate, with a little iron, alumina, and phosphoric acid.

Amongst other minerals found to give the orange band spectrum I may mention zircon, euxenite, tyrite, fergusonite, rhabdophane, cerussite, apatite, galliferous blende, argentiferous galena, anglesite, harmotome, allanite, cerite, magnesite, oolite from Bath, &c.

Coral.

106. In my former paper (88) I mentioned that a specimen of pink coral contained about a half per cent. of yttria, judging from the very strong yttrium spectrum it

gave in the vacuum tube. Professor MARTIN DUNCAN has identified this specimen as a *Gorgonia* of the genus *Melithæa*. Another recent coral, *Mussa sinuosa*, gave equally strong indications of yttrium. By the kindness of Professor DUNCAN I have since been enabled to submit a large number of corals to spectrum examination in the radiant matter tube. Nearly all showed more or less discontinuity in their phosphorescent spectra, but as in the yttrium spectrum research I obtained only two specimens giving a brilliant yttrium spectrum, so in the present quest I have found only two corals giving a strong orange-band spectrum. One is a *Pocillopora damicornis*, from Singapore and most of the Pacific Islands which have reefs, one of the old group of tabulate corals. A fragment of this coral, treated with sulphuric acid and examined in the radiant matter tube, gave as brilliant an orange-band spectrum as I had ever seen. The other is of the species *Symphyllia*, close to *Mussa*, a reef-builder from the same locality as the *Mussa* which gave so much yttria.

Sea-water.

107. These results induced me next to try sea-water. Ammonic oxalate and hydrate gave a white precipitate, which was filtered off and washed. The oxalate was then ignited, dissolved in nitric acid, and the solution supersaturated with ammonia and boiled. The resulting precipitate, tested in the radiant matter tube, showed the orange-band spectrum very well.

"X" in Strontium Minerals.

108. The orange-band spectrum in the radiant matter tube at first sight bore a close resemblance in the red region to the flame spectrum of strontium; the two spectra therefore were examined together, and on comparing them a near coincidence was observed between two lines in the orange. Was it possible that the sought-for element was strontium?

This led to an examination of the strontic nitrate used in the flame reaction. When converted into sulphate and tested in the radiant matter tube the experiment succeeded only too well. The orange-band spectrum came out brilliantly.

Other commercial strontium compounds were now tested. Yttria was found almost universally, but the orange band was capricious; the nitrate generally showed it well, caustic strontia sometimes, chloride as a rule not at all. These were from different makers. The source was enquired for, and in a few weeks my laboratory was filled with large specimens of Gloucestershire, Italian, and Sicilian cœlestine, and Scotch, Italian, and German strontianite, together with waste-products, mother-liquors, and every commercial salt of strontium. The kindness of the manufacturers was great, and I regret that the outcome was not more notable.

Italian cœlestine showed a good orange-band spectrum when crushed and examined in the tube without any chemical treatment. After getting the mineral into solution by fusion with sodic carbonate, &c., the x could be concentrated by fractionally precipitating with alkaline carbonates (coming down in the first fractions). The sulphate produced from this precipitate also showed the desired spectrum.

This sulphate was digested for some time in warm ammonic carbonate, and now the old distressing anomalies re-commenced. On most occasions, when working roughly on a scale of a few grammes, all the x was found in the filtrate on evaporation and ignition. When, however, I took identically the same material, and worked it up more carefully, in pounds or hundredweights, it sometimes gave nothing at all, sometimes only a *ridiculus mus* on the smallest sized filter, got from a mountain of raw material. This was at first accounted for by the want of homogeneity of the mineral. The real explanation, however, was not discovered till long after (115).

A quantitative estimation was attempted of the amount of x substance got from Italian cœlestine. 630 grms. gave 1·525 grm., or 0·24 per cent. Analysis showed this to be chiefly strontic sulphate, and the atomic weight of the metal was close to that of strontium.

Is "x" a Mixture?

109. For a considerable time strontium minerals and salts only were worked upon, these being considered the cheapest and most fruitful sources of x. A considerable quantity of material was thus accumulated, showing the desired spectrum with great brilliancy. When, however, attempts were made to separate the spectrum-forming body from the accompanying elements, as strontium, calcium, &c., all the foregoing anomalies were displayed. Ultimately two portions of substance were produced—a precipitate (113) containing the supposed new element, and a filtrate, containing the strontium, calcium, and other impurities. Neither the precipitate nor filtrate tested in the usual manner showed the orange band anything like so well as the material before such separation, and indeed at this stage of the experiments it frequently vanished altogether.

Some of the filtrate and precipitate were now mixed together, treated with sulphuric acid, and tested as before: they gave the orange-band spectrum as brightly as did the original substance. The ammonia precipitate was too small for analysis, but judging from its origin it might contain any or all of the rare earths. Chemical analysis showed nothing but a calcium salt in the filtrate.

110. Could it be that the union of two bodies was necessary to give this spectrum, and that calcium was one of these? Could the other constituent be of the nature of an acid such as boric, or a halogen like fluorine?

Many experiments were tried to test this hypothesis. Pure Iceland spar was dissolved in acid, a little of the above described precipitate added, and the mixture tested in the usual way. The orange band appeared again.

Every conceivable mixture was now made of lime with other bodies, but whilst I frequently obtained faint indications of orange band there was never sufficient to satisfy me that I had artificially formed the spectrum-bearing body; the traces observed were evidently due to the all pervading presence of the sought for body.

So far all had been contradictory and disheartening. Analogy with the yttrium results failed to throw light to guide through the gloom. The hypothesis that the body sought was an earth, widely diffused in minute quantities only, and that its anhydrous sulphate gave a phosphorescent spectrum in the radiant matter tube, had guided me a certain distance and then led me widely astray. A new factor must now be taken into account—the presence of a calcium compound appeared to be necessary. An earthy body which, when treated and tested in the usual manner, fails to show the faintest glow of an orange-band spectrum, can by admixture with calcic sulphate be made to yield a pure and brilliant spectrum, rivalling in clearness and beauty that given by yttric sulphate.

111. Of the two components of the phosphorescing body—calcium and x—which is the necessary and which is the variable factor?

This question did not appear difficult to answer. In the first case the calcium must be kept the constant, and x be made the variable quantity.

A piece of pure colourless Iceland spar—the sulphate from which had been proved to phosphoresce normally with a greenish-blue light, without bands or concentration in any part of the spectrum (164)—was dissolved in hydrochloric acid, and mixed with about 10 per cent. of various metallic sulphates. Sulphuric acid was added, the mixture evaporated to dryness, ignited, and tested in a radiant matter tube. The bodies thus used to replace x, in addition to the more common earths, were the earths from samarskite—enumerated in the first part of this paper (40)—in as pure a state as I could get them, together with various earthy precipitates, oxalates, &c., obtained from different minerals during the preceding operations.

These experiments resulted in an *embarras de richesse*. Whereas, hitherto, I had considered the orange-band-forming body rare and sparsely distributed I now found it sharing with yttrium the attribute of ubiquity. The answer to my question was too full, and left me again in doubt as to whether calcium or x was the variable quantity.

The yttrium spectrum turned up in this series of experiments about as frequently as the orange-band spectrum. I knew that in such cases yttrium was present as an impurity; might it not be that the almost universal occurrence of the orange-band spectrum was equally caused by a minute but varying quantity of x in the earths under test?

112. I took them one by one and submitted them to further severe chemical treatment, fractionally precipitating them, in cold dilute solution, with weak ammonia, or fractionally crystallising their oxalates from nitric acid. As the purification progressed the orange-band spectrum generally lessened in intensity till in the case of many earths it faded out altogether, and in most of them it gave evident indications of

being extraneous to the earth itself. In some instances, however, the spectrum increased in intensity; moreover, when the purified earth showed any diminution of the orange band the eliminated impurity always showed the orange band in an exalted degree. I drew from these experiments the inference that x was a definite element, as widely distributed, or nearly so, as yttrium, but requiring admixture with a calcium compound to bring out its phosphorescent properties.

113. Next I had to ascertain if the calcium could be replaced by any analogous body. In this case, therefore, the x was kept constant whilst the calcium was replaced. An ammonia precipitate (109) from a rich accumulation of orange band substance was chosen as the x. Tested in the usual manner, by itself, it showed nothing, but mixed with lime it gave the orange-band spectrum very bright and pure.

The metals used to mix with it were in the form of sulphates—strontium, barium, glucinum, zirconium, thorium, magnesium, zinc, cadmium, lead, copper, silver, cerium, lanthanum, didymium, aluminium, manganese, tin, bismuth, antimony; also silicic, titanic, tantalic, tungstic, molybdic, and niobic acids. More than half of these bodies possessed the property of conferring "orange-band" phosphorescence on the precipitate under examination, although by themselves they evinced no power of giving a phosphorescent spectrum.

Explanation of foregoing Anomalies.

114. In this manner the remarkable fact was established, that the x I sought was an earth which of itself could give little or no phosphorescent spectrum in the radiant matter tube, but became immediately endowed with this property by admixture with some other substance, which substance likewise by itself had no power of phosphorescing with a discontinuous spectrum.

Of the great number of bodies used to mix with the earth x in these experiments, which acted best? It was not easy to try comparative experiments at this early stage; ultimately I came to the conclusion that lime, if not the best, was as good as any.

115. These results afford a full explanation of the anomalies which had so long hampered my endeavours to repeat on a large scale experiments which, when working with small quantities, had given good results (99, 101, 108). The preliminary experiments were intended to ascertain whether the desired orange band was present or not. Natural impatience led to hurried operations and defective washing of precipitates, and thus some of the necessary *lime* was left with the phosphorescing body. The subsequent larger operations were performed in a more systematic manner with the object of securing as large a yield of substance as possible. The precipitates were thoroughly washed, the lime was more completely thrown out, and the sought-for earth, although obtained, refused to reveal itself by the spectroscope and radiant matter tube.

The contradictory behaviour of the sulphate to water (99) was now easily explained. The insoluble crystals, which from the brilliancy of their phosphorescent spectrum I had at first mistaken for the nearly pure sulphate of x, were merely calcic or strontic sulphate contaminated with perhaps not more than the one ten-thousandth part of x sulphate which it had carried down with them on crystallising.

"X" in Cerite.

116. In the corresponding yttrium research I was aided materially by the fact that the sought-for earth did not give an absorption spectrum (42). This enabled me to throw out a large number of obscurely known elements, and I therefore early endeavoured to ascertain whether the supposed new earth, x, did or did not give an absorption spectrum. At first I could not decide one way or the other. I frequently obtained a good orange-band spectrum when the solutions gave no trace of absorption spectrum, whilst on other occasions the solution showed good didymium and other bands. Gradually, however, it was noticed that whenever the didymium absorption bands were strong the orange-band spectrum was also particularly brilliant. Moreover, amongst the earths enumerated in par. 113 as mixed with lime in the quest for x, I have mentioned that some of them gave the orange-band spectrum with increased intensity; the earths of the cerium group were the most noteworthy, and these considerations made it probable that here would be found the location of x.

117. On a former occasion, when searching for the citron-band-yielding earth, and examining cerite (22 to 25), I made use of the potassic-sulphate method of separating the two great sub-groups, viz., the cerium and the yttrium earths; the former giving insoluble, and the latter soluble, double sulphates. I said (23):—

"The precipitated double sulphates were dissolved in hydrochloric acid, and the earths precipitated as oxalates. After ignition and treatment with sulphuric acid, the mixed ceria, lanthana, and didymia were tested in the radiant matter tube, but the merest trace only of citron band was visible."

A repetition of the above experiment produced similar results. The contents of the tube were now removed, mixed with lime and excess of sulphuric acid, ignited, and again tested in the tube. This time the orange-band spectrum came out very brilliantly, showing in a striking manner the necessity of supplementing x with some other earth to bring out its phosphorescing properties.

The cerium group, to which x was now almost certainly traced, consists of cerium, lanthanum, didymium, samarium, and perhaps yttrium-α (136). The other metals, enumerated in par. 101 as being precipitated by potassic sulphate, were found not to phosphoresce with a discontinuous spectrum, either alone or when mixed with lime.

Analysis of Cerite.

118. The first necessity was to get the earths ceria, lanthana, and the mixture hitherto called didymia, in a pure state, for my so-called pure earths of this group all showed the orange band in more or less degree.

About 14 lbs. of cerite were finely ground, made into a thick paste with strong sulphuric acid, and heated to drive off excess of acid. The mass became of a white or pale grey colour. This was digested in cold water, filtered, and the residue well washed with cold water.

To the filtrate oxalic acid was added, which precipitated all the earths, with any lime, &c., that might be present, as oxalates. It saves time at first only to aim at a partial separation of the mixed earths, and for this purpose it is well to proceed as follows:—The dried oxalates are boiled with strong nitric acid till completely decomposed, the nitrates are evaporated to dryness, mixed with three times their weight of nitre, and fused at the lowest temperature at which nitrous fumes come off; the residue is digested in water, filtered, and washed. The insoluble residue, of a pale yellow colour, consists of ceric oxide and basic ceric nitrate, with a little of the other oxides, whilst the filtrate contains the bulk of the lanthanum, didymium, and samarium.

Separation of Ceria, Lanthana, Didymia, and Samaria.

119. To free the lanthanic, didymic, and samaric nitrates from the last traces of cerium it is necessary to fuse them again very gently with three or four times their weight of potassic nitrate, at a temperature just sufficient to cause slight decomposition. The operation of fusing must be repeated on the evaporated filtrate many times to throw out all the cerium.

The ceric oxide, or basic nitrate obtained, is freed from any didymium by re-treatment with nitric acid and fusion as above; the presence of didymium being indicated by its brown colour or by the absorption spectrum of the solution.

120. The separation from each other of lanthana, didymia, and samaria is a most laborious process, and the amounts of these earths, obtainable in anything like a pure state, is small, compared with the mass of material worked up. The solution of the nitrates of these elements is made perfectly neutral, diluted to such a strength as to contain about 1 per cent. of the oxides, and a very dilute solution of ammonia is added, about 0·1 gramme NH_3 in 500 cubic centimetres, the precipitation being conducted in large vessels, as ordinary Winchester quart bottles. The first precipitates formed are rich in samarium, and also contain much didymium; these are followed by didymium, with some lanthanum and samarium; and the final precipitates consist almost wholly of lanthanum. By this method there are obtained three portions of hydrates, which must be again worked up separately by precipitation; the first for samarium (133), the second for didymium (127), and the third for lanthanum (125), the process of fractional precipitation being repeated on each portion fifty or a hundred times.

121. The separation of the last traces of didymium from the samarium can be accomplished only by fractional precipitation, an operation so tedious that probably few chemists will be inclined to undertake it. The second portion of hydrates, consisting chiefly of didymium, is purified from the small quantities of samarium and lanthanum by fusing with potassic nitrate, as explained above for the traces of cerium (118); to separate the lanthanum the oxalates are dissolved in warm strong nitric acid and allowed to cool, when didymic oxalate nearly free from lanthanum is obtained; after repeating several times, the last trace of lanthanum remains in the solution.

122. To separate the small quantity of didymium from the lanthanum obtained in the final precipitates with ammonia, the only method is to continue the process of fractionation; the lanthanic oxide finally obtained should be pure white, any trace of yellow showing that didymium is still present.

123. As cerite contains small quantities of the yttria earths, these must be separated from cerium, didymium, &c., by making a cold solution of the sulphates and adding finely-powdered potassic sulphate in quantity more than sufficient to saturate the solution, allowing the mixture to stand (with frequent agitation) for a few days; filtering, and washing the filtrate with a saturated solution of potassic sulphate. The filtrate contains the yttria earths, and for their complete separation it is advisable to repeat the operation with potassic sulphate three or four times. The insoluble residues, consisting of a double potassic sulphate with either cerium, didymium, or other member of this group, are boiled with sodic hydrate, filtered, well washed, re-dissolved in nitric acid, precipitated with oxalic acid, and the oxalates ignited, leaving the earths lanthana, didymia, or samaria to be finally purified as described further on.

124. The ceric oxide obtained in the manner just described was white. A considerable thickness of a strong solution did not show a trace of absorption spectrum. The atomic weight of the metal was taken and yielded the number =141·1

The number given by Bühric* . . . . . =141·2

„ „ Robinson†. . . . . =140·2

Many older determinations‡ range from 138 to 139.

This ceric oxide gave no orange-band spectrum in the radiant matter tube, either with or without the addition of lime.

Purification of Lanthana.

125. The lanthana obtained in the manner described above (120, 122) was more difficult to purify than ceria. Long after the lanthana appeared pure, it gave in the

* Journ. prakt. Chem., (2), xii., 209.

† 'Chemical News,' vol. l., p. 251, Nov. 28, 1884.

‡ *Ibid.*, vol. xlix., p. 282, June 27, 1884.

radiant matter tube a good orange-band spectrum when mixed with lime and treated as usual, although without lime it gave no spectrum.

So long as the lanthana showed the didymium absorption bands I could not be certain whether the orange-band spectrum belonged to it or to didymium, therefore the tedious process of fractionation with very weak ammonia in cold dilute solutions was repeated for some weeks. The first precipitates were lanthana containing most of the didymia, whilst the final precipitates were lanthana almost if not quite free from didymia, according to the quantity originally present. After several hundred fractional precipitations repeated over and over again, a little lanthana was got which failed to show the didymium absorption bands. As the purification progressed the phosphorescent orange-band spectrum became fainter, until finally a lanthana was obtained which, mixed with lime and treated in the usual manner, gave no orange-band spectrum whatever. This lanthana was snow-white, and had an atomic weight of 138·3. MARIGNAC gives for lanthana 138·6, BRAUNER 138·28, CLÈVE 138·22.

Purification of Didymia.

126. The earth formerly called didymia is now known to be a mixture of didymia and samaria. The didymia which I prepared by the method described above, when mixed with lime and sulphuric acid, and tested in the radiant matter tube, gave the orange-band spectrum as brightly as I had ever seen it. It was not, however, quite free from the accompanying samaria, and systematic operations were now commenced with the object of obtaining the didymia and the samaria in a state of purity,—that is to say, in such a condition that one of them should show no orange-band spectrum at all, whilst the other should give the spectrum in its highest degree of intensity.

I did not attempt the two purifications simultaneously on the same material. One earth only was taken in hand at a time, and by repeated fractionations and the most profuse sacrifice of material, I was at last enabled to obtain a little of the desired earth quite free from admixture.

127. I took didymia first. About 1000 grms. of the earth, partially purified as described (120, 121), were dissolved in a large excess of strong nitric acid. To the nearly boiling liquid a hot saturated solution of oxalic acid was carefully added, and constantly stirred, until the precipitate, which at first rapidly disappeared, just refused to dissolve. A drop or two of nitric acid was now added to render the solution clear, and the liquid set aside to cool, when brilliant pink-coloured prisms of didymic oxalate (containing nitric acid) crystallised out. These crystals contained nearly all the didymium and samarium, whilst the mother-liquor contained the greater part of the lanthanum—reserved for the preparation of pure lanthana (125).

The crystals of didymic oxalate were ignited and again converted into nitrate, and the above-described partial crystallisation as oxalate repeated five or six times, in each case rejecting the mother-liquor as contaminated with lanthanum.

128. The final oxalates—the ultimate cumulation of the portions least soluble in nitric acid—were next converted into nitrates, and the excess of acid driven off. The anhydrous salt was dissolved in fifty times its weight of water, and fractionally precipitated by ammonia in the following manner:—A large quantity of ammonia was first prepared of the dilution (1 to 5000) used in the previous fractionation (120), and 500 c.c. of this was gradually added, with constant stirring, to Winchester quarts about three-fourths full of the dilute didymic nitrate. In about half an hour another 500 c.c. of ammonia was again added, and this operation was repeated at intervals till the Winchester quarts were full. The bulk of the samaria comes down in the first precipitates, which are filtered off and set aside for the preparation of pure samaria (133).

To the filtrate, containing didymium, with a little samarium and lanthanum, ten successive quantities of about 200 c.c. each of dilute ammonia were added to each Winchester quart at intervals of about an hour, and after violent agitation allowed to subside. The clear supernatant liquid was now poured off, evaporated to about half its bulk, and then, when cold, again poured back into the precipitate, and the operation of precipitating with dilute ammonia was likewise repeated. By this means the greater portion of the samarium present was obtained in the precipitate, whilst the didymium left in solution contained a less proportion of samarium.

129. After a time a balance seemed to be established between the affinities at work, when the earths would appear in the same proportion in the precipitate and the solution. At this stage they were thrown down by ammonia, and the precipitated earths set aside to be worked up by the fusion of their anhydrous nitrates so as to alter the ratio between them, when fractionation by ammonia could be again employed.

Samaric nitrate decomposes by heat before didymic nitrate. The nitrates were mixed with four times their weight of potassic nitrate, and the whole kept fused in a crucible till about three-fourths of the earthy nitrates were decomposed. The cooled mass was then dissolved in water, filtered, and the solution evaporated to dryness, and again submitted to fusion. This was repeated several times.

The basic nitrates insoluble in water were dissolved in nitric acid, and put through the operation of fractional precipitation with ammonia, for samaria (133), in the manner just described above (128).

130. To remove the last traces of samarium which might have survived this treatment, the solution of nitrates which had longest resisted decomposition by fusion was now mixed with excess of potassic sulphate. The precipitated double sulphates were subjected to long washing with a saturated solution of potassic sulphate, in which the samarium salt is more soluble than the didymium salt. They were then reconverted into nitrates, and the precipitation and washing with potassic sulphate repeated several times. Finally, the didymium salt was converted into oxalate, and re-crystallised many times from nitric acid, to eliminate any trace of lanthanum that might still contaminate it. Pure didymia is of a very deep chocolate-brown colour.

These proceedings are tedious enough even in their narration, but no mere words can enable the reader to realise the wearisome character of these operations when repeated day by day, month after month, on long rows of Winchester quart bottles.

131. I commenced the purification of didymia in the latter part of the year 1883, and the operations have been going on since almost daily in my laboratory. At intervals of some weeks the didymia in the then stage of purification was tested in the radiant matter tube, a little lime having previously been added to bring out the discontinuous phosphorescence. During the first month the intensity of the orange-band spectrum scarcely diminished. After this it began to fade, but the last traces of orange band were very stubborn, and not till the last few weeks could I obtain a didymia to show no trace of the orange-band spectrum; and this result has not been accomplished without sacrifice. My 1000 grammes have dwindled away bit by bit, till now less than half a gramme represents all my store.

132. Whilst in the midst of the operations of purifying didymium and samarium I had the pleasure to receive a visit from Prof. Clève, to whom we owe so much of our knowledge of the chemistry of the samarskite and cerite metals, and especially of didymium and samarium. He gave me not only most valuable information, and suggestions respecting the work I was then engaged upon, but on his return to Upsala he munificently presented me with specimens of lanthana, didymia, samaria, yttria, and erbia—specimens at that time considered to represent a state of purity. According to any chemical tests these earths would be deemed absolutely pure, but the test of the phosphorescent spectrum proved too severe a trial, and the didymia, lanthana, and samaria all showed the orange band—the lanthana faintly, the didymia more strongly, and the samaria brightest of all. A subsequent lot of "samarium-free" didymia, sent by Prof. Clève, also gave a strong orange-band spectrum, though the samarium present probably did not amount to more than the one hundred thousandth part of the didymium.

Purification of Samaria.

133. The foregoing experiments left little doubt that x, the orange-band-forming body, was samarium; the last problem was, therefore, to get this earth in a pure state. The general plan of operations was the same as I adopted in getting didymium free from samarium, only attention was now directed to the portions richest in samarium which had been formerly set aside (128, 129). On fractionation in highly dilute solutions with very weak ammonia the first precipitates are richer in samaria than the last. These first precipitates were re-converted into nitrates, and fractionation again proceeded with.

Fusion of the nitrates with potassic nitrate (129), or precipitation by, and washing with, potassic sulphate (130), are of no use in the final purification of samarium. When the object is to separate a little samarium from a large quantity of didymium,

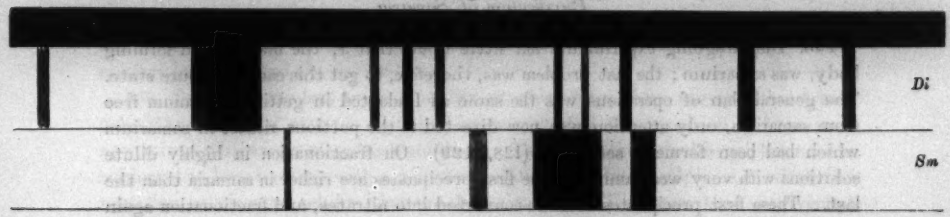
fusing the nitrates will effect the purpose, but I have not found the converse to hold good. The potassic sulphate method cannot separate the last traces of didymium from samarium, for the didymic double sulphate, not being quite insoluble, would wash out along with the first portions of the samaric salt. I have found no method better than fractionation with ammonia, and Professor CLÈVE tells me that is his experience.

Towards the end of the operations, when the samaria is getting pure, it is useful to precipitate it as a double sulphate with potassic sulphate, and wash it well for some time, to remove any traces of earths of the yttria and other groups which might have been present and become accumulated with the samaria (123).

134. During fractional precipitation with ammonia an experienced eye can judge roughly what is the preponderating earth present, by the appearance of the precipitate as it comes down. When much samaria is present, with but little didymia and lanthana, the precipitate forms immediately. When there is much didymia, and little samaria and lanthana, the precipitate forms almost as quickly as in the first case, but does not settle so rapidly. With much didymia, and a fair quantity of lanthana, the precipitate forms more slowly than before and settles sluggishly. When there is much lanthana and little didymia the precipitate takes a long time to settle, the liquid remaining opalescent for days. These peculiarities are due in great measure to the varying basicity of the elements, samarium being the least basic and lanthanum being the most basic, didymium occupying an intermediate position.

In freeing samarium from the last portions of didymium the only test available to detect the presence of the latter metal is the absorption spectrum. The best plan is to provide a strong solution of the samaric nitrate in a flask, to act as a lens, and to concentrate the light of a gas-flame by its means on to the slit of a low dispersion spectroscope. Long after the light colour of the ignited oxide shows that the didymium is getting small in quantity, its absorption bands will be so strong as almost to obliterate the fainter samarium spectrum.

Fig. 2.



135. The fractionation should be persevered in till no didymium bands are seen in the absorption spectrum. After this point is reached I prefer to keep on fractionating for some time longer, if the material will hold out, so as to make assurance doubly

sure. The colour of samaria, as pure as I have been able to prepare it, is white with the faintest possible tinge of yellow. The absorption spectrum of samarium salts is much more feeble than the spectrum of didymium salts.

The accompanying drawing (fig. 2) shows the absorption spectra of solutions of didymic and of samaric nitrates. It will be observed that the strongest bands of the samarium absorption spectrum are almost covered by strong absorption bands of didymium. Unless, therefore, the samarium is decidedly in excess, it will be difficult for any but a very practised observer to detect its presence. Fortunately the marvellous delicacy of the phosphorescent spectrum of samarium renders any other spectrum test of little value.

136. I have already mentioned (117) that the cerite earths are supposed to contain a fifth member, which has been provisionally called Yα.* Not much is known respecting the properties of this earth, but from the little I can glean it would appear to become concentrated with the samarium, from which a partial separation may be effected either by continuing the operation of fractional precipitation or by taking advantage of the different solubilities of their double potassic sulphates in potassic sulphate; the potassio-samaric sulphate being almost insoluble in a saturated solution of potassic sulphate, whilst the corresponding salt of Yα is soluble in 100 to 200 volumes of the same solution. By persevering in this mode of treatment I ultimately obtained a small quantity of a white earth which gave no samarium spectrum in the radiant matter tube. Whether or no it was Yα I cannot say, as the quantity obtained was insufficient to enable me to determine its atomic weight.

The Phosphorescent Spectrum of Samarium.

137. Pure samaric sulphate by itself gives a very feeble spectrum. Some of the pure salt was heated to redness,† sealed in a radiant matter tube, and carefully exhausted. The coil was adjusted so as to give a powerful spark; the room was well

Fig. 3.

darkened, and the eye kept shielded from extraneous light. It was difficult to hit the exact moment of exhaustion between the disappearance of gas and non-conductivity, but by careful watching at the spectroscope a point was reached at which the

* Marignac, 'Comptes Rendus,' vol. 90, p. 899; 'Chemical News,' vol. 41, p. 250.

† Samaric sulphate is not decomposed at the temperature employed.

phosphorescence appeared. The spectrum consists of a faint band in the red, then a sharp orange line (146, 165), next a wide ill-defined orange band, and finally an equally ill-defined green band. The appearance is shown in fig. 3.

138. When, however the samaria is mixed with lime (114) before examination in the radiant matter tube, the change is very striking, and the spectrum is, if anything, more beautiful than that of yttrium. The bands are not so numerous, but the contrasts are sharper. Examined with a somewhat broad slit, and disregarding the fainter bands, which require care to bring them out, the spectrum is seen to consist of three bright bands,—red, orange, and green,—nearly equidistant, the orange being the brightest. With a narrower slit the orange and green bands are seen to be double, and on closer examination faint wings are seen, like shadows to the orange and green bands. In this spectrum the sharp orange-coloured line (137) of pure samaric sulphate is absent.

139. The bands are best seen in a spectroscope of low dispersion, and with not too narrow a slit. In appearance they are more analogous to the absorption bands seen in solutions of didymium than to the lines given by spark spectra. Examined with a high magnifying power all appearance of sharpness generally disappears : the scale measurements given below must therefore be looked upon as approximate only; the centre of each band may be taken as accurately determined within the unavoidable errors of experiment, but it is impossible to define their edges with much precision.

140. The accompanying cut (fig. 4) gives as good an idea of the spectrum of

Fig. 4.

samarium-calcium as is possible in black and white. The numbers along the top are the squared reciprocals of wave-lengths, and are on the same scale as the diagram of the yttrium spectrum (71) given in my Bakerian Lecture already quoted. The phosphorescing mixture in the tube consisted of 20 parts of pure samaria and 80 parts of lime. They were converted into nitrates in a platinum capsule, and then decomposed by excess of sulphuric acid and ignited at a dull red heat. If sulphuric acid is added in the first instance there is a difficulty in getting the earths completely converted.

The least refrangible band seen is a very faint red, which extends from $\frac{1}{\lambda^2}$ 2310 to 2400. Here a much stronger red band begins extending to 2494. The first component of the bright orange band begins at 2739 and ends at 2762. Between 2762 and 2798 is a dark interval, and then the second component of the orange band

is seen extending from 2798 to 2818. This band is stronger and more sharply defined than the preceding band. A faint yellow wing extends from the second orange band to 2942. There is now an intensely black interval reaching to 3025; here a faint yellowish-green light is seen extending to 3149, where the green band commences and extends to 3164. Here a fainter green wing begins, and extends to 3270. On this wing a very narrow faint green band is seen, having its centre at 3190. There is then another dark space, after which three ill-defined blue and violet bands are seen, too faint to measure accurately.

141. Preliminary experiments (114) had shown me that lime was one of the best materials to mix with samaria in order to bring out its phosphorescent spectrum, but it was by no means the only body which would have the desired effect. More accurate observations were now taken with pure materials mixed together in definite quantities. The bodies employed were those enumerated in par. 113. Of these the following induced no phosphorescence:—zirconium, cerium, didymium, copper, silver, manganese, and tin; silicic, titanic, tungstic, molybdic, niobic, and tantalic acids.

142. The other substances which I tried caused the samarium to give good phosphorescence with a discontinuous spectrum. There is a general resemblance between these spectra, but nearly all of them differ one from another in details. In the following descriptions I will take the calcium-samarium spectrum (140) as the standard of comparison:—

Strontium and barium, when mixed with samarium, give almost identical spectra; the red band is similar to the one produced by calcium, but the orange bands have become a blurred orange space, with outline ill-defined on the side towards the red, and sharper at the more refrangible side. There is no trace of division in the orange space; but the green, which with calcium is narrow and single, becomes with barium and strontium a well-defined luminous double green band, with a sharp black separating interval.

Beryllium and samarium give a very faint phosphorescent spectrum, consisting of a red, orange, and green band only; the green occupying the position of the second green band given by calcium-samarium.

143. Thorium and samarium give a very similar spectrum to the one produced by barium or strontium with samarium, a slight difference being observable in the orange, which shows signs of separation into two components.

Thorium, as oxide or sulphate, by itself gives no phosphorescence (26, 28), and in fact, as I have already shown, renders the vacuum tube non-conducting;* samaria

* "This earth is, however, remarkable for its very strong attraction for the residual gas in the vacuum tube. On putting thorina in a tube furnished with well-insulated poles, whose ends are about a millimetre apart in the centre, and heating strongly during exhaustion, the earth, on cooling, absorbs the residual gas with such avidity that the tube becomes non-conducting, the spark preferring to pass several inches in air rather than strike across the space of a millimetre separating the two poles."—Proc. Roy. Soc., No. 213, 1881, Vol. XXXII., p. 209.

likewise gives scarcely any phosphorescence (137). It is, therefore, somewhat remarkable that a mixture of these two bodies should bring out the samarium spectrum so brilliantly.

Magnesium and samarium give the red band and second orange band sharply; the first orange is, however, obliterated, and its place occupied by a faint, broad, ill-defined band, extending to the orange band. The green band is very wide, faint, and ill-defined.

Zinc and samarium give the red band as usual, but there is only one orange band, which is sharp, and occupies the position of the interval between these bands in the calcium and samarium spectrum. The green band is widened out, and occupies the position of the second green band of the thorium-samarium spectrum.

The cadmium-samarium spectrum is similar to that of zinc-samarium, only the green band is still wider, and extends further towards the red end.

144. Lead with samarium gives a very brilliant spectrum. The red is clear and sharp, and has a narrow faint wing on each side. The orange is one wide band, with no trace of division, whilst the green is sharp and duplicated as in the thorium-samarium spectrum, to which, indeed, it bears a great resemblance. The centres of the green bands are at $\frac{1}{\lambda^2}$ 3133 and 3199.

Lanthanum and samarium phosphoresce brightly, and give a very sharp spectrum consisting of three bands, closely resembling the zinc-samarium spectrum; the red, however, not being so bright as the other two bands. On the contrary, cerium or didymium mixed with samarium gives no phosphorescence.

Aluminium and samarium give a spectrum resembling the corresponding calcium one as to the red and double orange, but having a very broad, somewhat faint, green band, with a black division in the middle occupying the position of the bright green band of calcium-samarium.

Bismuth and samarium gave a somewhat faint spectrum, almost identical with the lanthanum-, zinc-, cadmium-, and glucinum-samarium spectrum.

The antimony-samarium spectrum is almost identical with that with bismuth, the spectrum being, however, much fainter.

145. The samarium spectra, modified by other metals as above described, may be divided into three groups. The first group comprises the spectra given when glucinum, magnesium, zinc, cadmium, lanthanum, bismuth, or antimony is mixed with the samarium. It consists simply of three coloured bands, red, orange, and green; as a typical illustration I will select the lanthanum-samarium spectrum (fig. 5). The centres of the bands are—red 2429, orange 2808, and green 3177.

The second type of spectrum gives a single red and orange and a double green band. This is produced when barium, strontium, thorium, or lead are mixed with samarium. The lead-samarium spectrum (fig. 6) illustrates this type. The centres of the bands of this spectrum are—red 2437, orange 2830, green 3133 and 3199.

The third kind of spectrum is given by calcium mixed with samarium. Here the red and green are single, and the orange double. Aluminium would also fall into this class were it not that the broad ill-defined green band is also doubled. The calcium-samarium spectrum, already illustrated in fig. 4 (140), is a good illustration of this type.

Fig. 5.

Fig. 6.

Mixed Samarium and Yttrium Spectra.

146. It was interesting to ascertain what spectrum a mixture of samarium and yttrium would give. A mixture of 90 parts of samaria to 10 of yttria was treated with sulphuric acid and then ignited, and afterwards examined in the radiant matter tube. The result was as remarkable as it was unexpected. Scarcely a trace of the yttrium spectrum could be detected. The powder phosphoresced with moderate intensity, but the spectrum was almost the facsimile of that given by pure samaric sulphate (137, fig. 3), except that the sharp orange line, which in the spectrum of pure samaric sulphate is only just visible, had gained sufficiently in intensity to be measurable, and was found to lie at 2693. I next tried a mixture of samaria 80, and yttria 20. The spectrum was identical with the one last observed, with one striking difference—the 2693 line now shines out with great brilliancy of a fine orange-red colour, as sharp as a gas line, and so unlike the bands usually met with in the spectra of phosphorescent earths as to suggest the explanation that some other spectrum-forming body was present in the mixture.

The next tube experimented with contained 70 parts of samaria and 30 parts of yttria. The spectrum was identical with the one last observed. The sharp orange line was present in full intensity.

147. Mixtures were now made of samaria 60, yttria 40; samaria 60·63, yttria 39·37 (equivalent proportions of the two earths); in this last a slight division could be

detected in the green band. The following experiments were continued; mixtures were prepared in the following proportions:—

Samaria.	Yttria.
58	42
56	44
54	46
53	47
52	48
51	49
50	50
49	51
48	52
47	53
46	54
45	55

The results were almost identical with those yielded by the previous mixtures. The green band gradually condensed into two bands at $\frac{1}{\lambda^2}$ 3133 and 3199 as the proportion of yttria increased; the first green band was stronger than the other, and the wide orange band began to show signs of a wing on the side next the green. At this stage the spectrum nearly resembled the lead-samarium spectrum (145, fig. 6). Very little of the yttria spectrum could be detected, and the brilliant orange line stood out sharply in the whole series.

In the next mixture—samaria 44, yttria 56—a faint band $\left(\frac{1}{\lambda^2}\ 3038\right)$, was visible by the side of the two green bands, in the position occupied by the yellow band of yttria.

148. The experiments were continued, and the following mixtures were made and and carefully mapped:—

Samaria.	Yttria.
43	57
42	58
41	59
40	60
39	61

The faint line first observed in samaria 44, yttria 56, gradually got stronger, and there were also signs of a double green band in about the position occupied by the yttrium green.

149. The change which had been rapidly coming over the spectra since samaria 43, yttria 57, was now nearly accomplished, and in the next trial mixture—samaria 35,

yttria 65—the only indication that could now be found of the samarium spectrum was seen in the two faint green bands next to the citron line of yttria, a faint yellow band, and the new orange line, which shone out as brightly and sharply as ever.

The series was continued :—

Samaria.	Yttria.
30	70
25	75
20	80
10	90
5	95

In each case a careful map of the spectrum was taken on the $\frac{1}{\lambda^2}$ scale, and each spectrum was found to be practically identical. Slight differences in the relative intensities of some of the bands were observed ; but the general character of the spectrum was unaltered. The yttria spectrum now shone forth with scarcely any band of samaria, except that, standing out beyond the other bands in sharpness and brilliancy, the 2693 line was present in all.

150. It will be remarked that a sudden change occurs between the two mixtures, samaria 45, yttria 55, and samaria 35, yttria 65 ; on one side the spectrum pertains to that of samaria, whilst on the other side the spectrum is that of yttria, the new orange line running equally across them all.

Fig. 7.

Fig. 8.

151. The diagrams of some of these intermediate spectra shown in the accompanying figures illustrate how sudden is the transformation from the samarium to the yttrium spectrum. Fig. 7 is the spectrum of a mixture of 44 parts samaria and 56 parts yttria, and except for the increased strength of the orange line, and a faint trace of the red and yellow yttria bands, it is the pure samarium spectrum. Fig. 8 is the

spectrum of 42 samaria and 58 yttria, and is built up of some of the component bands of the spectrum of each earth: whilst in fig. 9, the spectrum of 39 samaria and 61 yttria is seen to be almost the yttria spectrum.

Fig. 9.

152. I have already shown (86) that one part of yttrium can be detected spectroscopically in the presence of a million parts of calcium, and the reaction is almost as sensitive if other earths are taken instead of lime. Again, the spectrum test for samarium is, if possible, even more delicate (156 to 163), one part being detectible when mixed with more than two million parts of calcium.

Fig. 10.

153. The accompanying diagram (fig. 10) shows at a glance the remarkable effect these two earths have of masking each other's spectrum reactions. The ordinates above the horizontal zero line represent the percentages of samaria in the mixtures examined, and the lower ordinates show the percentages of yttria present. The dots

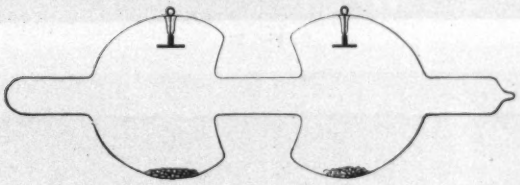
on the horizontal line show the actual mixtures experimented with (146 to 149). The part to the left covers the mixtures showing a preponderance of the samarium spectrum, and the right-hand portion shows the mixtures which give preponderance to the yttrium spectrum only. The intermediate portion covers the narrow borderland in which the fainter mixed spectra were observed.

In no mixture does the resulting spectrum contain the complete spectrum of either earth, many bands of each being suppressed.

154. It was of interest to ascertain whether the orange line 2693 could be evoked by mechanically forming a mixture which, when prepared chemically, gave the line strongly.

A radiant matter tube was made in the form of a double bulb, as shown in Fig. 11.

Fig. 11.



In one half was introduced ignited samaric sulphate, and in the other half the same weight of yttric sulphate, each being finely powdered. The tube was well exhausted and the powders separately examined with the spectroscope. Each gave its characteristic spectrum (72, 137). The double bulb was now sealed off, and the powders well mixed by violent agitation. On careful examination it was seen that the spectrum shown by the mixture was simply that of the two components superposed, and had no resemblance to the spectrum shown by a similar mixture prepared by solution and chemical action (fig. 13.)

The Action of Calcium on the Samarium-Yttrium Spectrum.

155. The addition of calcium to mixtures of samarium and yttrium has the effect of greatly developing the samarium spectrum, and, *pari passu*, of impairing the sharpness of the bands of the yttria spectrum. This exaltation of the samarium bands is not, however, effected without a corresponding loss. The brilliantly sharp orange line 2693 entirely disappears.

Fig. 12 is a drawing of the spectrum given by a mixture of equal parts of samaria, yttria, and lime. On comparing it with fig. 13, which represents the spectrum given by the same mixture without the lime, it will be seen that the action is as might have

been anticipated from previous experiments. The lime has divided the samarium orange band and strengthened the red, whilst it has increased the brilliancy of the yttrium citron and green, and brought out the blue and violet bands.

Fig. 14 is a representation of the spectrum given by a mixture of samaria 2, yttria 54, and lime 44. Even with this large amount of yttria present the samarium spectrum greatly overpowers that of yttrium. Without the lime very little samarium would be visible in this mixture (149).

Fig. 12.

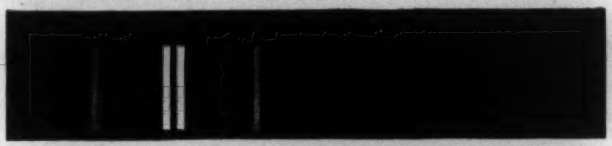


Fig. 13.

Fig. 14.

The Delicacy of the Spectrum Test for Samarium.

156. The foregoing observations had prepared me for the exceeding delicacy of this spectroscopic test for samarium. Experiments were now commenced with the object of getting some approach to a quantitative estimate of how small a quantity of samarium could be detected.

A solution of specially purified calcic nitrate (79) which was found to contain neither yttrium nor samarium by the radiant matter test, was standardised, so that one part of calcium was contained in 50 parts of solution.

157. A standard solution of samaric nitrate was made containing 1 part of samarium in 100,000 parts of solution.

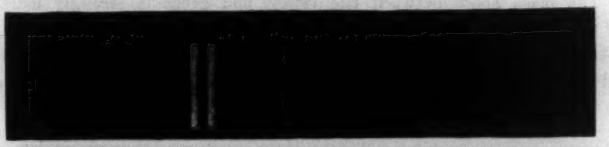
These solutions were mixed in the proportion of 1 part samarium to 100 parts of

calcium. The spectrum (fig. 15) was very brilliant, and but little inferior in sharpness to the spectrum given by a 50 per cent. mixture.

Fig. 15.

158. A mixture was now prepared containing 1 part of samarium to 1000 parts of calcium. Very little difference could be detected between the spectrum of this mixture and that of the last. The bands were, however, a little less sharp. Fig. 16 shows the appearance of this spectrum.

Fig. 16.

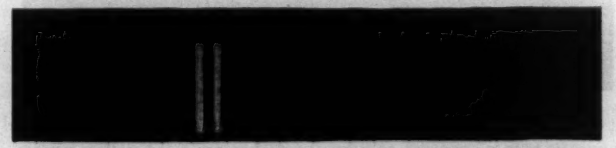


159. A mixture containing 1 part of samarium to 10,000 parts of calcium. The resulting spectrum is shown in fig. 17. The bands are now getting fainter, the second

Fig. 17.

green band is fading out, and the continuous spectrum of calcic sulphate is getting brighter.

Fig. 18.



160. The next mixture tried contained 1 part of samarium in 100,000 parts of calcium. The appearance of the spectrum is shown in fig. 18. Here the green is

almost gone, being overshadowed by the continuous spectrum of calcium which has spread over it. The red band has likewise almost disappeared in the greater brightness of the continuous red of the calcic spectrum. The double orange band is still very prominent, and the black space, 2942, between it and the green is very marked.

161. The next mixture, 1 part of samarium to 500,000 parts of calcium, gives a spectrum which is fainter than the last, but the orange bands are still distinctly

Fig. 19.

visible. The black space between the yellow and green is strongly marked, but narrower than before. Fig. 19 shows the appearance of this spectrum.

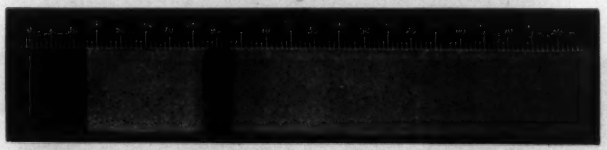
162. A mixture of 1 part of samarium in 1,000,000 parts of calcium was next subjected to experiment. In this the samarium spectrum is very feeble, and the

Fig. 20.

orange bands are only to be seen with difficulty. Now the most striking characteristic of this spectrum is the black space which still cuts out the greater portion of the yellow. Fig. 20 represents this spectrum.

163. A mixture of one of samarium in 2,500,000 parts of calcium was now taken.

Fig. 21.



In the spectrum shown by this mixture the bands of samarium have entirely gone, and its presence now is apparent only by the darkening in the yellow portion of what otherwise would be a continuous spectrum. Fig. 21 shows this appearance.

164. Finally the calcium spectrum by itself was examined. It is continuous, with no break, lines, or bands in it.

The Anomalous Line $\frac{1}{\lambda^2}$ 2693.

165. On several occasions I have spoken of an orange line, 2693, which by its brilliancy and sharpness is a prominent object in most of the samarium-yttrium spectra. With samaric sulphate it is exceedingly faint. With samaria containing 5 per cent. of yttria it is very little brighter; with 10 per cent. of yttria it gains a little; with 15 per cent. it is brighter still, and with a mixture of 80 parts samaria and 20 parts yttria it is at its maximum intensity. It continues to be the most striking feature in the spectra of the various mixtures of samaria and yttria until the proportion becomes samaria 3, yttria 97, when it begins to get less bright, and only when pure yttria is reached does it altogether vanish.

The accompanying diagram (fig. 22) shows the rise and fall in intensity of this bright line, with the proportions of samaria and yttria experimented on.

Fig. 22.

166. It is noteworthy that so long as this bright line is a component of the spectrum, the other bands manifest decidedly less intensity than they do in other phosphorescent spectra where this line is absent. Many of the bands usually present in the samaria and yttria spectra are also suppressed. The profound modification in the spectra of samaria and yttria developed by their mixture is, I believe, without precedent in spectrum analysis. The molecules of each earth, samaria and yttria, which separately are capable of vibrating rhythmically and of giving a characteristic phosphorescent spectrum, when mixed yield a spectrum approaching in character that of samaria or of yttria, according as one or other earth preponderates, and produce the dominant line 2693.

167. In this and the former paper on Radiant Matter Spectroscopy, much stress has been laid on the sensitiveness of the Radiant Matter test for indicating the

presence of samarium and yttrium; but it might be argued, from the anomalies that arise when *both* these elements occur together, that in reality the Radiant Matter test, however delicate, is one not to be depended upon. For instance, it might reasonably be asked what inference is to be drawn in the case of certain minerals treated with sulphuric acid and tested in the vacuum tube, and found to give only a feeble spectrum? Does this prove the absence of all but traces of either samarium or yttrium, or does it show the presence of both these earths in considerable quantity? The answer is simple. In spite of the perplexing anomalies that have come to light, and are described in this paper, regarding the influence of these two phosphorescing earths on each other, no single instance has occurred during the work connected with this subject in which, with the experience now gained, brilliant phosphorescence and a characteristic spectrum could not be evolved from any mixture containing both or either of the earths samaria and yttria. If, after treatment with sulphuric acid and ignition, the earthy mixture gives a pure spectrum of either yttria or samaria, and the line 2693 is absent, it is pretty safe to assume that the particular earth indicated is alone present. If, however, the spectrum is not quite characteristic, or the anomalous line 2693 is present, it is not sufficient to test the unknown mineral or mixture direct in the vacuum tube. It must first be treated chemically to separate the samaria and yttria (123, 133), and lime must be added before testing in the radiant matter tube (138), when the spectrum immediately makes its appearance if either of these earths be present in the smallest quantity. Although I say lime is to be added, many other substances perform the same office of eliciting the spectrum, such as baryta, lead, &c. (142—145); but my chief experience has been with lime, and I have always found it to give uniform results under varied conditions.

One important lesson taught by the many anomalies unearthed in these researches is, that inferences drawn from spectrum analysis *per se* are liable to grave doubt, unless at every step the spectroscopist goes hand in hand with the chemist. Spectroscopy may give valuable indications, but chemistry must after all be the court of final appeal.

168. Chemistry, however, by itself would have been helpless to solve the difficulties had it not been possible to appeal at every step to the radiant matter tube and to the spectroscope. The problems to be solved are so new as to be entirely outside the experience of laboratory work. A double orange-coloured band shows itself in a faint emission spectrum obtained under novel circumstances. On further examination the band, or one not far from it, is seen to occur in minerals of very divergent kinds and apparently irrespective of their chemical constitution or locality, as well as in laboratory reagents and chemicals of assured purity. This band is sometimes accompanied by bands in other parts of the spectrum, and occasionally shifts its place to the right or to the left. Frequently the orange band disappears and a citron-coloured band takes its place. Chemical research continued for a longer time than most chemical researches require fails to throw any light on the subject. These being the conditions

of the problem, the very last explanation likely to occur to the enquirer would be that these elusive shifting bands were due to the presence of two elements almost universally distributed, and that these two elements should be yttrium and samarium,—yttrium one of the rarest of known elements, and samarium, almost unknown at the time its spectrum reaction was first discerned.

Description of the Coloured Lithograph.

PLATE 75.

The uppermost spectrum is that of yttric sulphate in the radiant matter tube. It is copied in colours from the author's Bakerian Lecture already quoted, and is placed at the head of the coloured spectra for convenience of comparison.

The other spectra are coloured copies of the illustrations in the text, and are designated by the same numbers.

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XV. *Researches on the Theory of Vortex Rings.*—Part II.

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The present communication forms a continuation of some researches the first part of which was published in Part I. of the Transactions for 1884.* In that paper was considered the case of a circular hollow with cyclic motion round it. In the following pages the more general case is investigated where the core is of different density from that of the surrounding fluid, has a hollow inside it, and circulations additional to that due to the rotational filaments actually present. The investigation

* References to this are in square brackets, thus [I. 5]. [T. F.] refers to a paper on "Toroidal Functions" in Phil. Trans., Vol. 172 (1881), containing the theory of the functions used.

is not merely one of mathematical interest, for the vortex atom theory of matter has—so far as it has yet been developed—shown such claims on our consideration that anything throwing light on it will be of value. The supposition of a dense core may possibly be necessary to account for the different masses of the various elements.

As soon as the existence of a core is postulated the ring at once becomes more complex, depending on the density (or even the arrangement of density) of its core, on its vorticity, and on the presence or absence of additional circulations. In what follows the vorticity has been taken uniform; this not only greatly simplifies the mathematical methods, but is also the case we should naturally choose first to investigate. In the general investigation the density is taken to be different from that of the surrounding fluid. The ring is supposed hollow, with an additional circulation round it, and another additional circulation round the outer boundary of the core. It is evident that the presence of the former circulation necessitates the perpetual existence of the hollow. It is shown that the presence of the latter circulation is necessary to render the ring stable when its density is greater than that of the rest of the fluid.

As in the former paper, the investigation is divided into three sections. The first is preliminary and deals with the necessary functions and their approximate values. The second is devoted to the consideration of the state of steady motion. Here the approximations are carried in the beginning so far as to include the second order of infinitesimals, but this necessitates in certain parts approximation to the fourth order. It has been carried to this order for future work; the reader however may, so far as the results of the present communication are concerned, without any loss of intelligibility, pass over the parts giving the calculation of the highest orders. The third section discusses the question of fluted vibrations, of pulsations, and of stability.

When the motion is steady the sectional centre * of the hollow lies outside that of the core. In general (§ 9) if C_1 is its position (with given outside boundary) when the inner additional circulation is very large, and C_2 when the same quantity is zero, C_1 is outside of C_2 and the position of C when the additional circulation is general, is the centre of gravity of masses proportional to the added circulation at C_1 and the circulation due to the core itself at C_2. When the hollow is just zero, the distance of C_2 from the sectional centre of the core bears to the sectional radius the ratio $5r/8a$ where r is the sectional radius, and a the radius of the ring. This, therefore, is the point where the hollow begins to form when the energy is sufficiently increased. If with the same outer boundary the mass of the core be lessened (or size of hollow increased) C_2 moves in and ultimately coincides with the centre of the outer section. The position of C_1 alters in the same manner, only in this case the hollow can never vanish.

If m be the volume of the core, Π the pressure at an infinite distance, μ the circulation when there are no additional ones, and d, d' the densities, then (§ 10) a hollow will

* By sectional centre is meant the centre of the cross-section; by apertural centre is meant the centre of the aperture.

begin to form when the radius of the ring is $4m\Pi/\mu^2(d+d')$. So long as the core is simply continuous the volume is constant, and therefore the sectional radius varies inversely as the square root of the radius of the ring. When there is no core it was shown in the former paper that the sectional radius of the hollow remained constant. In the general case, after a hollow is formed the sectional radius of the core changes more slowly, and the additional circulations add to this tendency. The outer section always decreases as the aperture increases, but when the hollow becomes large this decrease is very small, and the sectional radius of the core remains almost constant. The sectional radius of the hollow also increases with the aperture. In cases where a hollow begins to form the sectional radius at that time is equal to $\sqrt{1+\sigma}$ times its ultimate value; σ being the density of the core with respect to the surrounding fluid.

The expansibility of the ring due to the presence of a hollow has a marked effect on the variation of the velocity of translation with increasing aperture, the tendency being to make the variation smaller.

With an internal additional circulation the ring will possess internal energy comparable with that of the external fluid. It will, however, decrease as the whole energy is increased. This is of importance for the general theory of gases.

The fluted vibrations in general consist of two sets of two, travelling in opposite directions round the core, the modes being defined by the number of flutings. For a single continuous core there are two sets; for a hollow core without an internal additional circulation, there are three sets together with a standing wave (where the time of vibration is infinite); for two additional circulations we get four sets, the times being determined by a biquadratic, which I have not succeeded in solving in general terms. When there is no rotational core the motion is always stable. When there is a simple continuous core, whose density referred to the outer fluid is σ, and no additional inner circulation, the ratio of the outer circulation to that due to the core must be $> \sqrt{\left\{\frac{\sigma}{2}(2+\sigma)/(1+\sigma)\right\}}$. When there is no additional circulation, or no slip over the core, the ring cannot be always stable unless $\sigma < \sqrt{2}$. These conditions hold until a hollow has formed. When there is a hollow and no internal additional circulation, the simple ring usually considered is still stable. But if the core is denser than the surrounding fluid, it is always stable only when the outer additional circulation is larger than a certain critical value depending on the densities and the circulations. If it is less than this critical value the ring becomes unstable at some point as the aperture increases. When the density of the core is very large, of the order 10^p (p large) this critical value is $\sqrt{\frac{3}{2}}\,10^{p/3}$ times the circulation of the core itself.

The condition of stability when there is an inner additional circulation depends on the reality of the roots of a biquadratic equation, and the general conditions are not discussed, but the same property of the outer additional circulation preserving the stability clearly holds good. The motion is always stable for pulsations.

These conditions of stability only reach so far as fluted vibrations and pulsations are concerned. The question of the stability for twisted and beaded vibrations is not considered. J. J. THOMSON has proved that simply continuous rings of the same density as the rest of the fluid are stable for these kinds of vibrations. The general case yet remains to be investigated.

Some of the simpler results are here collected for the sake of reference.

$$\text{(distance of } C_1 \text{ from sectional centre)}/r=\frac{r}{4a}(1-x+\log 1/x) \quad . \quad . \quad (38)$$

$$\text{(distance of } C_2 \text{ from sectional centre)}/r=\frac{r}{4a}\left\{\tfrac{5}{2}-\frac{3x}{2}-\frac{x}{1-x}\log 1/x\right\} \quad . \quad (38)$$

$$ar^2(1-x)=\frac{m}{2\pi^2}$$

$$\frac{r^2}{r_0^2}\mu_2^2\rho=\mu_2^2\rho-(\mu_1+\mu')^2+\frac{\mu_1^2}{x}+2\mu'^2\frac{1-x-x\log 1/x}{(1-x)^2}+2\mu_1\mu'\frac{\log 1/x}{1-x} \quad . \quad . \quad (43)$$

$$\frac{a_1}{a}(1+\rho)=(1-x)\rho+(1+x)-\frac{2x}{1-x}\log 1/x \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

The general formula for the velocity of translation is given in eq. (44), the following are cases :—

No core

$$V=\frac{\mu_2}{4\pi a}\left(\log\frac{8a}{r}-\tfrac{1}{2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

Continuous core

$$V=\frac{\mu}{4\pi a}\left(\log\frac{8a}{r}+\tfrac{5}{6}\right)+\frac{11}{32\pi a}\frac{\mu'^2}{\mu_2\rho} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

Ordinary ring

$$V=\frac{\mu}{4\pi a}\left(\log\frac{8a}{r}-\tfrac{1}{4}\right)$$

For hollow with no added circulations

$$V=\frac{\mu}{8\pi a}\left\{\log\frac{64a^2}{rr'}-\frac{1+x^2}{(1-x)^2}\log 1/x\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

where

μ' = circulation due to vortex filaments
μ_1 = added inner circulation
μ_2 = whole outer circulation
ρ = ratio of density of outside fluid to density of core
m = volume of core
r = outer sectional radius

r' = inner sectional radius
x = $(r'/r)^3$
r_0 = value of r when the aperture is infinite
a = radius of ring
a_1 = radius of ring when the hollow begins to form
r_1 = sectional radius " " " "
V = velocity of translation

Section I.—*Preliminary.*

1. The motion we are about to consider is one of steady motion in a fluid, part of which is rotational. If ψ denote the stream function, ω the angular rotation, and ρ, z, the cylindrical co-ordinates of a point, then

$$\frac{\delta^2\psi}{\delta z^2}+\frac{\delta^2\psi}{\delta\rho^2}-\frac{1}{\rho}\frac{\delta\psi}{\delta\rho}=2\omega\rho$$

Now when the motion is steady the rotations are so arranged that the vorticity is a function of the stream line or $2\omega/\rho=f(\psi)$. For steady motion then we have

$$\frac{\delta^2\psi}{\delta z^2}+\frac{\delta^2\psi}{\delta\rho^2}-\frac{1}{\rho}\frac{\delta\psi}{\delta\rho}=\rho^2 f(\psi)$$

Before, therefore, it is possible to discuss the properties of any vortex ring it is necessary to know its vorticity. The case considered in the following pages is that where the vorticity is constant. The methods developed will however apply to any cases where the motion is arranged in anchor-ring shells, the vorticity in any shell being constant. Here then $f(\psi)$ is a constant $=A$ (say). A particular integral is then at once obtained, viz., $\psi=\frac{1}{8}A\rho^4$, and the general solution becomes [I. § 1].

$$\psi=\tfrac{1}{8}A\rho^4+\frac{1}{\sqrt{C-c}}\Sigma_0^\infty(A_nR_n+B_nT_n)\cos(nv+\alpha_n)$$

Since the translatory motion is uniform the problem may be reduced to one of steady motion by impressing on every point a velocity equal and opposite to V, the velocity of translation. The stream function for the fluid outside the core will then be of the form

$$\psi_2=-\tfrac{1}{2}V\rho^2+\frac{1}{\sqrt{C-c}}\Sigma_0^\infty A_n'R_n\cos nv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

whilst for the portion of fluid constituting the core it is of the form

$$\psi_1=\tfrac{1}{8}A\rho^4+\frac{1}{\sqrt{C-c}}\Sigma_0^\infty(A_nR_n+B_nT_n)\cos nv \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

We shall assume—an assumption to be justified by the result—that

$$(A_s R_s + B_s T_s)/(A_{s-1} R_{s-1} + B_{s-1} T_{s-1})$$

is of order k, or of the first order of small quantities by which the approximation proceeds. In this case A_s/A_{s-1} is of order k^2, and B_s/B_{s-1} of zero order, in other words A_s/A_0 of order k^{2s}, and B_s of the same order as B_0, but there is no means at present of determining the order of A_0, or A'_0 with reference to B_0.

2. The most general case considered is where the core is hollow, and has therefore an inner free surface, and an outer surface common with the fluid moving irrotationally. The cross sections of these surfaces will approximate to circles. Take the critical circle (radius a) of our curvilinear coordinates to be that belonging to the mean circle of the inner cross section, and for this mean circle let k be k_1. Its actual form can then be represented by the equation

$$k = k_1(1 + \alpha_2 \cos 2v + \alpha_3 \cos 3v + \dots) \quad \dots\dots\dots \quad (3)$$

where α_s is of order k^s.

The outer surface can be represented by the form

$$k = k_2(1 + \beta_1 \cos v + \beta_2 \cos 2v + \beta_3 \cos 3v + \dots) \quad \dots\dots\dots \quad (4)$$

where, as in the former case, β_s is of order k^s.

It might be thought that it would be possible to represent the outer surface by an equation of the first form, and the inner surface by one of the second form. But we have no right to do this, for the inner surface might not contain the critical circle, and it would then be impossible to represent it by the second form. To assume that form for it would, therefore, be equivalent to assuming that the inner surface contains the critical circle of the outer. Now, the outer must evidently contain that of the inner, hence the equations above given can actually represent them.

The mean circle approximating most closely to the outer surface will not be that represented by k_2, but one which does not belong to the system k at all. It will be necessary to know the distance of its centre from that of the inner mean circle. Now [T. F., Eq. 6] if R, r, denote the radius of the axial circle, and of the cross-section respectively of a tore (u),

$$R/r = C \qquad a/r = S$$

Hence for the inner surface

$$\left.\begin{aligned} &\text{radius of cross-section} = a/S = 2ak_1 \\ &\text{radius of ring} \qquad\quad = aC/S = a(1 + 2k_1^2) \end{aligned}\right\} \text{to second order.}$$

To determine the similar quantities for the outer surface let it meet the plane of the ring in the points (k', k''), then

$$k'=k_2(1-\beta_1) \qquad k''=k_2(1+\beta_1) \qquad \text{(to the second order).}$$

Hence

$$\text{radius of cross-section}=\tfrac{1}{2}\{(R''+r'')-(R'-r')\}$$
$$\text{radius of ring}\quad=\tfrac{1}{2}\{(R'-r')+(R''+r'')\}$$

From these it follows easily that

$$\left.\begin{aligned}&\text{radius of section}=2ak_2\\&\text{radius of ring}\quad=a(1+2k_2^2+2k_2\beta_1)\\&\text{and distance between centres of mean cross sections}\\&\qquad=2a(k_2^2-k_1^2+k_2\beta_1)\end{aligned}\right\}\quad\ldots\ldots\quad(5)$$

the centre of the outer lying outside that of the inner.

The volume of the core will be

$$\iint 2\pi\rho\frac{dn}{du}\cdot\frac{dn'}{dv}dudv$$

taken over the section. Let us first find the volume of the surface bounded by

$$k=k_2(1+\beta_1\cos v\ldots)$$

Volume

$$=2\pi a^3\int_0^{2\pi}\int_u^{\infty}\frac{S}{(C-c)^3}dudv$$

$$=-\pi a^3\int_0^{2\pi}\left[\frac{1}{(C-c)^2}\right]_u^{\infty}dv$$

$$=\pi a^3\int_0^{2\pi}\frac{dv}{(C-c)^2}$$

Now

$$\frac{1}{(C-c)^3}=\frac{1}{C^3}\left(1+\frac{2c}{C}+\frac{3c^2}{C^2}+\ldots\right)$$
$$=4k^3(1+4k^2+4k\cos v+6k^2\cos 2v+\ldots)$$

Along the boundary $k=k_1(1+\beta_1\cos v+\beta_2\cos 2v)$.

Therefore to the second order in the bracket

$$\frac{1}{(C-c)^3}=4k_1^3\{1+4k_1^2+\tfrac{1}{2}\beta_1^2+6\beta_1k_1+p\cos v+q\cos 2v+\ldots\}$$

Hence volume

$$=8\pi^2\alpha^3k_1^2(1+4k_1^2+\tfrac{1}{2}\beta_1^2+6\beta_1k_1)$$

or volume of core

$$=8\pi^2\alpha^3\{k_2^2-k_1^2+4(k_2^4-k_1^4)+\tfrac{1}{2}\beta_1^2k_2^2+6\beta_1k_2^3\}=m \text{ (say)} \quad . \quad . \quad . \quad . \quad (6)$$

i.e., to lowest order

$$m=8\pi^2\alpha^3(k_2^2-k_1^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

3. The circulation of the ring, taking the velocity through the ring as positive, is

$$-2\iint\tfrac{1}{2}A\rho\frac{dn}{du}\frac{dn'}{dv}dudv$$

taken over the cross section. Hence by what has gone before, the circulation due to the actual vortex filaments of the core is

$$\mu'=-4\pi A\alpha^3\{k_2^2-k_1^2+4(k_2^4-k_1^4)+\tfrac{1}{2}\beta_1^2k_2^2+6\beta_1k_2^3\} \quad . \quad . \quad . \quad . \quad (8)$$

The outside stream function is

$$\psi_2=-\tfrac{1}{2}V\rho^2+\frac{1}{\sqrt{C-c}}\Sigma_0^\infty A_n'R_n\cos nv$$

Hence the circulation of the fluid outside the ring is [I., Eq. 4]

$$\mu_2=-\frac{\pi\sqrt{2}}{a}(A_0'+A_1'+A_2'+\ldots) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Again taking the circulation round the inner surface of the core, the circulation in the core additional to that due to its own rotation is

$$\mu_1=-\frac{\pi\sqrt{2}}{a}(A_0+A_1+A_2+\ldots)-4\pi A\alpha^3(k_1^2+4k_1^4) \quad . \quad . \quad . \quad . \quad (10)$$

For the sake of greater generality we shall suppose circulations, additional to that due to the core, as existing in the outer irrotationally moving fluid, and in the core itself. In the case where there is none added to the outside fluid

$$\mu_2=\mu_1+\mu'$$

whence to the lowest order of small quantities

$$A_0'=A_0+\frac{4A\alpha^4}{\sqrt{2}}k_2^2$$

If there are no added rotations A_0, A'_0 will therefore be of order k^2 with reference to Aa^4. By always taking them to be of this order therefore the approximation will still be true when there are added circulations.

4. It will be convenient here to consider the equation giving the pressure at any part of the core, including the case where small vibrations are superposed on the steady motion. The equations of motion are, d being the density, and taking the motion in the plane of z, x,

$$-\frac{1}{d}\frac{\delta p}{\delta x}=\frac{\delta u}{\delta t}+u\frac{\delta u}{\delta x}+w\frac{\delta u}{\delta z}$$

$$-\frac{1}{d}\frac{\delta p}{\delta z}=\frac{\delta w}{\delta t}+u\frac{\delta w}{\delta x}+w\frac{\delta w}{\delta x}$$

$$\frac{\delta u}{\delta z}-\frac{\delta w}{\delta x}=2\omega=-Ax$$

Therefore, integrating along a line in the plane of x, z,

$$-\frac{1}{d}\left(\frac{\delta p}{\delta x}dx+\frac{\delta p}{\delta z}dz\right)=\frac{\delta}{\delta t}(udx+wdz)+u\left(\frac{\delta u}{\delta x}dx+\frac{\delta u}{\delta z}dz\right)$$
$$+w\left(\frac{\delta w}{\delta x}dx+\frac{\delta w}{\delta z}dz\right)+Ax(udz-wdx)$$

or if v denote the velocity at any point and v' the velocity along the line of integrations,

$$-\frac{1}{d}\frac{\delta p}{\delta s}=\frac{\delta v'}{\delta t}+\tfrac{1}{2}\frac{\delta(v^2)}{\delta s}-A\frac{\delta\psi}{\delta s}$$

Therefore

$$\frac{p}{d}=f(t)-\dot{\phi}-\tfrac{1}{2}v^2+A\psi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where ϕ is the flow along any line in a plane through the axis of z up to the point in question. It is a function depending in general on the path of integration, but $\dot{\phi}$ is independent of this (is in fact due to the added irrotational motion) and is single valued when ψ is so. When ψ is many-valued (as, for instance, in the case of pulsations), $\dot{\phi}-A\psi$ is single-valued.

Section II.—*Steady motion.*

5. Suppose the approximations are carried so far as to include terms in $\cos nv$ in the stream functions and equations to the bounding surfaces. The constants to be determined will then be V, the $n-1$ quantities α, the n quantities β, and the coefficients A'_n, A_n, B_n, $3(n+1)$ in number—or in all $5n+3$ quantities. We may

regard A'_0, A_0 as determined by (eq. 9, 10) when the circulations are given, and B_0 will be determined by the fact that the pressure along the free surface is zero, whilst it is Π at an infinite distance. There remain therefore $5n$ constants to be determined. The conditions to be satisfied are the following :—

(1) ψ_2 must be constant along the outer boundary of the core. This gives n equations, viz., by equating to zero the coefficients of $\cos v \ldots \cos nv$ along the surface.

(2) ψ_1 constant along both the outer and inner boundaries of the core. This gives $2n$ equations.

(3) The pressure must be the same on both sides of the outer boundary. This gives n equations.

(4) The pressure along the inner surface must be constant. This gives also n equations. Hence on the whole the surface conditions give $5n$ equations, sufficient therefore to determine the $5n$ constants. It is therefore possible to approximate to any order desired.

It has already been noticed that A'_0, A_0 are to be regarded as of the order k^2 with respect to Aa^4. It will be seen later that when this is the case the B are of zero order with respect to the same quantity. Hence, if in the approximations account is taken of A_2, the term in which B occurs must be carried as far as $B_4 \cos 4v$. It will be necessary therefore to carry the latter terms to the fourth order, except in the case where the added circulation is very large compared with that due to the actual rotational motion. This case is therefore a much easier one to discuss than the more general one. We proceed to apply the conditions given above to determine ψ_2, ψ_1. Our method of procedure will be first to express the functions in terms of k, and the cosines of multiples of v; then to substitute the value of k along the surface, and reduce the expressions to a series of cosines of multiples of v, whose coefficients are functions of k_1 or k_2 as the case may be. The conditions above are then applicable at once.

6. *The function* ψ_2. It has been seen (1) that

$$\psi_2 = -\tfrac{1}{2}\mathrm{V}\rho^2 + \frac{1}{\sqrt{\mathrm{C}-c}}\Sigma_0^\infty \mathrm{A}'_n \mathrm{R}_n \cos nv$$

Now,

$$\tfrac{1}{2}\mathrm{V}\rho^2 = \tfrac{1}{2}a^2\mathrm{V}\left(\frac{\mathrm{S}}{\mathrm{C}-c}\right)^2 = \tfrac{1}{2}a^2\mathrm{V}\left\{\frac{1-k^2}{1+k^2-2kc}\right\}^2$$

$$= \tfrac{1}{2}a^2\mathrm{V}(1+2k^2+4k\cos v+6k^2\cos 2v)$$

For later purposes the value of $(\mathrm{C}-c)^{-1}$ must be carried to the fourth order, and then

$$\frac{1}{\sqrt{(C-c)}}=\frac{\sqrt{(2k)}}{\sqrt{(1+k^2-2k\epsilon)}}$$

$$=\sqrt{(2k)}\left\{1+\tfrac{1}{4}k^3+\tfrac{9}{64}k^4+(k+\tfrac{3}{8}k^3)\cos v+\left(\frac{3k^2}{4}+\tfrac{5}{16}k^4\right)\cos 2v\right.$$
$$\left.+\tfrac{5}{8}k^3\cos 3v+\tfrac{35}{64}k^4\cos 4v\right\} \quad (12)$$

Substituting the values of R given in [I., Eq. 11],

$$A'_0R_0+A'_1R_1\cos v+A'_2R_2\cos 2v$$
$$=-k^{-1}\{\tfrac{1}{2}L-1+\tfrac{1}{8}(L+1)k^2\}A'_0+\tfrac{1}{2}k^{-1}\{1-\tfrac{3}{8}(L-\tfrac{1}{2})k^2\}A'_1\cos v$$
$$+k^{-1}(1-\tfrac{5}{4}k^2)A'_2\cos 2v$$

Putting in these values and reducing, it will be found that

$$\left.\begin{aligned}\frac{1}{\sqrt{2}}\psi_2=&-\tfrac{1}{2}A'_0\{L-2+\tfrac{1}{4}(2L-1)k^2\}+\tfrac{1}{4}A'_1-\frac{a^2V}{2\sqrt{2}}\left(1+2k^2\right)\\&+\left\{-\tfrac{1}{2}A'_0(L-2)+\tfrac{1}{2}\frac{A'_1}{k^2}-\frac{2a^2V}{\sqrt{2}}\right\}k\cos v\\&+\left\{-\tfrac{3}{8}A'_0(L-2)+\tfrac{1}{4}\frac{A'_1}{k^2}+\frac{A'_2}{k^4}-\frac{3a^2V}{\sqrt{2}}\right\}k^2\cos 2v\end{aligned}\right\} \quad (13)$$

Now, along the outer surface

$$k=k_2(1+\beta_1\cos v+\beta_2\cos 2v)$$

whence

$$\left.\begin{aligned}L&=L_2+\tfrac{1}{4}\beta_1{}^2-\beta_1\cos v+(\tfrac{1}{4}\beta_1{}^2-\beta_2)\cos 2v\\k^2&=k_2{}^2\{1+\tfrac{1}{2}\beta_1{}^2+2\beta_1\cos v+(2\beta_2+\tfrac{1}{2}\beta_1{}^2)\cos 2v\}\end{aligned}\right\} \quad (14)$$

Suppose that when these values are substituted ψ_2 becomes $p_0+p_1\cos v+p_2\cos 2v$. This gives the value of ψ_2 along the outer surface, and since this is constant, p_1, p_2 both vanish.

Make these substitutions and reduce, then it will be found that

$$\left.\begin{aligned}\frac{1}{\sqrt{2}}p_0=&-\tfrac{1}{2}A'_0\left\{L_2-2+\tfrac{1}{4}(2L_2-1)k_2{}^2\right\}+\tfrac{1}{4}A_1-\tfrac{1}{2}\frac{a^2V}{\sqrt{2}}(1+2k_2{}^2)\\&-\tfrac{1}{2}\beta_1\left\{\tfrac{1}{2}A'_0(L_2-3)k_2+\tfrac{1}{2}\frac{A'_1}{k_2}+\tfrac{1}{4}A'_0\beta_1+\frac{2a^2V}{\sqrt{2}}k_2\right\}\\\frac{p_1}{\sqrt{2}}=&-\tfrac{1}{2}A'_0(L_2-2)k_2+\tfrac{1}{2}\frac{A'_1}{k_2}-\frac{2a^2V}{\sqrt{2}}k_2+\tfrac{1}{2}\beta_1A'_0=0\\\frac{p_2}{\sqrt{2}}=&-\tfrac{3}{8}A'_0(L_2-2)k_2{}^2+\tfrac{1}{4}A'_1+\frac{A'_2}{k_2{}^2}-\frac{3a^2V}{\sqrt{2}}k_2{}^2-\tfrac{1}{2}(\tfrac{1}{4}\beta_1{}^2-\beta_2)A'_0\\&+\tfrac{1}{2}\beta_1\left\{-\tfrac{1}{2}A'_0(L_2-3)k_2-\tfrac{1}{2}\frac{A'_1}{k_2}-\frac{2a^2V}{\sqrt{2}}k_2\right\}=0\end{aligned}\right\} \quad (15)$$

Hence

$$A'_1=\frac{4a^2V}{\sqrt{2}}k_2^2+\left(L_2-2-\frac{\beta_1}{k_2}\right)A'_0k_2^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

$$\frac{A'_2}{k_2^4}=-\tfrac{1}{2}A'_0\frac{\beta_2}{k_2^2}+\tfrac{1}{8}A'_0(L_2-2)+\frac{2a^2V}{\sqrt{2}}+\tfrac{1}{2}\frac{\beta_1}{k_2}\left\{A'_0(L_2-2)+\frac{4a^2V}{\sqrt{2}}-\tfrac{1}{4}A'_0\frac{\beta_1}{k_2}\right\} \quad (17)$$

Now by (9)

$$A'_0+A'_1+A'_2=-\frac{\mu_2 a}{\pi\sqrt{2}}$$

Hence to the second order

$$A'_0\left\{1+\left(L_2-2-\frac{\beta_1}{k_2}\right)k_2^2\right\}=-\frac{\mu_2 a}{\pi\sqrt{2}}-\frac{4a^2V}{\sqrt{2}}k_2^2$$

or

$$A'_0=-\frac{\mu_2 a}{\pi\sqrt{2}}+\left\{\frac{\mu_2 a}{\pi\sqrt{2}}\left(L_2-2-\frac{\beta_1}{k_2}\right)-\frac{4a^2V}{\sqrt{2}}\right\}k_2^2 \quad \ldots\ldots \quad (18)$$

Again, differentiating (13) with respect to k

$$\left.\begin{aligned}\frac{1}{\sqrt{2}}k\frac{\delta\psi_2}{\delta k}=&\tfrac{1}{2}A'_0\{1-(L-1)k^2\}-\frac{2a^2V}{\sqrt{2}}k^2\\&+\left\{-\tfrac{1}{2}A'_0(L-3)-\tfrac{1}{2}\frac{A'_1}{k^2}-\frac{2a^2V}{\sqrt{2}}\right\}k\cos v\\&+\left\{-\tfrac{3}{8}A'_0(2L-5)-\frac{2A'_2}{k^4}-\frac{6a^2V}{\sqrt{2}}\right\}k^2\cos 2v\end{aligned}\right\} \quad \ldots\ldots \quad (19)$$

Along the surface this becomes

$$\left.\begin{aligned}\frac{1}{\sqrt{2}}k\frac{\delta\psi_2}{\delta k}=&\tfrac{1}{2}A'_0\{1-(L_2-1)k_2^2\}\\&-\frac{2a^2V}{\sqrt{2}}k_2^2+\tfrac{1}{2}\beta_1\left\{-\tfrac{1}{2}A'_0(L_2-4)k_2+\tfrac{1}{2}\frac{A'_1}{k_2}-\frac{2a^2V}{\sqrt{2}}k_2\right\}\\&+\left\{-\tfrac{1}{2}A'_0(L_2-3)-\tfrac{1}{2}\frac{A'_1}{k_2^2}-\frac{2a^2V}{\sqrt{2}}\right\}k_2\cos v\\&+\left[-\tfrac{3}{8}A'_0(2L_2-5)-\frac{2A'_2}{k_2^4}-\frac{6a^2V}{\sqrt{2}}\right.\\&\left.+\tfrac{1}{2}\frac{\beta_1}{k_2}\left\{-\tfrac{1}{2}A'_0(L_2-4)+\tfrac{1}{2}\frac{A'_1}{k_2^2}-\frac{2a^2V}{\sqrt{2}}\right\}\right]k_2^2\cos 2v\end{aligned}\right\} \quad (20)$$

Further, to the third order of small quantities

$$\frac{1}{\sqrt{2}}\frac{\delta\psi_2}{\delta v}=-\left\{-\tfrac{1}{2}A'_0(L-2)+\tfrac{1}{2}\frac{A'_1}{k^2}-\frac{2a^2V}{\sqrt{2}}\right\}k\sin v$$

and along the surface

$$= -\left\{-\tfrac{1}{2}A'_0(L_2-2)+\tfrac{1}{2}\frac{A'_1}{k_2^2}-\frac{2a^3V}{\sqrt{2}}\right\}k_2\sin v$$
$$+ \text{terms of the fourth order.}$$

Hence by (15)

$$\frac{1}{\sqrt{2}}\frac{\partial\psi_2}{\partial v}=\tfrac{1}{2}A'_0\beta_1\sin v+\text{terms of the fourth order} \quad . \quad . \quad . \quad . \quad (21)$$

7. *The function* ψ_1.—The part depending on $A\rho^4$ and B must be carried to the fourth order. The part depending on the A_n is similar to the expression already deduced for ψ_2, without the terms in V.

$$\tfrac{1}{8}A\rho^4=\tfrac{1}{8}Aa^4\left(\frac{1-k^2}{1+k^2-2kc}\right)^4$$
$$=\tfrac{1}{8}Aa^4\{1+12k^2+42k^4+8k(1+6k^2)\cos v+20k^2(1+4k^2)\cos 2v$$
$$+40k^3\cos 3v+70k^4\cos 4v\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

$$\frac{1}{\sqrt{(C-c)}}B_0T_0=\frac{\pi k^{-\frac{1}{2}}}{4\sqrt{(C-c)}}B_0\{1+\tfrac{1}{4}k^2+\tfrac{1}{64}k^4\}$$
$$=\frac{\pi\sqrt{2}}{4}B_0\{1+\tfrac{1}{2}k^2+\tfrac{7}{32}k^4+k(1+\tfrac{5}{8}k^2)\cos v$$
$$+\tfrac{1}{4}k^2(3+2k^2)\cos 2v+\tfrac{5}{8}k^3\cos 3v+\tfrac{35}{64}k^4\cos 4v\}$$

$$\frac{1}{\sqrt{(C-c)}}B_1T_1\cos v=\frac{3\pi k^{\frac{1}{2}}}{8\sqrt{(C-c)}}(1-\tfrac{1}{8}k^2)\cos v$$
$$=\frac{3\pi\sqrt{2}}{8}B_1k\{\tfrac{1}{2}k+\tfrac{1}{8}k^3+(1+\tfrac{1}{2}k^2)\cos v+\tfrac{1}{2}k(1+\tfrac{7}{8}k^2)\cos 2v$$
$$+\tfrac{3}{8}k^2\cos 3v+\tfrac{5}{16}k^3\cos 4v\}$$

$$\frac{1}{\sqrt{(C-c)}}B_2T_2\cos 2v=\frac{15\pi k^{\frac{1}{2}}}{32\sqrt{(C-c)}}(1-\tfrac{1}{4}k^2)\cos 2v$$
$$=\frac{15\pi\sqrt{2}}{32}B_2k^2\{\tfrac{3}{8}k^2+\tfrac{1}{2}k\cos v+\cos 2v+\tfrac{1}{2}k\cos 3v+\tfrac{3}{8}k^2\cos 4v\}$$

$$\frac{1}{\sqrt{(C-c)}}B_3T_3\cos 3v=\frac{35\pi k^{\frac{1}{2}}}{64\sqrt{(C-c)}}B_3\cos 3v$$
$$=\frac{35\pi\sqrt{2}}{64}B_3k^3(\tfrac{1}{2}k\cos 2v+\cos 3v+\tfrac{1}{2}k\cos 4v)$$

$$\frac{1}{\sqrt{(C-c)}}B_4T_4\cos 4v=\pi\sqrt{2}\frac{5.7.9}{2^9}k^4\cos 4v$$

Hence $\frac{\psi_1}{\pi\sqrt{2}}$ (so far as it depends on B)=

$$\left.\begin{aligned}
&\tfrac{1}{4}B_0\left(1+\tfrac{1}{2}k^2+\tfrac{7}{32}k^4\right)+\tfrac{3}{16}B_1k^2\left(1+\tfrac{1}{4}k^2\right)+\tfrac{45}{256}B_2k^4+\tfrac{1}{8}\frac{Aa^4}{\pi\sqrt{2}}\left(1+12k^2+42k^4\right)\\
&+\left\{\tfrac{1}{2}\left(B_0+\tfrac{3}{4}B_1+\frac{4Aa^4}{\pi\sqrt{2}}\right)+\tfrac{1}{16}\left(\tfrac{1}{4}B_0+\tfrac{3}{8}B_1+\tfrac{15}{64}B_2+\frac{48Aa^4}{\pi\sqrt{2}}\right)k^2\right\}k\cos v\\
&+\left\{\tfrac{3}{16}\left(B_0+B_1+\tfrac{5}{8}B_2+\frac{40Aa^4}{3\pi\sqrt{2}}\right)+\tfrac{1}{16}\left(2B_0+\tfrac{21}{16}B_1+\tfrac{35}{64}B_2+\frac{160Aa^4}{\pi\sqrt{2}}\right)k^2\right\}k^2\cos 2v\\
&+\tfrac{1}{384}\left(5B_0+\tfrac{35}{8}B_1+\tfrac{105}{32}B_2+\tfrac{315}{256}B_3+\frac{160Aa^4}{\pi\sqrt{2}}\right)k^3\cos 3v\\
&+\tfrac{1}{1024}\left(\tfrac{315}{32}B_0+15B_1+\tfrac{495}{32}B_2+35B_3+\frac{5.7.9}{4}B_4+35\times 32.\frac{Aa^4}{\pi\sqrt{2}}\right)k^4\cos 4v
\end{aligned}\right\}\quad(23)$$

The next step is to substitute in this the value of k along the two surfaces, which will give the value of $\psi_1/\sqrt{2}$ along these surfaces. Suppose for the outer surface it becomes

$$p_0+p_1\cos v+p_2\cos 2v+p_3\cos 3v+p_4\cos 4v$$

The substitution $k=k_3(1+\beta_1\cos v+\beta_2\cos 2v+\ldots)$ being made it will be found that

$$\begin{aligned}
p_1=&\frac{\pi}{4}\left(B_0+\tfrac{3}{8}B_1+\frac{4Aa^4}{\pi\sqrt{2}}\right)k_3+\frac{\pi}{4}\left(\tfrac{3}{8}B_0+\tfrac{3}{8}B_1+\tfrac{15}{64}B_2+\frac{24Aa^4}{\pi\sqrt{2}}\right)k_3^3\\
&+\frac{3\pi}{16}\beta_1k_3^2\left(B_0+B_1+\tfrac{5}{8}B_2+\frac{40Aa^4}{3\pi\sqrt{2}}\right)+\frac{\pi}{4}\beta_1k_3^2\left(B_0+\tfrac{3}{8}B_1+\frac{12Aa^4}{\pi\sqrt{2}}\right)\\
&+\frac{\pi}{8}\beta_2k_2\left(B_0+\tfrac{3}{4}B_1+\frac{4Aa^4}{\pi\sqrt{2}}\right)
\end{aligned}$$

and the complete coefficient of $\cos v$ is found by adding the terms in A_2, viz., (15)

$$-\tfrac{1}{2}A_0(L_2-2)k_3+\tfrac{1}{2}\frac{A_1}{k_3}+\tfrac{1}{2}A_0\beta_1$$

This complete coefficient must vanish. Hence remembering that A_0 is of the second order with reference to Aa^4, it follows that

$$B_0+\tfrac{3}{4}B_1+\frac{4Aa^4}{\pi\sqrt{2}}=e_1\quad\ldots\ldots\ldots\quad(24)$$

where e_1 is of the second order of small quantities, and can therefore be put equal to zero when multiplied by quantities of the third or higher orders.

Again, it will be found that

$$p_2=\tfrac{5}{16}\left(B_0+B_1+\tfrac{5}{7}B_2+\frac{40A\alpha^4}{3\pi\sqrt{2}}\right)k_2^2+\tfrac{1}{16}\left(2B_0+\tfrac{11}{6}B_1+\tfrac{35}{6}B_3+\frac{160A\alpha^4}{\pi\sqrt{2}}\right)k_2^4$$
$$+\tfrac{1}{8}\left(B_0+\tfrac{5}{3}B_1+\frac{4A\alpha^4}{\pi\sqrt{2}}\right)\beta_1k_2+\tfrac{5}{32}\beta_1^2k_2^2\left(B_0+B_1+\tfrac{5}{3}B_2+\frac{40A\alpha^4}{3\pi\sqrt{2}}\right)$$
$$+\tfrac{5}{32}\beta_1k_2^3\left(\tfrac{5}{3}B_0+\tfrac{9}{4}B_1+\tfrac{15}{4}B_2+\tfrac{35}{4}B_3+\frac{80A\alpha^4}{\pi\sqrt{2}}\right)$$
$$+\tfrac{1}{8}\beta_1k_2^3\left(\tfrac{15}{8}B_0+\tfrac{9}{4}B_1+\tfrac{45}{16}B_2+\frac{72A\alpha^4}{\pi\sqrt{2}}\right)+\tfrac{1}{8}(2\beta_2+\tfrac{1}{2}\beta_1^2)k_2^2\left(B_0+\tfrac{5}{3}B_1+\frac{12A\alpha^4}{\pi\sqrt{2}}\right)$$
$$+\tfrac{1}{8}\beta_2k_2\left(B_0+\tfrac{5}{3}B_1+\frac{4A\alpha^4}{\pi\sqrt{2}}\right)$$

and the complete coefficient of cos $2v$ is found as before by adding

$$-\tfrac{1}{2}A_0(\tfrac{1}{4}\beta_1^2-\beta_2)-\tfrac{3}{8}A_0(L_2-2)k_2^2+\tfrac{1}{4}A_1+\frac{A_2}{k_2^2}-\tfrac{1}{4}\beta_1\left\{A_0(L_2-3)k_2+\frac{A_1}{k_2}\right\}$$

Remembering the remark as to the order of A_0, &c., it follows that

$$B_0+B_1+\tfrac{5}{3}B_2+\frac{40A\alpha^4}{3\pi\sqrt{2}}=e_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

where e_2 is of the second order of small quantities.

The coefficients of cos $3v$, cos $4v$ are easily found, viz.:—

$$p_3=\tfrac{1}{16}\left\{\tfrac{5}{3}B_0+\tfrac{9}{4}B_1+\tfrac{15}{4}B_2+\tfrac{35}{4}B_3+\frac{80A\alpha^4}{\pi\sqrt{2}}\right\}k_2^3=0$$

$$p_4=\tfrac{5}{32}\tfrac{5}{16}\left\{\tfrac{7}{5}B_0+3B_1+\tfrac{9}{5}B_2+7B_3+\tfrac{63}{4}B_4+\frac{224A\alpha^4}{\pi\sqrt{2}}\right\}k_2^4+\tfrac{5}{32}p_3\beta_1k_2^3=0$$

These equations give, to find the values of B to the *lowest* order.

$$\tfrac{7}{5}B_0+3B_1+\tfrac{9}{5}B_2+7B_3+\tfrac{63}{4}B_4+\frac{224A\alpha^4}{\pi\sqrt{2}}=0$$

$$\tfrac{5}{3}B_0+\tfrac{9}{4}B_1+\tfrac{15}{4}B_2+\tfrac{35}{4}B_3+\frac{80A\alpha^4}{\pi\sqrt{2}}=0$$

$$B_0+B_1+\tfrac{5}{3}B_2+\frac{40A\alpha^4}{3\pi\sqrt{2}}=0$$

$$B_0+\tfrac{5}{3}B_1+\frac{4A\alpha^4}{\pi\sqrt{2}}=0$$

These give the following values

$$\left.\begin{aligned} B_4 &= -\tfrac{9}{64}B_0 - \tfrac{2505}{3136}\cdot\frac{Aa^4}{\pi\sqrt{2}} \\ B_3 &= -\tfrac{3}{35}B_0 - \tfrac{252}{343}\cdot\frac{Aa^4}{\pi\sqrt{2}} \\ B_2 &= -\tfrac{3}{15}B_0 - \tfrac{64}{147}\cdot\frac{Aa^4}{\pi\sqrt{2}} \\ B_1 &= -\tfrac{2}{5}B_0 - \tfrac{4}{5}\cdot\frac{Aa^4}{\pi\sqrt{2}} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots \quad (26)$$

These values of the B greatly simplify the coefficients of $\cos v$ and $\cos 2v$. In that of $\cos v$ occurs the expression

$$\tfrac{4}{5}B_0 + \tfrac{3}{4}B_1 + \tfrac{1}{16}B_2 + \frac{24Aa}{\pi\sqrt{2}}$$

By means of the above equations it follows at once that its value is $18Aa^4/(\pi\sqrt{2})$. The coefficient of $(\cos v)/\sqrt{2}$ now becomes

$$-\tfrac{1}{2}A_0(L_2-2)k_2 + \tfrac{1}{3}\frac{A_1}{k_2} + \tfrac{1}{2}A_0\beta_1 + \frac{\pi}{4}c_1k_2 + \frac{9Aa}{2\sqrt{2}}k_2^3 + \frac{2Aa}{\sqrt{2}}\beta_1k_2^3.$$

So also the expression

$$4B_0 + \tfrac{21}{4}B_1 + \tfrac{35}{4}B_2 + \frac{320Aa^4}{\pi\sqrt{2}}$$

occurring in the coefficient of $\cos 2v$ is found to be $248Aa^4/(\pi\sqrt{2})$, and the coefficient becomes

$$-\tfrac{1}{2}A_0(\tfrac{1}{4}\beta_1^2-\beta_2) - \tfrac{3}{8}A_0(L_2-2)k_2^2 + \tfrac{1}{4}A_1 + \frac{A_2}{k_2^2} - \tfrac{1}{4}\beta_1\left\{A_0(L_2-3)k_2 + \frac{A_1}{k_2}\right\}$$
$$+\frac{3\pi}{16}c_2k_2^2 + \tfrac{31}{4}\frac{Aa^4}{\sqrt{2}}k_2^4 + \frac{\pi}{8}\beta_1k_2c_1 + \tfrac{27}{4}\frac{Aa^4}{\sqrt{2}}\beta_1k_2^3 + (\beta_2+\tfrac{1}{4}\beta_1^2)k_2^2\frac{2Aa^4}{\sqrt{2}}.$$

Now these coefficients vanish both at the outer and inner surfaces—their values for the inner will be found by writing k_1 for k_2, $\beta_1=0$ and α_2 for β_2. They are written down later in equations B.

The pressure conditions require a knowledge of $k\delta\psi/\delta k$. So far as it depends on the B it is given by (differentiating 23)

$$\frac{1}{\pi\sqrt{2}}k\frac{\delta\psi_1}{\delta k}$$
$$= \tfrac{1}{4}(k^2+\tfrac{7}{8}k^4)B_0 + \tfrac{3}{16}(2k^2+k^4)B_1 + \tfrac{45}{64}B_2k^4 + \frac{Aa^4}{\pi\sqrt{2}}(3k^2+21k^4)$$
$$+\left\{\tfrac{1}{4}B_0k(1+\tfrac{15}{8}k^2) + \tfrac{3}{8}B_1k(1+\tfrac{3}{8}k^2) + \tfrac{45}{64}B_2k^3 + \frac{Aa^4}{\pi\sqrt{2}}(k+18k^3)\right\}\cos v$$
$$+\left\{\tfrac{1}{8}B_0k^2(3+4k^2) + \tfrac{3}{8}B_1k^2(1+\tfrac{7}{4}k^2) + \tfrac{15}{16}B_2k^2 + \tfrac{35}{32}B_2k^4 + \frac{5Aa^4}{\pi\sqrt{2}}(k^2+8k^4)\right\}\cos 2v$$

the coefficient of $\cos 3v$, $\cos 4v$ vanishing by what has gone before.

Substituting for k, and putting in the terms involving A_2 from (20), the value along the outer surface is

$$\left.\begin{aligned}
&\frac{1}{\sqrt{2}}k\frac{\delta\psi_1}{\delta k}\\
&=\tfrac{1}{2}A_0\left\{1-(L_2-1)k_2^2\right\}+\tfrac{1}{2}\beta_1\left\{-\tfrac{1}{2}A_0(L_2-4)k_2+\tfrac{1}{2}\frac{A_1}{k_2}\right\}+\frac{\pi}{4}e_1k_2^2\\
&+\frac{\pi}{8}e_1\beta_1k_2+\frac{Aa^4}{\sqrt{2}}\left(2k_2^3+\tfrac{3}{8}k_2^4+\beta_1^2k_2^3+\tfrac{5}{4}\beta_1k_2^3\right)\\
&+\left\{-\tfrac{1}{2}A_0(L_2-3)k_2-\tfrac{1}{2}\frac{A_1}{k_2}+\frac{\pi}{4}e_1k_2+\frac{Aa^4}{\sqrt{2}}(\tfrac{37}{8}k_2^3+4\beta_1k_2^2)\right\}\cos v\\
&+\left[-\tfrac{1}{8}A_0(2L_2-5)k_2^2-\frac{2A_2}{k_2^2}+\tfrac{1}{2}\beta_1\left\{-\tfrac{1}{2}A_0(L_2-4)+\tfrac{1}{2}\frac{A_1}{k_2^2}\right\}k_2\right.\\
&\left.+(\beta_2+\tfrac{1}{4}\beta_1^2)k_2^2\frac{4Aa^4}{\sqrt{2}}+\frac{\pi}{8}e_1\beta_1k_2+\frac{3\pi}{8}e_2k_2^2+\frac{Aa^4}{\sqrt{2}}(31k_2^4+\tfrac{5}{4}\beta_1k_2^3)\right]\cos 2v
\end{aligned}\right\}\quad(27)$$

The lowest term in $-\delta\psi_1/\delta v$ is

$$\sqrt{2}\left\{-\tfrac{1}{2}A_0(L-2)k+\tfrac{1}{2}\frac{A_1}{k}+\frac{\pi}{4}e_1k+\tfrac{1}{2}\frac{Aa^4}{\sqrt{2}}k^2\right\}\sin v$$

Along the outer surface this is

$$\sqrt{2}\left\{-\tfrac{1}{2}A_0(L_2-2)k_2+\tfrac{1}{2}\frac{A_1}{k_2}+\frac{\pi}{4}e_1k_2+\tfrac{5}{2}\frac{Aa^4}{\sqrt{2}}k_2^2\right\}\sin v$$
$$+\text{ terms of the fourth order.}$$

By Eq. (A.2) below this is

$$-\tfrac{1}{2}\beta_1\left(A_0+\frac{4Aa^4}{\sqrt{2}}k_2^2\right)\sin v\quad\ldots\ldots\ldots\quad(28)$$

and is therefore of the third order.

8. *Pressure conditions.*—Along the inner surface the pressure is zero, and since it is a stream line the velocity must be constant. Denote it by U, then

$$U^2=\frac{1}{\rho^2}\left\{\left(\frac{\delta\psi_1}{\delta u}\right)^2+\left(\frac{\delta\psi_1}{\delta v}\right)^2\right\}\left(\frac{du}{dn}\right)^2$$
$$=\frac{(C-c)^4}{a^4S^2}\left\{\left(k\frac{\delta\psi_1}{\delta k}\right)^2+\left(\frac{\delta\psi_1}{\delta v}\right)^2\right\}$$

Now along this surface $a_1=0$, and therefore $\delta\psi_1/\delta v$ is of the fourth order, whilst $k\delta\psi_1/\delta k$ is of the second. Hence it may be neglected, and

$$a^2U=-\frac{(C-c)^2}{S}k\frac{\delta\psi_1}{\delta k}$$

therefore

$$-2a^2Uk=(1+5k^2-4k\cos v+2k^2\cos 2v)k\frac{\delta\psi_1}{\delta k}$$

or along the surface

$$-2a^2Uk_1(1+a_2\cos 2v)=(1+5k_1^2-4k_1\cos v+2k_1^2\cos 2v)k\frac{\delta\psi_1}{\delta k}.$$

Suppose for the moment

$$k\frac{\delta\psi_1}{\delta k}=q_0+q_1\cos v+q_2\cos 2v$$

then

$$\begin{aligned}
&q_0(1+5k_1^2)-2q_1k_1=-2a^2U_0k_1\\
&q_1-4q_0k_1=0\\
&q_2-2q_1k_1+2q_0k_1^2=-2a^2U_0k_1a_2
\end{aligned}$$

therefore

$$\begin{aligned}
q_0(1-3k_1^2)&=-2a^2U_0k_1\\
q_1-4k_1q_0&=0\\
q_2-6q_0k_1^2-a_2q_0&=0
\end{aligned}$$

These give the equations (C) below.

Before proceeding to introduce the last condition of equality of pressure on both sides of the outer boundary of the core, it will be convenient to collect here and partly discuss the equations already obtained. They are

A (no $\cos v$ in ψ_1)

$$\left.\begin{aligned}
&-\tfrac{1}{2}A_0(L_1-2)k_1+\tfrac{1}{2}\frac{A_1}{k_1}+\frac{\pi}{4}e_1k_1+\tfrac{3}{8}\frac{Aa^4}{\sqrt{2}}k_1^3=0 \quad (1)\\
&-\tfrac{1}{2}A_0(L_2-2)k_2+\tfrac{1}{2}\frac{A_1}{k_2}+\frac{\pi}{4}e_1k_2+\tfrac{3}{8}\frac{Aa^4}{\sqrt{2}}k_2^3+\tfrac{1}{2}\beta_1\left(A_0+\frac{4Aa^4}{\sqrt{2}}k_2^2\right)=0 \quad (2)
\end{aligned}\right\}$$

B (no $\cos 2v$)

$$\left.\begin{aligned}
&-\tfrac{3}{8}A_0(L_1-2)k_1^3+\tfrac{1}{4}A_1+\frac{A_2}{k_1^2}+\frac{3\pi}{16}e_2k_1^3+\tfrac{21}{4}\frac{Aa^4}{\sqrt{2}}k_1^4+\tfrac{1}{2}a_2\left(A_0+\frac{4Aa^4}{\sqrt{2}}k_1^2\right)=0\\
&-\tfrac{3}{8}A_0(L_2-2)k_2^3+\tfrac{1}{4}A_1+\frac{A_2}{k_2^2}+\frac{3\pi}{16}e_2k_2^3+\tfrac{21}{4}\frac{Aa^4}{\sqrt{2}}k_2^4+\tfrac{1}{2}\beta_2\left(A_0+\frac{4Aa^4}{\sqrt{2}}k_2^2\right) \quad (1)\\
&-\tfrac{1}{2}\beta_1\left\{\tfrac{1}{4}\beta_1\left(A_0-\frac{4Aa^4}{\sqrt{2}}k_2^2\right)+\tfrac{1}{2}A_0(L_2-3)k_2+\tfrac{1}{2}\frac{A_1}{k_2}-\frac{\pi}{4}e_1k_2-\tfrac{27}{8}\frac{Aa^4}{\sqrt{2}}k_2^3\right\}=0 \quad (2)
\end{aligned}\right\}$$

C (constant pressure inside)

$$\left.\begin{aligned}
&\tfrac{1}{2}A_0\{1-(L_1-1)k_1^2\}+\frac{\pi}{4}e_1k_1^2+\frac{2Aa^4}{\sqrt{2}}k_1^2+\tfrac{3}{8}\frac{Aa^4}{\sqrt{2}}k_1^4=-\frac{2a^2U\mu}{\sqrt{2}}(1+3k_1^2) &&(1)\\
&-\tfrac{1}{2}A_0(L_1+1)k_1-\tfrac{1}{2}\frac{A_1}{k_1}+\frac{\pi}{4}e_1k_1+\tfrac{11}{8}k_1^3\frac{Aa^4}{\sqrt{2}}=0 &&(2)\\
&-\tfrac{3}{8}A_0(2L_1+3)k_1^2-\frac{2A_2}{k_1^2}-\tfrac{1}{2}\alpha_2\left(A_0-\frac{4Aa^4}{\sqrt{2}}k_1^2\right)+\frac{3\pi}{8}e_2k_1^2+19\frac{Aa^4}{\sqrt{2}}k_1^4=0 &&(3)
\end{aligned}\right\}$$

Values of A_0, A_1.—Subtract C (2) from A (1) and divide by k_1

$$\tfrac{3}{2}A_0+\frac{A_1}{k_1^3}-\frac{Aa^4}{\sqrt{2}}k_1^2=0$$

therefore

$$A_1=-\tfrac{3}{2}A_0k_1^3+\frac{Aa^4}{\sqrt{2}}k_1^4 \quad \ldots\ldots\ldots\ldots \quad (29)$$

Now (10)

$$A_0+A_1+A_2=-\frac{\mu_1 a}{\pi\sqrt{2}}-\frac{4Aa^4}{\sqrt{2}}(k_1^2+4k_1^4)$$

Therefore to the fourth order

$$(1-\tfrac{3}{2}k_1^2)A_0=-\frac{\mu_1 a}{\pi\sqrt{2}}-\frac{Aa^4}{\sqrt{2}}k_1^2(4+17k_1^2)$$

or

$$A_0=-\frac{\mu_1 a}{\pi\sqrt{2}}(1+\tfrac{3}{2}k_1^2)-\frac{4Aa^4}{\sqrt{2}}k_1^2(1+\tfrac{23}{4}k_1^2)$$

or to the lowest orders respectively

$$\left.\begin{aligned}
A_0&=-\frac{\mu_1 a}{\pi\sqrt{2}}-\frac{4Aa^4}{\sqrt{2}}k_1^2\\
A_1&=\tfrac{3}{2}\frac{\mu_1 a}{\pi\sqrt{2}}k_1^3+\frac{7Aa^4}{\sqrt{2}}k_1^4
\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (29a)$$

Value of β_1.—In equations A divide (1) by k_1, (2) by k_2, and subtract

$$\tfrac{1}{2}A_0(L_2-L_1)+\tfrac{1}{2}A_1\frac{k_2^3-k_1^3}{k_1^3k_2^3}-\tfrac{3}{8}\frac{Aa^4}{\sqrt{2}}(k_2^2-k_1^2)-\tfrac{1}{2}\frac{\beta_1}{k_2}\left(A_0+\frac{4Aa^4}{\sqrt{2}}k_2^2\right)=0$$

Therefore

$$\begin{aligned}
\tfrac{1}{2}\frac{\beta_1}{k_2}\left(A_0+\frac{4Aa^4}{\sqrt{2}}k_2^2\right)&=\tfrac{1}{2}A_0(L_2-L_1)-\tfrac{3}{4}A_0\left(1-\frac{k_1^3}{k_2^3}\right)-\tfrac{1}{2}\frac{Aa^4}{\sqrt{2}}(k_2^3-k_1^3)\left(9-\frac{k_1^6}{k_2^6}\right)\\
&=-\tfrac{1}{2}A_0\left\{\log\frac{k_2}{k_1}+\tfrac{3}{4}\left(1-\frac{k_1^3}{k_2^3}\right)\right\}-\tfrac{1}{2}\frac{Aa^4}{\sqrt{2}}(k_2^3-k_1^3)\left(9-\frac{k_1^6}{k_2^6}\right)
\end{aligned}$$

or substituting for A_0

$$\frac{\beta_1}{k_2}\left\{-\frac{\mu_1 a}{\pi\sqrt{2}}+\frac{4A a^4}{\sqrt{2}}(k_2^2-k_1^2)\right\}=\left\{\frac{\mu_1 a}{\pi\sqrt{2}}-\frac{\mu' a}{\pi\sqrt{2}}\frac{k_1^2}{k_2^2-k_1^2}\left\{\log\frac{k_2}{k_1}+\frac{3}{2}\left(1-\frac{k_1^2}{k_2^2}\right)\right\}\right.$$
$$\left.+\frac{1}{4}\frac{\mu' a}{\pi\sqrt{2}}\left(9-\frac{k_1^2}{k_2^2}\right)\right.$$

whence

$$\frac{\beta_1}{k_2}=-\frac{\mu_1-\frac{k_1^2}{k_2^2-k_1^2}\mu'}{\mu_1+\mu'}\left\{\log\frac{k_2}{k_1}+\frac{3}{2}\left(1-\frac{k_1^2}{k_2^2}\right)\right\}-\frac{1}{4}\frac{\mu'}{\mu_1+\mu'}\left(9-\frac{k_1^2}{k_2^2}\right) \quad . \quad . \quad . \quad (30)$$

Case (1) when there is no added circulation

$$\frac{\beta_1}{k_2}=\frac{k_1^2}{k_2^2-k_1^2}\log\frac{k_2}{k_1}+\frac{7}{4}\frac{k_1^2}{k_2^2}-\frac{9}{4}$$

Case (2) when the added circulation is very large

$$\frac{\beta_1}{k_2}=-\left\{\log\frac{k_2}{k_1}+\frac{3}{2}\left(1-\frac{k_1^2}{k_2^2}\right)\right\}$$

Value of e_1.—Substituting for A_1 in equation A (1)

$$-\tfrac{1}{2}A_0(L_1-\tfrac{1}{2})+\frac{\pi}{4}e_1+5\frac{Aa^4}{\sqrt{2}}k_1^2=0$$

or

$$\frac{\pi}{4}e_1=-\tfrac{1}{2}\left(\frac{\mu_1 a}{\pi\sqrt{2}}-\frac{k_1^2}{k_2^2-k_1^2}\frac{\mu' a}{\pi\sqrt{2}}\right)L_1+\frac{\mu_1 a}{4\pi\sqrt{2}}+\frac{k_1^2}{k_2^2-k_1^2}\frac{\mu' a}{\pi\sqrt{2}}$$
$$=-\frac{a}{\pi\sqrt{2}}\left\{\tfrac{1}{2}\left(\mu_1-\frac{k_1^2}{k_2^2-k_1^2}\mu'\right)\log\frac{4}{R_1}-\tfrac{1}{4}\mu_1-\frac{k_1^2}{k_2^2-k_1^2}\mu'\right\} \quad . \quad . \quad . \quad (31)$$

Value of U_1.—Substituting in C (1), the surface velocity is given by

$$-\frac{2a^2U_1k_1}{\sqrt{2}}(1+3k_1^2)=\tfrac{1}{2}A_0(1+\tfrac{1}{2}k_1^2)+\frac{2Aa^4}{\sqrt{2}}k_1^2+\frac{3}{2}\frac{Aa^4}{\sqrt{2}}k_1^4$$

$$U_1=-\frac{k_1^{-1}}{a^2\sqrt{2}}\left\{\tfrac{1}{2}A_0(1-\tfrac{1}{2}k_1^2)+\frac{2Aa^4}{\sqrt{2}}k_1^2+\tfrac{1}{2}\frac{Aa^4}{\sqrt{2}}k_1^4\right\}$$
$$=\frac{k_1^{-1}}{2a^2\sqrt{2}}\left\{\frac{\mu_1 a}{\pi\sqrt{2}}(1-k_1^2)+\frac{4Aa^4}{\sqrt{2}}k_1^2(1+\tfrac{13}{4}k_1^2)-\frac{4Aa^4}{\sqrt{2}}k_1^2-13\frac{Aa^4}{\sqrt{2}}k_1^4\right\}$$
$$=\frac{\mu_1}{4\pi a k_1}(1-k_1^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

This is the velocity along the surface of the core, not just outside it.

Equations B (1) (2) and C (3) become when the values already obtained are substituted

$$\left.\begin{aligned}
&-\tfrac{3}{8}A_0(L_1-1)k_1^3+\frac{A_2}{k_1^2}+\frac{3\pi}{16}c_2k_1^3+8\frac{Aa^4}{\sqrt{2}}k_1^4+\tfrac{1}{2}a_2\left(A_0+\frac{4Aa^4}{\sqrt{2}}k_1^2\right)=0\\
&-\tfrac{3}{8}A_0(L_2-1)k_2^3+\tfrac{3}{8}A_0(k_2^3-k_1^3)+\frac{A_2}{k_2^2}+\frac{3\pi}{16}c_2k_2^3+8\frac{Aa^4}{\sqrt{2}}k_2^4-\tfrac{1}{2}\cdot\frac{Aa^4}{\sqrt{2}}(k_2^4-k_1^4)\\
&\qquad+\tfrac{1}{2}\beta_2\left(A_0+\frac{4Aa^4}{\sqrt{2}}k_2^2\right)+\tfrac{1}{2}\frac{\beta_1}{k_2}\left\{\tfrac{1}{2}A_0(k_2^3+3k_1^3)+\frac{Aa^4}{\sqrt{2}}(9k_2^4-k_1^4)\right.\\
&\qquad\qquad\left.-\tfrac{1}{2}\beta_1k_2\left(3A_0+\frac{4Aa^4}{\sqrt{2}}k_2^2\right)\right\}=0\\
&-\tfrac{3}{8}A_0(2L_1+3)k_1^3-\frac{2A_2}{k_1^2}+\frac{3\pi}{8}c_2k_1^3+19\frac{Aa^4}{\sqrt{2}}k_1^4-\tfrac{1}{2}a_2\left(A_0-\frac{4Aa^4}{\sqrt{2}}k_1^2\right)=0
\end{aligned}\right\}\quad(33)$$

Subtracting one-half the third from the first

$$\left.\begin{aligned}
&\tfrac{1}{16}A_0k_1^3+A_2\left(\frac{1}{k_1^2}+\frac{1}{k_2^2}\right)-\tfrac{3}{4}\frac{Aa^4}{\sqrt{2}}k_1^4+\tfrac{1}{4}a_2\left(3A_0+\frac{4Aa^4}{\sqrt{2}}k_1^2\right)=0\\
&\text{Eliminating } c_2 \text{ from the first and second}\\
&-\tfrac{3}{8}A_0\left\{L_1-L_2+1-\frac{k_1^3}{k_2^3}\right\}+A_2\left(\frac{1}{k_1^4}-\frac{1}{k_2^4}\right)+\frac{8Aa^4}{\sqrt{2}}(k_1^3-k_2^3)+\tfrac{1}{4}\frac{Aa^4}{\sqrt{2}}\frac{k_2^4-k_1^4}{k_2^2}\\
&\qquad+\tfrac{1}{2}\frac{a_2}{k_2^2}\left(A_0+\frac{4Aa^4}{\sqrt{2}}k_1^2\right)-\tfrac{1}{2}\frac{\beta_2}{k_2^2}\left(A_0+\frac{4Aa^4}{\sqrt{2}}k_2^2\right)\\
&\qquad-\tfrac{1}{2}\frac{\beta_1}{k_2}\left\{\tfrac{1}{2}A_0(k_2^3+3k_1^3)+\frac{Aa^4}{\sqrt{2}}(9k_2^4-k_1^4)-\tfrac{1}{4}\beta_1k_2\left(3A_0+\frac{4Aa^4}{\sqrt{2}}k_2^2\right)\right\}=0
\end{aligned}\right\}\quad(33a)$$

In order to determine V, the velocity of translation, and the quantities a_2, β_2, c_2, A_2, it will be necessary to consider the remaining condition, viz., of equality of pressure on both sides of the outer boundary of the core. We take the general case where the densities of the fluid in the outside and inside are different. In this case, even when there is no additional circulation in the outer fluid, the velocities on the two sides of the surface will be different. Let dashed letters refer to the outer fluid; also let p denote pressure, d density, and let Π = pressure at an infinite distance. Then

$$\frac{p'}{d'}=\text{const}-\tfrac{1}{2}(\text{vel})^2$$

therefore

$$p'=\Pi-\tfrac{1}{2}d'.U^2.$$

If U_2 denote the velocity along the surface inside the core

$$p=\text{const}-\tfrac{1}{2}d.U_2^2+Ad.\psi_2$$

also

$$0 = \text{const} - \tfrac{1}{2}d\mathrm{U}_1^2 + \mathrm{A}d.\psi_1$$

therefore

$$\begin{aligned} p &= \tfrac{1}{2}d\mathrm{U}_1^2 + \mathrm{A}d(\psi_2 - \psi_1) - \tfrac{1}{2}d.\mathrm{U}_2^2 \\ &= \Pi - \tfrac{1}{2}d'\mathrm{U}'^2_2 \end{aligned}$$

along the boundary. Suppose for the moment that

$$\begin{aligned} \mathrm{U}_2^2 &= l + m\cos v + n\cos 2v \\ \mathrm{U}_2'^2 &= l' + m'\cos v + n'\cos 2v \end{aligned}$$

Then the equations of condition are

$$\left.\begin{aligned} \Pi - \tfrac{1}{2}d'l' &= \tfrac{1}{2}d\mathrm{U}_1^2 + \mathrm{A}d(\psi_2 - \psi_1) - \tfrac{1}{2}dl \\ d'm' &= dm \\ d'n' &= dn \end{aligned}\right\} \quad \ldots\ldots \quad (34)$$

Now

$$\begin{aligned} \mathrm{U}^2 &= \frac{\mathrm{S}^2}{\rho^4}\left\{\left(\frac{\delta\psi}{\delta u}\right)^2 + \left(\frac{\delta\psi}{\delta v}\right)^2\right\} \\ &= \frac{(\mathrm{C}-c)^4}{a^4\mathrm{S}^2}\left\{\left(k\frac{\delta\psi}{\delta k}\right)^2 + \left(\frac{\delta\psi}{\delta v}\right)^2\right\} \end{aligned}$$

Further

$$\frac{(\mathrm{C}-c)^2}{\mathrm{S}} = \frac{(1+k^2-2kc)^2}{2k(1-k^2)} = \frac{1+5\kappa^2-4k\cos v+2k^2\cos 2v}{2k}$$

and along the surface this becomes

$$\frac{1}{2k_2}\{1+5k_2^2+\tfrac{1}{2}\beta_1^2-4k_2\cos v+(\tfrac{1}{2}\beta_1^2-\beta_2+2k_2^2)\cos 2v\}$$

Also substituting in (20) the values of A'_1, A'_2 determined from (15)

$$\begin{aligned} \frac{1}{\sqrt{2}}k_2\frac{\delta\psi_2}{\delta k} &= \tfrac{1}{2}\mathrm{A}'_0\{1-(\mathrm{L}_2-1)k_2^2\} - \frac{2a^2\mathrm{V}}{\sqrt{2}}k_2^2 + \tfrac{1}{2}\mathrm{A}'_0\beta_1(k_2-\tfrac{1}{2}\beta_1) \\ &\quad + \left\{-\tfrac{1}{2}\mathrm{A}'_0(2\mathrm{L}_2-5) - \frac{4a^2\mathrm{V}}{\sqrt{2}} + \tfrac{1}{2}\mathrm{A}'_0\frac{\beta_1}{k_2}\right\}k_2\cos v \\ &\quad + \left\{-\mathrm{A}'_0(\mathrm{L}_2-1\tfrac{1}{8}) - \frac{10a^2\mathrm{V}}{\sqrt{2}} + \mathrm{A}'_0\frac{\beta_2}{k_2^2} - \frac{\beta_1}{k_2}\left(\tfrac{1}{2}\mathrm{A}'_0\overline{2\mathrm{L}_2-5} + \frac{4a^2\mathrm{V}}{\sqrt{2}}\right)\right\}k_2^2\cos 2v \end{aligned}$$

Hence

$$\begin{aligned} \frac{(\mathrm{C}-c)^2}{a^2\mathrm{S}}\left(k\frac{\delta\psi_2}{\delta k}\right) &= \frac{1}{a^2k_2\sqrt{2}}\left[\tfrac{1}{2}\mathrm{A}'_0\{1+(3\mathrm{L}_2-4)k_2^2\} + \frac{6a^2\mathrm{V}}{\sqrt{2}}k_2^2 - \tfrac{1}{2}\mathrm{A}'_0\beta_1k_2\right. \\ &\quad + \left\{-\mathrm{A}'_0(\mathrm{L}_2-\tfrac{1}{2})k_2 - \frac{4a^2\mathrm{V}}{\sqrt{2}}k_2 + \tfrac{1}{2}\mathrm{A}'_0\beta_1\right\}\cos v \\ &\quad + \left\{\tfrac{1}{2}\mathrm{A}'_0(\tfrac{1}{2}\beta_1^2+\beta_2) + \tfrac{1}{2}\mathrm{A}'_0(2\mathrm{L}_2-1\tfrac{1}{4})k_2^2 - \frac{2a^2\mathrm{V}}{\sqrt{2}}k_2^2\right. \\ &\qquad \left.\left.-\beta_1k_2\left(\mathrm{A}'_0\overline{\mathrm{L}_2-\tfrac{5}{8}} + \frac{4a^2\mathrm{V}}{\sqrt{2}}\right)\right\}\cos 2v\right] \end{aligned}$$

Also (21)

$$\frac{(C-c)^2}{a^2S}\frac{\delta\psi}{\delta v}=\frac{1}{2\sqrt{2}}\frac{1}{k_2a^2}A'_0\beta_1\sin v$$

which in the square gives

$$\frac{1}{16a^4k_2^2}A'^2_0\beta_1^2(1-\cos 2v)$$

For the first approximation we shall keep as far as $\cos v$. In this case

$$\frac{(C-c)^2}{a^2S}k\frac{\delta\psi}{\delta k}=\frac{1}{a^2k_2\sqrt{2}}\left[\tfrac{1}{2}A'_0-\left\{A'_0(L_2-\tfrac{1}{2})+\frac{4a^2V}{\sqrt{2}}-\tfrac{1}{2}A'_0\frac{\beta_1}{k_2}\right\}k_2\cos v\right]$$

and in the square

$$\frac{1}{2a^4k_2^2}\left[\tfrac{1}{4}A'^2_0-A'_0\left\{A'_0(L_2-\tfrac{1}{2})+\frac{4a^2V}{\sqrt{2}}-\tfrac{1}{2}A'_0\frac{\beta_1}{k_2}\right\}k_2\cos v\right]$$

or, to lowest orders

$$l'=\frac{1}{8a^4k_2^2}A'^2_0$$

$$m'=-\frac{1}{2a^4k_2}\left\{A'_0(L_2-\tfrac{1}{2})+\frac{4a^2V}{\sqrt{2}}-\tfrac{1}{2}A'_0\frac{\beta_1}{k_2}\right\}A'_0$$

So, to the first three orders of k, we get from (27)

$$\frac{1}{\sqrt{2}}k_2\frac{\delta\psi_1}{\delta k}=\tfrac{1}{2}A_0+\frac{2Aa^4}{\sqrt{2}}k_2^2+\left\{-\tfrac{1}{2}A_0(L_2-3)-\frac{A_1}{2k_2^2}+\frac{\pi}{4}e_1+\frac{Aa^4}{\sqrt{2}}(\tfrac{3}{2}k_2^2+4\beta_1k_2)\right\}k_2\cos v$$

Substituting for e_1 and A_1 from (A.2) and (29)

$$\frac{1}{\sqrt{2}}k_2\frac{\delta\psi_1}{\delta k}=\tfrac{1}{2}\left(A_0+\frac{4Aa^4}{\sqrt{2}}k_2^2\right)+\left\{\tfrac{1}{2}A_0\left(1+3\frac{k_1^2}{k_2^2}\right)+\frac{Aa^4}{\sqrt{2}}\left(9-\frac{k_1^4}{k_2^4}\right)k_2^2-\tfrac{1}{2}\frac{\beta_1}{k_2}\left(A_0-\frac{4Aa^4}{\sqrt{2}}k_2^2\right)\right\}k_2\cos v$$

whence

$$\frac{(C-c)^2}{a^2S}k_2\frac{\delta\psi_1}{\delta k}=\frac{1}{a^2k_2\sqrt{2}}\left[\tfrac{1}{2}\left(A_0+\frac{4Aa^4}{\sqrt{2}}k_2^2\right)+\left\{-\tfrac{3}{2}A_0\left(1-\frac{k_1^2}{k_2^2}\right)+\frac{Aa^4}{\sqrt{2}}\left(1-\frac{k_1^4}{k_2^4}\right)k_2^2-\tfrac{1}{2}\frac{\beta_1}{k_2}\left(A_0-\frac{4Aa^4}{\sqrt{2}}k_2^2\right)\right\}k_2\cos v\right]$$

and the square is

$$=\frac{1}{2a^4k_2^2}\left[\tfrac{1}{4}\left(A_0+\frac{4Aa^4}{\sqrt{2}}k_2^2\right)^2+\left(A_0+\frac{4Aa^4}{\sqrt{2}}k_2^2\right)\left\{-\tfrac{3}{2}A_0\left(1-\frac{k_1^2}{k_2^2}\right)+\frac{Aa^4}{\sqrt{2}}\left(1-\frac{k_1^4}{k_2^4}\right)k_2^2-\tfrac{1}{2}\frac{\beta_1}{k_2}\left(A_0-\frac{4Aa^4}{\sqrt{2}}k_2^2\right)\right\}k_2\cos v\right]$$

Hence, applying the conditions (34)

$$\Pi-\frac{d'}{16a^4k_2^3}A'^2_0=\tfrac{1}{2}d.U_1^2+Ad(\psi_2-\psi_1)-\frac{d}{16a^4k_2^3}\left(A_0+\frac{4Aa^4}{\sqrt{2}}k_2^2\right)^2$$

$$-\frac{d'}{2a^4k_2}\left\{A'_0(L_2-\tfrac{1}{2})+\frac{4a^2V}{\sqrt{2}}-\tfrac{1}{2}A'_0\frac{\beta_1}{k_2}\right\}A'_0$$

$$=\frac{d}{2a^4k_2}\left(A_0+\frac{4Aa^4}{\sqrt{2}}k_2^2\right)\left\{-\tfrac{3}{2}A_0\left(1-\frac{k_1^2}{k_2^2}\right)+\frac{Aa^4}{\sqrt{2}}\left(1-\frac{k_1^6}{k_2^6}\right)k_2^2-\tfrac{1}{2}\frac{\beta_1}{k_2}\left(A_0-\frac{4Aa^4}{\sqrt{2}}k_2^2\right)\right\}$$

Now (29a) to the lowest order

$$A_0+\frac{4Aa^4}{\sqrt{2}}k_2^2=-\frac{\mu_1 a}{\pi\sqrt{2}}+\frac{4Aa^4}{\sqrt{2}}(k_2^2-k_1^2)$$

$$=-(\mu_1+\mu')\frac{a}{\pi\sqrt{2}}$$

and

$$A'_0=-\frac{\mu_2 a}{\pi\sqrt{2}} \quad \text{from (18)}$$

$$U_1=\frac{\mu_1}{4\pi a k_1} \quad \text{from (32)}$$

$$\psi_2-\psi_1=Aa^4(k_2^2-k_1^2)-\frac{\sqrt{2}}{2}A_0(L_2-L_1) \quad \text{from (15, 23)}$$

$$=-\frac{\mu' a}{4\pi}-\frac{1}{\sqrt{2}}A_0(L_2-L_1)$$

Substituting, the above equations become

$$\left.\begin{aligned}&\Pi-\frac{1}{32\pi^2a^2k_2^3}\{\mu_2^2d'-(\mu_1+\mu')^2d\}=\frac{\mu_1^2}{32\pi^2a^2k_1^2}d-Ad\left\{\frac{\mu' a}{4\pi}+\frac{1}{\sqrt{2}}A_0(L_2-L_1)\right\}\\&\frac{\mu_2 ad'}{2\pi^2}\left\{-\mu_2 a(L_2-\tfrac{1}{2})+4a^2\pi V+\tfrac{1}{2}\mu_2 a\frac{\beta_1}{k_2}\right\}\\&\qquad=-\frac{(\mu_1+\mu')ad}{2\pi^2}\left\{-\tfrac{3}{2}\pi\sqrt{2}A_0\left(1-\frac{k_1^2}{k_2^2}\right)+\pi Aa^4\left(1-\frac{k_1^4}{k_2^4}\right)k_2^2\right.\\&\qquad\qquad\left.-\frac{\pi\sqrt{2}}{2}\frac{\beta_1}{k_2}\left(A_0-\frac{4Aa^4}{\sqrt{2}}k_2^2\right)\right\}\end{aligned}\right\} \quad (35)$$

The latter is

$$\tfrac{1}{2}\mu_2^2d'\left(\frac{\beta_1}{k_2}-2L_2+1\right)a+4\mu_2a^2\pi d'V$$

$$=-(\mu_1+\mu')d\left\{\frac{3a}{2}\left(\mu_1-\frac{k_1^2}{k_2^2-k_1^2}\mu'\right)\frac{k_2^2-k_1^2}{k_2^2}-\tfrac{1}{2}\mu' a\left(1+\frac{k_1^2}{k_2^2}\right)+\frac{a}{2}\frac{\beta_1}{k_2}\left(\mu_1-\frac{k_2^2+k_1^2}{k_2^2-k_1^2}\mu'\right)\right\}$$

therefore

$$\left.\begin{aligned} 4\pi a d' \mu_2 \mathrm{V} = &-\tfrac{1}{2}\mu_2{}^2 d' \left(\frac{\beta_1}{k_2} - 2\mathrm{L}_2 + 1\right) \\ &-(\mu_1+\mu')d\left\{\frac{3}{2k_2{}^2}(\mu_1 k_2{}^2 - k_1{}^2 - \mu' k_1{}^2) - \tfrac{1}{4}\mu' \frac{k_2{}^2+k_1{}^2}{k_2{}^2}\right\} \\ &-\tfrac{1}{2}\left(\mu_1 - \frac{k_2{}^2+k_1{}^2}{k_2{}^2-k_1{}^2}\mu'\right) d.(\mu_1+\mu')\frac{\beta_1}{k_2} \end{aligned}\right\} \quad . \quad . \quad (36)$$

This gives V, the velocity of translation.

The first equation is

$$\Pi = \frac{1}{32\pi^2 a^2 k_2{}^2}\{\mu_2{}^2 d' - (\mu_1+\mu')^2 d\} + \frac{\mu_1{}^2}{32\pi^2 a^2 k_1{}^2} d + \frac{1}{16\pi^2 a^2}\left\{\mu' - 2\left(\mu_1 - \frac{k_1{}^2}{k_2{}^2-k_1{}^2}\mu'\right)(\mathrm{L}_2 - \mathrm{L}_1)\right\}\frac{\mu' d}{k_2{}^2 - k_1{}^2}$$

or

$$32\pi^2 a^2 \Pi = \{\mu_2{}^2 d' - (\mu_1+\mu')^2 d\}\frac{1}{k_2{}^2} + \frac{\mu_1{}^2 d}{k_1{}^2} + 2\left\{\mu' - 2\left(\mu_1 - \frac{k_1{}^2}{k_2{}^2-k_1{}^2}\mu'\right)(\mathrm{L}_2 - \mathrm{L}_1)\right\}\frac{\mu' d}{k_2{}^2 - k_1{}^2}$$

Another equation between a, k_1, k_2 is found from the fact that the volume of the core is constant, *i.e.*, (7)

$$8\pi^2 a^3 (k_2{}^2 - k_1{}^2) = \text{volume} = m \text{ say}$$

therefore

$$\frac{4m\Pi}{a} = \{\mu_2{}^2 d' - (\mu_1+\mu')^2 d\}\left(1 - \frac{k_1{}^2}{k_2{}^2}\right) + \mu_1{}^2 d. \frac{1 - \frac{k_1{}^2}{k_2{}^2}}{\frac{k_1{}^2}{k_2{}^2}} + 2\left\{\mu' - 2\left(\mu_1 - \frac{\frac{k_1{}^2}{k_2{}^2}}{1 - \frac{k_1{}^2}{k_2{}^2}}\mu'\right)(\mathrm{L}_2 - \mathrm{L}_1)\right\}\mu' d.$$

Write

$$k_1{}^2/k_2{}^2 = x$$

Then

$$\frac{4m\Pi}{a} = \{\mu_2{}^2 d' - (\mu_1+\mu')^2 d\}(1-x) + \mu_1{}^2 d\left(\frac{1}{x} - 1\right) + 2\mu' d\left\{\mu' - \left(\mu_1 - \frac{\mu' x}{1-x}\right)\log x\right\} \quad (37)$$

To find α_2, β_2, A_2, e_2, we must take account of one order higher and the terms in $\cos 2v$. This gives another equation, which, with the previous ones (33*a*), will be sufficient to determine them. As we do not require them for our present purposes we shall postpone their consideration to another occasion.

It remains to discuss the results already obtained, viz., the three expressions, which give β_1, V, and the relation between a and the k's (or R and r_1, r_2).

The formulæ are, writing x for $k_1{}^2/k_2{}^2$

$$(\mu_1+\mu')\frac{\beta}{k_2} = -\tfrac{1}{2}\mu_1\left\{\log\frac{1}{x} + 3(1-x)\right\} - \tfrac{1}{2}\mu'\left\{\tfrac{9}{2} - \tfrac{7}{2}x - \frac{x}{1-x}\log\frac{1}{x}\right\} \quad . \quad . \quad (30)$$

$$4\pi a\mu_2 d'\mathrm{V}=\tfrac{1}{2}\mu_2{}^2 d'\left\{-\frac{\beta_1}{k_2}+2\mathrm{L}_2-1\right\}$$
$$-\tfrac{1}{2}(\mu_1+\mu')d\left\{3(\mu_1\overline{1-x}-\mu' x)-\tfrac{1}{2}\mu'(1+x)-\left(\mu_1-\frac{1+x}{1-x}\mu'\right)\frac{\beta_1}{k_2}\right\} \quad . \quad (36)$$

$$\frac{4\pi\Pi}{a}=\{\mu_2{}^2 d'-(\mu_1+\mu')^2 d\}(1-x)+\mu_1{}^2 d\left(\frac{1}{x}-1\right)$$
$$+2\mu'\left\{\mu'+\left(\mu_1-\frac{x}{1-x}\mu'\right)\log\frac{1}{x}\right\}d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

Discussion of results.

9. (β_1). The expression for β_1 gives the relative position of the internal hollow to the outer boundary of the core. The centre of the cross-section of the outer boundary is outside that of the inner at a distance (by 5)

$$=2a(k_2{}^2-k_1{}^2+\beta_1 k_2)=2ak_2{}^2\left(1-x+\frac{\beta_1}{k_2}\right)$$

This is, as we shall see, in general negative. Hence the centre of the inner lies outside that of the outer, at a distance whose ratio (y) to the radius ($2ak_2$) of the outer is

$$y=-k_2\left(1-x+\frac{\beta_1}{k_2}\right)$$

Substituting for β_1/k_2

$$(\mu_1+\mu')y=k_2\left[\tfrac{1}{2}\mu_1\left\{\log\frac{1}{x}+3(1-x)\right\}+\tfrac{1}{2}\mu'\left\{\tfrac{9}{4}-\tfrac{7}{4}x-\frac{x}{1-x}\log\frac{1}{x}\right\}-(\mu_1+\mu')(1-x)\right]$$
$$=k_2\left\{\tfrac{1}{2}\mu_1\left(1-x+\log\frac{1}{x}\right)+\tfrac{1}{2}\mu'\left(\tfrac{5}{4}-\tfrac{3}{4}x-\frac{x}{1-x}\log\frac{1}{x}\right)\right\}$$
$$=\mu_1 y_1+\mu' y' \text{ (say)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

where y_1, y' are the values of y when μ_1 is very large, and zero respectively. If therefore we know the points at which the centres lie when the added circulation is very large, and when it is zero, the actual position is the centre of gravity of two masses proportional to the circulations, placed at the corresponding centres. It is only necessary therefore to discuss the values of y_1, y' separately.

When there is no added circulation

$$y'=\tfrac{1}{2}\left(\frac{5-3x}{2}-\frac{x}{1-x}\log\frac{1}{x}\right)k_2$$

When there is no hollow, $x=0$, $y'=\frac{5}{4}k_2$. This is the point at which, if the pressure be diminished, or the ring be increased large enough, the hollow will begin to form.

Again, when k_2 is uniform,

$$\frac{dy'}{dx}=-\frac{3}{4}+\frac{1}{1-x}-\frac{1!}{(1-x)^2}\log\frac{1}{x}$$

which is always negative, or y' continually decreases as x increases; while when $x=1$, $y'=0$, so that y' is always positive.

Hence, keeping the outside boundary the same, if the mass of the core be gradually diminished (or the hollow increased) the centre of the hollow moves in, from a point $(\frac{3}{4}k_2)^{\text{th}}$ of the radius, towards the centre of the outer boundary, coinciding with this last in the limit.

When the added circulation is very large

$$y_1=\tfrac{1}{2}\Big(1-x+\log\frac{1}{x}\Big)k_2$$

This is always positive. When $x=1$, or there is no rotational motion, $y_1=0$, as x decreases, y increases, and the hollow moves outwards to the boundary. From the formula itself we should gather that when the radius of the hollow is decreased to a certain small amount, it will be in contact with the outer boundary, and if this were possible the hollow would slip out of the inside of the core into the fluid, and there would be two rings formed, one with a hollow only and circulation round it, and another with rotational core and no added circulation. But this cannot be asserted until we have learnt the connexion between k_2 and a, from equation (37), for in the actual case it will be impossible to reduce k_1 below a certain limit to be determined. It is easy to see that $y_1>y'$.

An idea of the magnitudes of the quantities involved can best be obtained from a numerical example. Take for instance the case of a ring 10 cm. radius, and radius of cross-section 1 cm. Then $k_2=\frac{1}{10}$. Take the three cases where the radii of the cross-section of the hollow are respectively $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, we find

$x=\frac{1}{16}$	$y'=\cdot055$	$y_1=\cdot093$
$x=\frac{1}{4}$	$y'=\cdot041$	$y_1=\cdot053$
$x=\frac{9}{16}$	$y'=\cdot023$	$y_1=\cdot026$

The limit when $y_1=1$ is found from

$$\log\frac{1}{x}=39+x$$

or writing

$$x=e^{-39-\xi}$$

$$\xi=e^{-39}=10^{-16}\times\cdot115$$

i.e.

$$x=10^{-16}\times\cdot115$$

a sufficiently small quantity.

10. *Relation between* a, k_1, k_2.—When there is no hollow the volume of the core remains constant. In other words, $a^3k^3_2$ is constant. When a hollow exists the relation is found from the equation

$$\frac{4m\Pi}{a}=\{\mu_2{}^2d'-(\mu_1+\mu')^2d\}(1-x)+\mu_1{}^2d\left(\frac{1}{x}-1\right)+2\mu'\left\{\mu'+\left(\mu_1-\frac{x}{1-x}\mu'\right)\log\frac{1}{x}\right\}d \quad (37)$$

combined with that of constant volume, viz. :

$$m=8\pi^2a^3(k_2{}^2-k_1{}^2)$$

The first gives the ratio k_1/k_2, the second then gives k_1 or k_2 in terms of a, and a is determined by the energy of the motion, which is considered below.

In the case where there is no core,

$$\mu_1=\mu'=0,\ m=0,\ k_2=k_1$$

and

$$\Pi=\frac{\mu^2{}_2d'}{32\pi^2a^2k^2{}_2}$$

or

$$2ak_2=\frac{\mu_2}{2\pi}\sqrt{\frac{d'}{2\Pi}} \quad \ldots\ldots\ldots\ldots \quad (39)$$

or, the sectional radius of the hollow remains constant. This agrees with the result in [I. § 9].

It is clear that when there is an added circulation this radius cannot become very small. For we may write the equation

$$\mu_1d\left\{\frac{\mu_1}{x}+2\mu'\log\frac{1}{x}\right\}=\frac{4m\Pi}{a}-\{\mu_2{}^2d'-(\mu_1+\mu')^2d\}(1-x)+\mu_1{}^2d$$
$$-2\mu'^2d+2\mu'^2\frac{x}{1-x}\log\left(\frac{1}{x}\right)d$$

Now as x decreases from 1 to 0, the right hand side continually decreases. Hence the greatest value is when $x=1$

Therefore

$$\mu_1d\left(\frac{\mu_1}{x}+2\mu'\log\frac{1}{x}\right)<\frac{4m\Pi}{a}+\mu_1{}^2d$$
$$>\frac{4m\Pi}{a}-\mu_2{}^2d'+\{(\mu_1+\mu')^2+\mu_1{}^2-2\mu'^2\}d$$

Hence

$$x>\frac{\mu_1{}^2d+2\mu_1\mu'xd\log\frac{1}{x}}{\mu_1{}^2d+\frac{4m\Pi}{a}}>\frac{\mu_1{}^2d}{\mu_1{}^2d+\frac{4m\Pi}{a}}$$

This, therefore, is an inferior limit to x.

Now we saw above that $y_1=\frac{1}{3}\left(1-x+\log\frac{1}{x}\right)k_2$.

In the case where x is very small, the value of y depending on the added circulation is

$$\frac{\mu_1 y_1}{\mu_1+\mu'}=\frac{\mu_1 k_2}{2(\mu_1+\mu')}\log\frac{1}{x}$$

$$\text{and } x>\frac{\mu_1{}^2 d}{\mu_1{}^2 d+\frac{4m\Pi}{a}}$$

therefore

$$\frac{\mu_1 y_1}{\mu_1+\mu'}<\frac{\mu_1 k_2}{2(\mu_1+\mu')}\log\left(1+\frac{4m\Pi}{\mu_1{}^2 da}\right).$$

When $\mu_1{}^2 da$ is large compared with $4m\Pi$

$$y<\frac{2m\Pi k_2}{\mu_1{}^2 da}.$$

Hence, the centre of the hollow never approaches near the outer boundary.

Two or three cases further may be usefully considered. When there are no added circulations, $\mu_1=0$, $\mu_2=\mu'$, and

$$\frac{4m\Pi}{a}=\mu^2\left\{(1-x)d'+(1+x)d-\frac{2xd}{1-x}\log\frac{1}{x}\right\} \quad . \quad . \quad . \quad . \quad . \quad (40)$$

This gives the condition when a continuous core begins to develop a hollow. In this case $x=0$, and

$$\frac{4m\Pi}{a}=\mu^2(d+d')$$

or

$$a=\frac{4m\Pi}{\mu^2(d+d')} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (41)$$

Now a depends on the energy and increases with it. Hence as the energy increases a point is at last reached at which a continuous ring begins to develop a hollow. The sectional radius r_1 is then given by

$$8\pi^2\Pi r_1{}^2=\mu^2{}_2(d+d')$$

If a_1 be the value of a when a hollow is just formed, then in general

$$\frac{a_1}{a}(d+d')=(1-x)d'+(1+x)d-\frac{2xd}{1-x}\log 1/x,$$

or if $d'/d=\rho$

$$\frac{a_1}{a}(1+\rho)=(1-x)\rho+1+x-\frac{2x}{1-x}\log 1/x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

So also if r_0 be the radius of the hollow when $a=\infty$ and r for any other value of x

$$\frac{r^2}{r_0^2}\mu_2^2\rho=\mu_2^2\rho-(\mu_1+\mu')^2+\frac{\mu_1^2}{x}+2\mu'^2\frac{1-x+x\log x}{(1-x)^2}+2\mu_1\mu'\frac{\log 1/x}{1-x} \quad . \quad (43)$$

Where also $8\pi^2\Pi r^2{}_0=\mu^2{}_2\alpha'$.

Hence $r^2{}_0(1+\rho)=r_1{}^2\rho$, or when the hollow first forms the sectional radius is $\sqrt{\left(\frac{1+\rho}{\rho}\right)}$ its ultimate value.

It will be important to know how the section of the core alters as the aperture increases. If r be the radius of the section, $r=2ak_2$. Also $m=8\pi^2a^3(k^2{}_2-k^2{}_1)$ $=2\pi^2ar^2(1-x)$.

Hence

$$\frac{da}{a}+2\frac{dr}{r}-\frac{dx}{1-x}=0.$$

Differentiating (37) and substituting for $dx/(1-x)$ it will be found that

$$\left[\mu_1{}^2\frac{(1-x)^2}{x^2}+2\mu_1\mu'\frac{1-x}{x}-4\mu'^2+2\mu'\left(\frac{1+x}{1-x}\mu'-\mu_1\right)\log\frac{1}{x}\right]\frac{da}{a}$$

$$=-2\left[\left\{\mu_2{}^2\rho-(\mu_1+\mu')^2\right\}(1-x)+\mu_1{}^2\frac{1-x}{x^2}+2\mu'\left(\mu_1\frac{1-x}{x}-\mu'\right)+\frac{2\mu'^2}{1-x}\log\frac{1}{x}\right]\frac{dr}{r}.$$

Here the left hand factor is always positive, and the right hand factor is clearly positive when x is small whatever the relations between the circulations and densities. Hence *the section always decreases as the aperture increases, when the interior hollow is small compared with the outside.*

When x is nearly $=1=1-z$ say, the equation becomes

$$z^2(\mu_1{}^2+\mu_1\mu'+\tfrac{1}{3}\mu'^2)\frac{da}{a}=-2z\left\{\mu_2{}^2\frac{\alpha'}{\alpha}+2(\mu_1{}^2-\mu_1\mu'+\tfrac{1}{3}\mu'^2)z\right\}\frac{dr}{r}$$

Hence, when z is small, so also is dr/da, and it vanishes with z. In other words, *when the hollow is large compared with the outer boundary, the section of the ring decreases very slowly as the aperture increases.* When there is no core at all we have seen that it remains absolutely constant. In all cases where there is a hollow the section decreases more slowly than if the core were continuous.

It remains to see how the radius of the hollow itself changes with increasing aperture. Here $r'^2=xr^2$. Hence

$$2\frac{dr'}{r}=\frac{dx}{x}+2\frac{dr}{r}$$

$$=\frac{1-x}{x}\left(\frac{da}{a}+\frac{2dr}{r}\right)+\frac{2dr}{r}$$

$$=\frac{1}{x}\left\{(1-x)\frac{da}{a}+2\frac{dr}{r}\right\}$$

Substituting for dr/r

$$2\frac{dr'}{r'}=\frac{1}{x}\frac{\{\mu_2{}^2\rho-(\mu_1+\mu')^2\}(1-x)^2-2\mu_1\mu'(1-x)+2\mu'^2(1+x)+2\mu'\left(\mu_1-\frac{2x}{1-x}\mu'\right)\log\frac{1}{x}}{\mathrm{D}}\frac{da}{a}$$

Where D is the coefficient of $-2dr/r$ in the equation found above.

The maximum radius of the hollow will be when x has the value given by

$$\{\mu_2{}^2\rho-(\mu_1+\mu')^2\}(1-x)^2-2\mu_1\mu'(1-x)+2\mu'^2(1+x)+2\mu'\left(\mu_1-\frac{2x}{1-x}\mu'\right)\log\frac{1}{x}=0$$

This is satisfied by $x=1$.

It is easy to show that the expression on the left increases as x decreases, and therefore that dr/da is always positive. Hence

The radius of the hollow continually increases, as the aperture grows, up to coincidence with the outer boundary.

11. *Velocity of translation.* The velocity is given by the formula in (36). If the value of β_1/k_2 be substituted in the second term it becomes

$$4\pi a\mu_2 d'\mathrm{V}=\tfrac{1}{2}\mu_2{}^2 d'\left(2\mathrm{L}_2-1-\frac{\beta_1}{k_2}\right)+\tfrac{1}{2}\left(\mu_1-\frac{x}{1-x}\mu'\right)\left(\mu_1-\frac{1+x}{1-x}\mu'\right)d\log\frac{1}{x}$$

$$-\tfrac{1}{4}d\left\{3(1-x)\mu_1{}^2+\tfrac{1}{2}(7-13x)\mu_1\mu'+\frac{7x^2-10x+7}{2(1-x)}\mu'^2\right\}\quad\ldots\quad(44)$$

Case I. Where there is no core $d=0$, $\mu_1=\mu'=0$, $x=1$, $\beta_1=0$.

$$\mathrm{V}=\frac{\mu_2}{8\pi a}(2\mathrm{L}-1)\quad\ldots\quad(45)$$

which agrees with the result obtained in [I. 21].

Case II. Continuous core. $\mu_1=0$, $k_1=0$, $x=0$, $\beta_1/k_2=-\frac{9}{4}$

$$4\pi a\mu_2 d'\mathrm{V}=\tfrac{1}{2}\mu_2{}^2 d'\left(2\mathrm{L}+\frac{5}{4}\right)-\frac{7}{8}\mu'^2 d\quad\ldots\quad(46)$$

In the simplest case where there is no added circulation at all $\mu_2=\mu'$.

$$V=\frac{\mu'}{8\pi a}\left(2L+\frac{5}{4}-\frac{7}{4}\frac{d}{d'}\right)$$
$$=\frac{\mu'}{4\pi a}\left(L-\tfrac{1}{4}\right) \text{ when } d=d'.$$

In general, when x is small, the most important term is that involving $\log\frac{1}{x}$. It is given by

$$4\pi a\mu_2 d'V=\tfrac{1}{2}\mu_2^2 d'\left\{2L_2+\frac{\mu_1-\frac{x}{1-x}\mu'}{\mu_1+\mu'}(L_1-L_2)\right\}+\tfrac{1}{2}\left(\mu_1-\frac{x}{1-x}\mu'\right)\left(\mu_1-\frac{1+x}{1-x}\mu'\right)(L_1-L_2)d$$

$$8\pi a\mu_2 V=\mu_2^2\left\{\left(1+\frac{\mu'}{\mu_1+\mu'}\frac{1}{1-x}\right)L_2+\left(1-\frac{\mu'}{\mu_1+\mu'}\frac{1}{1-x}\right)L_1\right\}$$
$$+(\mu_1+\mu')^2\left(1-\frac{\mu'}{\mu_1+\mu'}\frac{1}{1-x}\right)\left(1-\frac{2\mu'}{\mu_1+\mu'}\cdot\frac{1}{1-x}\right)\frac{d}{d'}\log\frac{k_2}{k_1} \quad . \quad (47)$$

When there is no added circulation at all, and $d=d'$

$$V=\frac{\mu}{8\pi a}\left\{L_2+L_1-\frac{1-2x-x^2}{(1-x)^2}(L_1-L_2)\right)\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

As an example of the use of the formulæ let us find the alteration in V, in this simplest case, when the radius of the ring is doubled, supposing originally that the radius of the ring $=10$ cm., outside radius $=1$ cm., and inside $=\frac{1}{3}$ cm.

Since the volume is unaltered, if κ_2, κ_1 be the new values of k_2, k_1

$$8(\kappa_2^2-\kappa_1^2)=k_2^2-k_1^2=\tfrac{1}{400}-\tfrac{1}{1600}=\tfrac{3}{1600}$$

also since

$$\frac{4m\Pi}{a}=2\mu^2\left(1-\frac{x}{1-x}\log\frac{1}{x}\right)d$$

$$1-2\frac{\kappa_1^2}{\kappa_2^2-\kappa_1^2}\log\frac{\kappa_2}{\kappa_1}=\tfrac{1}{2}\left(1-2\frac{k_1^2}{k_2^2-k_1^2}\log\frac{k_2}{k_1}\right)$$

or

$$\frac{1}{z-1}\log_e z=\tfrac{1}{2}+\tfrac{1}{2}\log_e z$$

where

$$z=(\kappa_2/\kappa_1)^2$$

therefore

$$\log_{10} z=\cdot731(z-1)\log_{10} e$$
$$=\cdot3174(z-1)$$

whence

$$z=1\cdot85$$

But

$$\kappa_1^2(z-1)=\tfrac{3}{12800}$$

and

$$\kappa_1^2=\frac{3}{12800\times\cdot85}$$

Now

$$\frac{V'}{V}=\frac{\tfrac{1}{2}}{\dfrac{L'_2-\frac{1}{4}\frac{1}{(z-1)^2}\log z-\frac{1}{4}\frac{z+1}{(z-1)}}{L_2-\frac{1}{4}\frac{x^2}{(1-x)^2}\log 1/x-\frac{1}{4}\frac{1+x}{1-x}}}$$

$$=\cdot594.$$

If the pressure had been so great that there was no hollow at all, then

$$8\kappa^2=k^2=\tfrac{1}{400}$$

and

$$\frac{V'}{V}=\frac{\frac{1}{2}L'_2-\frac{1}{4}}{L_2-\frac{1}{4}}=\cdot625.$$

These cases are however not comparable, their masses being so different.

12. *Energy.*—The energy will be composed of two parts, external and internal. Denote them by E_2, E_1 respectively, and let

$$E=E_1+E_2$$

We need only the lowest order of terms in E_1. We can easily, therefore, obtain it by supposing the core to be bounded by concentric circles, and the velocity at any point to be, V in the direction of translation, and U perpendicular to the radius to the centre of the cross-section. The energy will not contain terms in V U. Hence, R being the radius of the axis of the ring,

$$E_1=\tfrac{1}{2}.2\pi Rd.\int_{r_1}^{r_2}2\pi r dr U^2+\tfrac{1}{2}md V^2$$

$$=2\pi^2 Rd.\int_{r_1}^{r_2}\left(\frac{A_0}{\sqrt{2.ar}}+\frac{Aar}{2}\right)^2 r dr+\tfrac{1}{2}md V^2$$

$$=2\pi^2 Rd\left\{\tfrac{1}{2}\frac{A_0^2}{a^2}\log\frac{r_2}{r_1}+\frac{A_0 A}{2\sqrt{2}}(r_2^2-r_1^2)+\frac{A^2a^2}{16}(r_2^4-r_1^4)\right\}+\tfrac{1}{2}md V^2$$

Now

$$A_0 = -\frac{\mu_1 a}{\pi\sqrt{2}} + \frac{\mu' a}{\pi\sqrt{2}} \frac{r_1^2}{r_2^2 - r_1^2}$$

$$Aa = -\frac{\mu'}{4\pi a^2(k_2^2 - k_1^2)} = -\frac{\mu'}{\pi(r_2^2 - r_1^2)}$$

Hence

$$E_1 = \tfrac{1}{4} R d \left\{ \left(\mu_1 - \frac{x}{1-x}\mu'\right)^2 \log \frac{1}{x} + 2\mu' \left(\mu_1 - \frac{x}{1-x}\mu'\right) + \tfrac{1}{2}\mu'^2 \frac{1+x}{1-x} \right\} + \tfrac{1}{2} m d V^2 \quad . \quad . \quad (49)$$

Here the terms in μ' are small compared with the others, even in the case where x is nearly unity. Hence the principal part of E_1 is

$$E_1 = \tfrac{1}{4} R d \mu_1^2 \log \frac{1}{x} = \tfrac{1}{2} d \mu_1^2 a (L_1 - L_2)$$

To the same order E_2 has been found in the paper "On the Steady Motion of a Hollow Vortex" [I., § 12]. Using the result there obtained

$$E_2 = 2\pi^2 r_2 [U'_2 \psi_0 + \tfrac{1}{2} U'_2 V (R^2 - \lambda a^2 + \tfrac{1}{2} r^2) + \tfrac{1}{2} R r V^2] d'$$

where

$$U'_2 = \frac{\mu_2}{4\pi a k_2} = \frac{\mu_2}{2\pi r_2}$$

$$\psi_0 = \frac{\mu_2 a}{2\pi}(L_2 - 2)$$

Further, since $\lambda = 1 - 4(L_2 - 2)k_2^2$, $r_2 = 2ak_2$, $R = a(1 + 2k_2^2)$*

$$R^2 - \lambda a^2 + \tfrac{1}{2} r^2 = 4(L_2 - \tfrac{1}{2}) a^2 k_2^2 = (L_2 - \tfrac{1}{2}) r_2^2$$

Hence

$$E_2 = 2\pi^2 r_2 \left[\frac{\mu_2^2 a}{4\pi^2 r_2} (L_2 - 2) + \frac{\mu_2 V}{4\pi} r_2 (L_2 - \tfrac{1}{2}) + \tfrac{1}{2} R r_2 V^2 \right] d'$$

$$= M' \left[\frac{\mu_2^2 (1 - 2k_2^2)}{4\pi^2 r_2^2} (L_2 - 2) + \frac{\mu_2 V}{4\pi R} (L_2 - \tfrac{1}{2}) + \tfrac{1}{2} V^2 \right] \quad . \quad . \quad . \quad . \quad (50)$$

where M' is the mass of the outer fluid displaced by the ring. The most important term in this is

$$E_2 = M' \frac{\mu_2^2}{4\pi^2 r_2^2} (L_2 - 2) = \tfrac{1}{2} d' \mu_2^2 a (L_2 - 2)$$

E_1 is in general of order k^2 with respect to this, unless there is an added inner circulation μ_1.

When there is no added internal circulation, the energy of the ring itself is small

* See erratum at end.

compared with the whole. When, however, there is such a circulation, and when the hollow is small, the internal energy is large, and may be larger than the external. As, however, the aperture increases—or as we have already seen the hollow increases—the internal energy will rapidly diminish. The effect therefore of increasing the whole energy is to actually decrease that of the core.

The external energy will, as in the ordinary theory, increase with the aperture.

Section III.—*Small Vibrations.*

13. The vibrations here considered are (1) when the core is fluted, and (2) when it pulsates. The normal modes can be represented by principal displacements of the form $\alpha_n \cos nv$, together with a series of terms whose amplitudes are (*with respect to* k) infinitesimally smaller than α_n. The time of vibration will be a function of a, k_1, k_2, and may therefore be supposed expressed as a series in terms of k_1, k_2. It will be the aim in what follows to obtain the first, or principal term, in this expansion, in other words,—in functions of k_1, k_2—, k_1 k_2 will be regarded as small quantities of the first order. To this order it will be found that we need only consider the principal term in the normal mode concerned.

Let the form of the surface at any time be given by

$$\left.\begin{aligned} k&=k_1(1+\alpha_n \cos nv+\gamma_n \sin nv)\\ k&=k_2(1+\beta_n \cos nv+\delta_n \sin nv)\end{aligned}\right\} \quad \ldots\ldots\ldots \quad (51)$$

where α, β, γ, δ are functions of the time.

The motion is then determined by the stream functions already obtained, with an added stream function, or velocity potential, to give the small motions.

In general, the stream function for the vibrations will be many valued. It has however been shown (I., § 13) that in the case of fluted vibrations this portion is of order k^{n+1}, and may be neglected. This cannot, however, be done in the case of pulsations, and for that the velocity potential must be employed.

Let χ_1, χ_2 be the additional stream functions.

Then

$$\chi_1=\frac{1}{\sqrt{(C-c)}}\{(A_nR_n+B_nT_n)\cos nv+(CR_n+D_nT_n)\sin nv\}+\epsilon$$

$$\chi_2=\frac{1}{\sqrt{(C-c)}}\{A'_nR_n\cos nv+C'_nR_n\sin nv\}+\epsilon'$$

where ϵ, ϵ' are uniform along the boundary.

The expression for the pressure at any point of the core has been already found (11) viz.:

$$\frac{p}{d}=C+f(t)-\dot{\phi}-\tfrac{1}{2}(\text{vel})^2+A\psi.$$

Since it will be necessary to obtain the value of $\dot{\phi}$, it will be necessary to obtain the value of ψ at any time; that is when the boundaries are given by equation (51). This will consist of two portions, one determined by the normal motion at the boundary, and the other (ϵ) by the fact that the circulation remains unaltered.

14. To find the function χ it is first necessary to know the velocity normal to the boundary at any time. Now since the boundary is not the circle, it is in steady motion, the function ψ_0 will itself produce a normal motion. This must first be found. Let θ be the angle which the boundary at any point makes with the circle $u=u$, then

$$\tan\theta=\frac{dn}{dn'}=\frac{du}{dv}=-\frac{1}{k}\frac{dk}{dv}$$

or since

$$k=k_1(1+\alpha\cos nv+\gamma\sin nv)$$

$$\tan\theta=n(\alpha\sin nv-\gamma\cos nv) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52)$$

Now the normal velocity outwards is

$$=-\frac{1}{\rho}\frac{\delta\psi}{\delta v}\frac{dv}{dn}\cos\theta-\frac{1}{\rho}\frac{\delta\psi}{\delta u}\frac{du}{dn}\sin\theta$$

where $\psi=\psi_0+\chi$, ψ_0 being the stream function already found. Also, remembering that θ is small, the normal velocity is

$$=-\frac{dn}{dt}\cos\theta-\mathrm{U}\sin\theta$$

$$=2ak_1(\dot{\alpha}\cos nv+\dot{\gamma}\sin nv)-n\mathrm{U}(\alpha\sin nv-\gamma\cos nv)$$

Therefore, along the boundary

$$-\frac{1}{\rho}\frac{\delta\chi}{dv}\frac{dv}{dn}=(2ak_1\dot{\alpha}+n\mathrm{U}\gamma)\cos nv+(2ak_1\dot{\gamma}-n\mathrm{U}\alpha)\sin nv$$

Then the two sets of conditions give (letters with one dash referring to the inner surface, and with two dashes to the outer surface)

$$\left.\begin{aligned}
\mathrm{AR}'_n+\mathrm{BT}'_n&=-a^3\mathrm{U}_1\surd(2k_1)\alpha+\frac{a^3}{n}(2k_1)^{\frac{1}{2}}\dot{\gamma}\\
\mathrm{AR}''_n+\mathrm{BT}''_n&=-a^3\mathrm{U}_2\surd(2k_2)\beta+\frac{a^3}{n}(2k_2)^{\frac{1}{2}}\dot{\delta}\\
\mathrm{CR}'_n+\mathrm{DT}'_n&=-a^3\mathrm{U}_1\surd(2k_1)\gamma-\frac{a^3}{n}(2k_1)^{\frac{1}{2}}\dot{\alpha}\\
\mathrm{CR}''_n+\mathrm{DT}''_n&=-a^3\mathrm{U}_2\surd(2k_2)\delta-\frac{a^3}{n}(2k_2)^{\frac{1}{2}}\dot{\beta}
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (53)$$

whilst for the motion outside the core

$$\left.\begin{aligned} AR''_n &= -a^2U'_2\sqrt{(2k_2)}\beta+\frac{a^2}{n}(2k_2)^{\frac{1}{2}}\dot{\delta} \\ CR''_n &= -a^2U'_2\sqrt{(2k_2)}\delta-\frac{a^2}{n}(2k_2)^{\frac{1}{2}}\dot{\beta} \end{aligned}\right\} \quad \ldots\ldots \quad (54)$$

Denote the right hand members of these by ξ, η, ξ', η', η_1, η'_1, then solving

$$A=\frac{\xi T''_n-\eta T'_n}{R'_nT''_n-R''_nT'_n}$$

$$B=\frac{-\xi R''_n+\eta R'_n}{R'_nT''_n-R''_nT'_n}$$

with similar expressions for C, D, in ξ', η'.

Suppose now

$$\epsilon=\frac{1}{\sqrt{C-c}}\Sigma\{(A_nR_n+B_nT_n)\cos nv+(C_nR_n+D_nT_n)\sin nv\}$$

It has to be determined from the fact that ϵ is uniform along the boundary, *i.e.*, when $k=k_1(1+\alpha_n\cos nv+\gamma_n\sin nv)$, &c., and that the whole circulation remains unchanged.

If U denote the velocity along u, at a point of the boundary; V the velocity along v as determined by ψ alone, and θ the inclination of the boundary to u (and therefore a small angle); then the circulation is

$$\mu=\frac{\pi\sqrt{2}}{a}(A+\Sigma A_n)+\int(U\cos\theta+V\sin\theta)ds$$

Now

$$U\cos\theta+V\sin\theta=\left(U_0+\frac{dU_0}{dk}\delta k\right)\left(1-\frac{\theta^2}{2}\right)+\frac{dV}{dk}\delta k.\theta.$$

$$=U_0+\frac{dU_0}{dk}\delta k+\frac{dV}{dk}\delta k\theta.$$

Now θ is of order δk. Also $\frac{dV}{dk}$ is of higher order than we require; for both reasons therefore it may be neglected, and

$$\int(U\cos\theta+V\sin\theta)ds=\mu+\frac{dU_0}{dk}\int_0^{2\pi}k(\alpha\cos nv+\gamma\sin nv)\frac{adv}{C-c}$$

since

$$\frac{dn}{dv}=\frac{a}{C-c}$$

Also

$$\frac{dU_0}{dk}ak\int_0^{2\pi}\frac{\alpha\cos nv+\gamma\sin nv}{C-c}dv$$
$$=ak\frac{dU_0}{dk}\int_0^{2\pi}\alpha\cos^2 nv\frac{2e^{-nu}}{S}dv$$
$$=\frac{2\pi akae^{-nu}}{S}\cdot\frac{dU_0}{dk}$$
$$=4\pi aak^{n+2}\frac{dU_0}{dk}$$

and can therefore be neglected except in the case of pulsations.

Hence $A+\Sigma A_n R_n=0$ to our order of approximations. The terms in ϵ can therefore be neglected except in pulsations, *i.e.*, $n=0$.

15. The next step is to determine $\dot{\phi}$. Now ϕ is the flow along any line up to the point in question. Choose this line to be the two portions

(1) straight line $v=\pi$ from $u=u_1$ to $u=u$
(2) circle $u=u$ from $v=v$ to $v=\pi$

The part of ϕ depending on ψ_0 depends on the path of integration, but is constant, and therefore will disappear in $\dot{\phi}$. Consequently the part depending on χ is the only portion needed. Hence

$$\phi=\int_{u_1}^{u}\left[\frac{1}{\rho}\frac{d\chi}{dv}\frac{dv}{dn'}\frac{dn}{du}\right]_{v=\pi}du+\int_{v}^{\pi}\left[\frac{1}{\rho}\frac{d\chi}{du}\frac{du}{dn}\frac{dn'}{dv}\right]_{u=u}dv$$
$$=\int_{u_1}^{u}\left[\frac{1}{\rho}\frac{d\chi}{dv}\right]_{v=\pi}du+\int_{v}^{\pi}\left[\frac{1}{\rho}\frac{d\chi}{du}\right]_{u=u}dv$$
$$=\phi_1+\phi_2\ (\text{say})$$

Now

$$\chi=\frac{1}{\sqrt{(C-c)}}(L\cos nv+M\sin nv)\ (\text{say})$$

Hence

$$\phi_1=(-)^n n\int_{u_1}^{u}\frac{\sqrt{(C+1)}}{aS}Mdu$$
$$=(-)^{n+1}\frac{n}{a}\int_{k'}^{k}\sqrt{\left(\frac{2}{k}\right)}Mdk$$
$$=(-)^{n+1}\frac{n}{a}\int_{k'}^{k}\sqrt{\left(\frac{2}{k}\right)}(CR_n+DT_n)dk$$

Whence, as the lowest terms only are required, and as $R_n\propto k^{-\frac{2n+1}{2}}$ and $T_n\propto k^{\frac{2n-1}{2}}$

$$\phi_1=(-)^n\left[\frac{\sqrt{(2k)}}{a}(CR_n-DT_n)\right]_{u_1}^{u}$$

Again,

$$\frac{d\chi}{du}=\sqrt{(2k)}\left\{\left(\frac{d\mathrm{L}}{du}-\tfrac{1}{2}\mathrm{L}\right)\cos nv+\left(\frac{d\mathrm{M}}{du}-\tfrac{1}{2}\mathrm{M}\right)\sin nv\right\}$$

Therefore

$$\phi_2=\frac{\sqrt{(2k)}}{a}\int_v^{\pi}\left\{\left(\frac{d\mathrm{L}}{du}-\tfrac{1}{2}\mathrm{L}\right)\cos nv+\left(\frac{d\mathrm{M}}{du}-\tfrac{1}{2}\mathrm{M}\right)\sin nv\right\}dv$$

$$=\frac{\sqrt{(2k)}}{na}\left\{-\left(\frac{d\mathrm{L}}{du}-\tfrac{1}{2}\mathrm{L}\right)\sin nv+\left(\frac{d\mathrm{M}}{du}-\tfrac{1}{2}\mathrm{M}\right)(\cos nv-(-1)^n)\right\}$$

Now

$$\mathrm{L}=\mathrm{AR}_n+\mathrm{BT}_n$$

Therefore

$$\frac{d\mathrm{L}}{du}-\tfrac{1}{2}\mathrm{L}=\mathrm{A}\left(\frac{d\mathrm{R}_n}{du}-\tfrac{1}{2}\mathrm{R}_n\right)+\mathrm{B}\left(\frac{d\mathrm{T}_n}{du}-\tfrac{1}{2}\mathrm{T}_n\right)$$

$$=\mathrm{A}\{(n^2-\tfrac{1}{4})\mathrm{SP}_n-\tfrac{1}{2}\mathrm{R}_n\}+\mathrm{B}\{-(n^2-\tfrac{1}{4})\mathrm{SQ}_n-\tfrac{1}{2}\mathrm{T}_n\}$$

$$=\mathrm{AR}_n\left\{\frac{2n\mathrm{SP}_n}{\mathrm{P}_{n+1}-\mathrm{P}_{n-1}}-\tfrac{1}{2}\right\}-\mathrm{BT}_n\left\{\frac{2n\mathrm{SQ}_n}{\mathrm{Q}_{n-1}-\mathrm{Q}_{n+1}}+\tfrac{1}{2}\right\}$$

Now $\mathrm{P}_n/\mathrm{P}_{n+1}$ and $\mathrm{Q}_n/\mathrm{Q}_{n-1}$ are both of order k.

Also,

$$(2n+1)\mathrm{P}_{n+1}-4n\mathrm{CP}_n+(2n-1)\mathrm{P}_{n-1}=0$$

Therefore

$$2n\frac{\mathrm{P}_n}{\mathrm{P}_{n+1}}=\frac{2n+1}{2\mathrm{C}}+\frac{2n-1}{2\mathrm{C}}\frac{\mathrm{P}_{n-1}}{\mathrm{P}_{n+1}}$$

or to the lowest order

$$2n\frac{\mathrm{P}_n}{\mathrm{P}_{n+1}}=(2n+1)k$$

So

$$2n\frac{\mathrm{Q}_n}{\mathrm{Q}_{n-1}}=(2n-1)k$$

and therefore to the same order

$$\frac{d\mathrm{L}}{du}-\tfrac{1}{2}\mathrm{L}=n(\mathrm{AR}_n-\mathrm{BT}_n). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (55)$$

and similarly

$$\frac{d\mathrm{M}}{du}-\tfrac{1}{2}\mathrm{M}=n(\mathrm{CR}_n-\mathrm{DT}_n)$$

Hence

$$\phi_1+\phi_2=\text{const.}+\frac{\sqrt{(2k)}}{a}\{-(\mathrm{AR}_n-\mathrm{BT}_n)\sin nv+(\mathrm{CR}_n-\mathrm{DT}_n)\cos nv\} \quad . \quad (56)$$

The part of ϕ_1 containing u disappearing with a corresponding term in ϕ_2, as it ought consistently with the fact that this part of ϕ is independent of the path of integration.

The values of $\dot{\phi}$ are needed along the inner and outer surfaces. Hence substituting the values of A, B, C, D already found, along the inner surface

$$\dot{\phi}=\frac{\sqrt{(2k_1)}}{a}\left\{-\frac{\dot{\xi}(R'_nT''_n+R''_nT'_n)-2\dot{\eta}R'_nT'_n}{R'_nT''_n-R''_nT'_n}\sin nv+\frac{\dot{\xi}'(R'_nT''_n+R''_nT'_n)-2\dot{\eta}'R'_nT'_n}{R'_nT''_n-R''_nT'_n}\cos nv\right\}$$

Now the lowest terms in R_n, T_n are proportional respectively to $k^{-\frac{2n+1}{2}}$, $k^{\frac{2n-1}{2}}$. Hence to the same order

$$\dot{\phi}=\frac{\sqrt{2}}{a}\left\{-\left(\frac{k_2^{2n}+k_1^{2n}}{k_2^{2n}-k_1^{2n}}\dot{\xi}\sqrt{k_1}-\frac{2(k_1k_2)^n}{k_2^{2n}-k_1^{2n}}\dot{\eta}\sqrt{k_2}\right)\sin nv+(\ldots)\cos nv\right\}$$

We shall put for the future

$$\frac{k_2^{2n}+k_1^{2n}}{k_2^{2n}-k_1^{2n}}=p,\quad \frac{2(k_1k_2)^n}{k_2^{2n}-k_1^{2n}}=q$$

where it is to be noticed that

$$p^2-q^2=1$$

With this notation

$$\dot{\phi}=\frac{\sqrt{2}}{a}\{-(p\dot{\xi}\sqrt{k_1}-q\dot{\eta}\sqrt{k_2})\sin nv+(p\dot{\xi}'\sqrt{k_1}-q\dot{\eta}'\sqrt{k_2})\cos nv\} \quad . \quad . \quad (57)$$

So along the outside boundary

$$\dot{\phi}=\frac{\sqrt{2}}{a}\{-(q\dot{\xi}\sqrt{k_1}-p\dot{\eta}\sqrt{k_2})\sin nv+(q\dot{\xi}'\sqrt{k_1}-p\dot{\eta}'\sqrt{k_2})\cos nv\} \quad . \quad . \quad (58)$$

whilst for the outside boundary in the outside fluid

$$\dot{\phi}=\frac{\sqrt{(2k_2)}}{a}(-\dot{\eta}_1\sin nv+\dot{\eta}_1'\cos nv) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (59)$$

16. *The pressure conditions.*—The expression for the pressure at any point of the core has been already found (11), viz. :—

$$\frac{p}{d}=\text{const}+f(t)-\dot{\phi}-\tfrac{1}{2}v^2+A\psi$$

At a point near a surface this is

$$\frac{p}{d}=\text{const}+f(t)-\dot{\phi}-\tfrac{1}{2}\left(U+\frac{\delta U}{\delta k}\delta k+\frac{S}{\rho^2}\frac{\delta\chi}{\delta u}\right)^2-\tfrac{1}{2}\left(\frac{S}{\rho^2}\frac{\delta\psi}{\delta v}+\frac{S}{\rho^2}\frac{\delta\chi}{\delta v}\right)^2+A\left(\psi_0+\frac{\delta\psi}{\delta k}\delta k+\chi\right)$$

$$=\text{const}+f(t)-\dot{\phi}-\tfrac{1}{2}U^2-U\left(\frac{\delta U}{\delta k}\delta k+\frac{1}{2a^2k}\frac{\delta\chi}{\delta u}\right)+A(\psi_0-2a^2U\delta k+\chi)$$

U being the tangential velocity along the original surface.

Now

$$U=-\frac{1}{2a^2}\frac{\partial\psi}{\partial k}$$

Hence outside

$$a^2U=-\frac{A_0'}{2\sqrt{2k}}$$

$$\frac{dU_1'}{dk}=-\frac{U_1'}{k} \quad \ldots\ldots\ldots\ldots \quad (60)$$

and inside

$$a^2U=-\frac{A_0}{2\sqrt{2k}}-Aa^4k$$

$$\frac{dU}{dk}=-\frac{U}{k}-2Aa^2 \quad \ldots\ldots\ldots\ldots \quad (61)$$

also

$$\frac{\delta\chi}{du}=\sqrt{(2k)}\left\{\left(\frac{dL}{du}-\tfrac{1}{2}L\right)\cos nv+\left(\frac{dM}{du}-\tfrac{1}{2}M\right)\sin nv\right\}$$

whence the part of the pressure depending on v is

$$\frac{p}{d}=-\dot{\phi}-U\left\{-\left(\frac{U}{k}+2Aa^2\right)\delta k+\frac{n}{a^2\sqrt{(2k)}}(AR_n-BT_n)\cos nv\right.$$
$$\left.+\frac{n}{a^2\sqrt{(2k)}}(CR_n-DT_n)\sin nv\right\}+A(\chi-2a^2U\delta k)$$

Along the inside surface this vanishes. Therefore

$$\frac{1}{a}\{p\dot{\xi}\sqrt{(2k_1)}-q\dot{\eta}\sqrt{(2k_2)}\}+U_1(U_1+2Aa^2k_1)\gamma-\frac{nU_1}{2a^2k_1}\{p\xi'\sqrt{(2k_1)}-q\eta'\sqrt{(2k_2)}\}$$
$$-A\left\{\frac{4a^2k_1^2}{n}\dot{\alpha}+4a^2k_1U_1\gamma\right\}=0$$

$$-\frac{1}{a}\{p\dot{\xi}'\sqrt{(2k_1)}-q\dot{\eta}'\sqrt{(2k_2)}\}+U_1(U_1+2Aa^2k_1)\alpha-\frac{nU_1}{2a^2k}\{p\xi\sqrt{(2k_1)}-q\eta\sqrt{(2k_2)}\}$$
$$+A\left\{\frac{4a^2k_1^2}{n}\dot{\gamma}-4a^2k_1U_1\alpha\right\}=0$$

Substituting for ξ, η, &c., these become

$$-\frac{4Aa^2k_1^2}{n}\dot{\alpha}+\frac{4a^2k_1^2}{n}p\dot{\gamma}+\{U_1(U_1-2Aa^2k_1)+nU_1^2p\}\gamma$$
$$+\left(2ak_2qU_2-2a\frac{k_2^2}{k_1}qU_1\right)\dot{\beta}-\frac{4a^2k_2^2}{n}q\dot{\delta}-nU_1U_2\frac{k_2}{k_1}q\delta=0$$

with a corresponding expression from the second equation. In these equations alter the meaning of the α, so as to write α for $k_1^2\alpha$, γ for $k_1^2\gamma$, β for $k_2^2\beta$, δ for $k_2^2\delta$. Further, write u, v, w for $U_1/(2ak_1)$, $U_2/(2ak_2)$, $U'_2/(2ak_2)$ respectively. Then these equations become

$$\left.\begin{aligned} -A\alpha\dot{\alpha}+p\ddot{\gamma}+\{n(np+1)u-nA\alpha\}u\gamma+nq(v-u)\dot{\beta}-q\ddot{\delta}-n^2uvq\delta=0 \\ p\ddot{\alpha}+\{n(np+1)u-nA\alpha\}u\alpha+A\alpha\dot{\gamma}-q\ddot{\beta}-n^2quv\beta-nq(v-u)\dot{\delta}=0 \end{aligned}\right\} \quad (62)$$

Along the outside surface, we find the corresponding expressions by interchanging k_1, k_2; α, β; γ, δ; u, v, *i.e.*, put $-p$ for p and $-q$ for q.

Then

$$\frac{P}{d}=\frac{4a^2}{n}[\{-p\ddot{\beta}+(\overline{-np+1}|v-A\alpha)nv\beta+A\alpha\dot{\delta}+q\ddot{\alpha}+n^2uvq\alpha-nq(v-u)\dot{\gamma}\}\cos nv$$
$$+\{-A\alpha\dot{\beta}-p\ddot{\delta}+(\overline{-np+1}|v-A\alpha)nv\delta+nq(v-u)\dot{\alpha}+q\ddot{\gamma}+n^2uvq\gamma\}\sin nv]$$

Again

$$\frac{P}{d'}=-\frac{\sqrt{(2k_2)}}{a}(\dot{\eta}'_1\cos nv-\dot{\eta}_1\sin nv)-U'_2\left(\frac{dU'_2}{dk}\delta k+\frac{1}{2a^2k}\frac{\delta\chi'}{\delta u}\right)+\text{const}$$
$$=-\frac{\sqrt{(2k_2)}}{a}(\dot{\eta}'_1\cos nv-\dot{\eta}_1\sin nv)$$
$$-U'_2\left\{-U'_2(\beta\cos nv+\delta\sin nv)+\frac{n}{2a^2k_2}\sqrt{(2k_2)}(\eta_1\cos nv+\eta_1\sin nv)\right\}$$

Substituting for the η from (54), and altering the meaning of $\alpha, \beta, \gamma, \delta$ as before

$$\frac{P}{d'}=\frac{4a^2}{n}[\{\ddot{\beta}+n(n+1)w^2\beta\}\cos nv+\{\dot{\delta}+n(n+1)w^2\delta\}\sin nv]$$

Hence, since the pressure is the same on both sides the boundary

$$\left.\begin{aligned} \dot{q}\ddot{\alpha}+n^2quv\alpha-nq(v-u)\dot{\gamma}-p\ddot{\beta}+n\{(-np+1)v-A\alpha\}v\beta+A\alpha\dot{\delta} \\ =\frac{d'}{d}\{\ddot{\beta}+n(n+1)w^2\beta\} \\ nq(v-u)\dot{\alpha}+q\ddot{\gamma}+n^2quv\gamma-A\alpha\dot{\beta}-p\ddot{\delta}+n\{(-np+1)v-A\alpha\}v\delta \\ =\frac{d'}{d}\{\ddot{\delta}+n(n+1)w^2\delta\} \end{aligned}\right\}$$

When the waves are travelling in the positive direction round the ring, δk will be of the form

$$\delta k_1=L\sin(nv+\lambda t)$$
$$\delta k_2=M\sin(nv+\lambda t)$$

or

$$\alpha=L\sin\lambda t \quad , \quad \gamma=L\cos\lambda t$$
$$\beta=M\sin\lambda t \quad , \quad \delta=M\cos\lambda t$$

Substituting these values, and writing ρ for d'/d, the four equations of condition reduce to the two

$$\left.\begin{aligned} &L\{p\lambda^2+A\alpha\lambda-n(np+1)u^2+nA\alpha u\}-Mq(\lambda-nu)(\lambda+nv)=0\\ &qL(\lambda+nu)(\lambda-nv)-M\{(p+\rho)\lambda^2-A\alpha\lambda-n(np-1)v^2-nA\alpha v-n(n+1)w^2\rho\}=0\end{aligned}\right\}\quad(62a.)$$

Whence the equation to determine λ is

$$\left.\begin{vmatrix} p\lambda^2+A\alpha\lambda-n(np+1)u^2+nA\alpha u & , & q(\lambda-nu)(\lambda+nv)\\ q(\lambda+nu)(\lambda-nv) & , & (p+\rho)\lambda^2-A\alpha\lambda-n(np-1)v^2-nA\alpha v-n(n+1)w^2\rho\end{vmatrix}=0\right\}\quad(63)$$

17. Case I.—No rotational core. Here the periods are given by

$$\lambda^2-n(n+1)w^2=0.$$

The time of vibration is therefore

$$\left.\begin{aligned}&\frac{2\pi}{w\sqrt{n(n+1)}}=\frac{(4\pi ak_2)^2}{\mu_2\sqrt{n(n+1)}}\\ \text{or by (39)}\quad&=\frac{\mu_2 d'}{2\Pi\sqrt{n(n+1)}}\end{aligned}\right\}\quad\ldots\ldots\ldots\quad(64)$$

and is therefore the same whatever the size of the ring. Moreover, when n is large, the time varies inversely as n. The ring is always stable for vibrations of this mode. The expression for the time does not agree with that found in the former paper [I. § 13]. The reason of this is stated in a note at the end of the present paper.

Case II. *Continuous case.*—Here $\mu_1=0$, $k_1=0$, $\alpha=\gamma=0$, $p=1$, $q=0$, and the equations of motion reduce to two

$$\left.\begin{aligned}&-(1+\rho)\ddot{\beta}-\{n(n-1)v^2+n(n+1)w^2\rho+nA\alpha v\}\beta+A\alpha\dot{\delta}=0\\ &-A\alpha\dot{\beta}-(1+\rho)\ddot{\delta}-\{n(n-1)v^2+n(n+1)w^2\rho+nA\alpha v\}\delta=0\end{aligned}\right\}$$

Here putting

$$\delta k=L\sin(nv+\lambda t)$$

$$(1+\rho)\lambda^2-A\alpha\lambda-\{n(n-1)v^2+n(n+1)w^2\rho+nA\alpha v\}=0.$$

Since the core is continuous

$$\mu'=-4\pi Aa^3k_2^2$$

therefore

$$A\alpha=-\frac{\mu'}{4\pi a^2k_2^2}=-2v$$

whence

$$\lambda^2+\frac{2v}{1+\rho}\lambda=\frac{n\{(n-3)v^2+(n+1)w^2\rho\}}{1+\rho}$$

and

$$\lambda=\frac{-v\pm\sqrt{\{v^2+n(1+\rho)((n-3)v^2+(n+1)w^2\rho)\}}}{1+\rho} \quad . \quad . \quad . \quad . \quad . \quad (65)$$

In order that the motion may be stable the expression under the root must always be positive, whatever value n has, $=>1$. The least value is when $n=1$. Hence the condition of stability is

$$2w^2\rho(1+\rho)>(1+2\rho)v^2$$

or

$$\mu_2^2>\mu'^2\frac{1+2\rho}{2(1+\rho)\rho} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (66)$$

In the ordinary theory where $\rho=1$ and $\mu_2=\mu'$ the ring is therefore always stable. When there is no additional circulation the condition becomes $\rho>1/\sqrt{2}$ or $d<d'\sqrt{2}$. The ordinary simple ring will therefore be stable even when the density of its core is as large as $\sqrt{2}$ times the surrounding fluid. In any case, whatever the density of the core may be, the ring will be stable provided it has an additional outside circulation given by (66). In general the two values of λ will be of opposite sign, and will correspond to waves travelling with and against the cyclic motion. The positive root gives that in the same direction and the negative in the opposite. In the simplest cases of equal density and no added circulation the roots are $-nv$ and $(n-1)v$.

18. Case III.—*No internal added circulation*, with a hollow. Here $\mu_1=0$, $u=0$, and the equation for λ gives one root zero, and the cubic

$$(1+p\rho)\lambda^3+A\alpha\rho\lambda^2-\{n(n-p)v^2+np(n+1)w^2\rho+nA\alpha pv+A^2\alpha^2\}\lambda$$
$$-nA\alpha\{(np-1)v^2+(n+1)w^2\rho+A\alpha v\}=0$$

in which since

$$\mu'=-4\pi A\alpha^3(k_2^3-k_1^3)$$

$$A\alpha=-\frac{2v}{1-x}$$

Denote the cubic by

$$\lambda^3+b\lambda^2+c\lambda+d=0$$

We shall first investigate the signs and finiteness of the different coefficients, and then pass on to the question of the reality of the roots.

$$b=\frac{A\alpha\rho}{1+p\rho}=-\frac{2v\rho}{1-x}\times\frac{1-x^n}{1-x^n+(1+x^n)\rho}$$

Since

$$p=\frac{k_2^{2n}+k_1^{2n}}{k_2^{2n}-k_1^{2n}}=\frac{1+x^n}{1-x^n}$$

Further denote $1+x+\ldots+x^{n-1}$ by X_n and $(1-x^n)+(1+x^n)\rho$ by f_n. It is to be noticed that since x lies between 0 and 1, these quantities are both positive and finite.

With this notation

$$b=-\frac{2\rho X_n}{f_n}v$$

and is always negative and finite.

The second coefficient is

$$c=-\frac{1}{1+p\rho}\{np(n+1)w^2\rho+n(n-p)v^2+nAapv+A^2a^2\}$$

After some algebraic reduction this may be put in the form

$$c=-\frac{1}{f_n}\left[(1+x^n)n(n+1)w^2\rho+(1-x)v^2\Sigma_0^{n-1}\left\{\left(n-\frac{2r+3}{2}\right)^2+r^2+3r+\frac{7}{4}\right\}x^r\right]$$

Here c is always finite and negative.

The third coefficient is

$$d=-\frac{nAa}{1+p\rho}\{(n+1)w^2\rho+(np-1)v^2+Aav\}$$
$$=\frac{2nX_n}{f_n}\left[(n+1)(w^2\rho-v^2)+\left\{\frac{2n}{1-x^n}-\frac{2}{1-x}\right\}v^2\right]v$$

which again, after some algebraic reduction, becomes

$$d=\frac{2nX_n}{f_n}\left\{(n+1)(w^2\rho-v^2)+\frac{2v^2}{X_n}\Sigma_0^{n-2}(n-r-1)x^r\right\}v$$

This is always finite, and if $w^2\rho>v^2$ it is also always positive. But if $w^2\rho<v^2$ it becomes negative for values of n less than a particular value, depending on the value of $w^2\rho/v^2$.

The conditions that

$$\lambda^3+b\lambda^2+c\lambda+d=0$$

shall have real roots are

i. $b^2>3c$

ii. $b^2c^2-4c^3>(4b^3-18bc+27d)d$

We have already seen that c is always negative. Hence the first condition is satisfied under all circumstances.

The second condition is

$$-4c^3+b^2c^2+18bcd-4b^3d>27d^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (67)$$

Here every term is positive, except when $w^2\rho<v^2$. Treating this as an equality, and writing $w^2\rho/v^2\equiv y$, we get a cubic equation in y. Let the roots of this be α, β, γ in order of magnitude. The inequality may then be written

$$(y-\alpha)(y-\beta)(y-\gamma)>0$$

Hence, either $y>\alpha$ or between β and γ.

When the central hollow is very small, x is also very small. If $x=0$

$$b=-\frac{2\rho}{1+\rho}v$$

$$c=-\frac{1}{1+\rho}\{n(n+1)w^2\rho+(n^2-3n+4)v^2\}$$

$$d=\frac{2n}{1+\rho}\{(n+1)w^2\rho+(n-3)v^2\}$$

In this put for the moment $n(n+1)w^2\rho/v^2+n^2-3n=2\xi$

Then

$$c=-\frac{2}{1+\rho}(\xi+2)v^2$$

$$d=\frac{4}{1+\rho}\xi v^2$$

and the above condition becomes

$$\frac{2}{1+\rho}(\xi+2)^3+\frac{\rho^2(\xi+2)^3}{(1+\rho)^3}+\frac{18\rho}{1+\rho}\xi(\xi+2)-27\xi^2+\frac{8\rho^2}{(1+\rho)^2}\xi>0$$

or

$$\xi^3-\left(8+4\rho-\frac{1}{2(\rho+1)}\right)\xi^2+\left(14+16\rho+4\rho^2-\frac{2}{1+\rho}\right)\xi+\frac{4(4+4\rho+\rho^2)}{2(1+\rho)}>0$$

whence

$$(\xi-4-2\rho)^2\left(\xi+\frac{1}{2(\rho+1)}\right)>0.$$

In this case the two roots α and β of the cubic in y become equal, and the condition of stability is

$$n(n+1)w^2\rho+n(n-3)v^2+\frac{v^2}{\rho+1}>0$$

This is always the case when $n\geq 3$.

When $n=2$

$$6w^2\rho > \left(2-\frac{1}{\rho+1}\right)v^2$$

or

$$w^2\rho > \frac{2\rho+1}{6(\rho+1)}v^2 \quad \ldots\ldots\ldots \quad (68)$$

When $n=1$

$$2w^2\rho > \left(2-\frac{1}{\rho+1}\right)v^2$$

or

$$w^2\rho > \frac{2\rho+1}{2(\rho+1)}v^2 \quad \ldots\ldots\ldots\ldots \quad (69)$$

To $n=1$ corresponds a vibration of the ring as a whole about its state of steady motion, whereby it describes a circle, *without change in cross section*, round its mean position. Instability therefore for $n=1$ actually means instability in steadiness, whereas instability for $n=2$ means instability of form.

For a ring of the same density as the fluid, and just at the point of forming its hollow, the condition of stability of steadiness is $w^2 > \frac{3}{4}v^2$, and for stability of form $w^2 > \frac{1}{4}v^2$.

For a ring with a very dense core, the corresponding conditions are $w^2\rho > \frac{1}{2}v^2$ and $w^2\rho > \frac{1}{6}v^2$.

When the hollow is very large x is nearly equal to unity. In the case $x=1$

$$b = -nv$$

$$c = -n(n+1)w^2$$

$$d = \frac{n^2}{\rho}\{(n+1)w^2\rho - 2v^2\}v$$

and the equation to find $\lambda/v=y$ becomes, writing $w^2/v^2=z$

$$y^3 - ny^2 - n(n+1)zy + n^2\left\{(n+1)z - \frac{2}{\rho}\right\} = 0 \quad \ldots\ldots \quad (70)$$

When ρ is infinitely large (density of core infinitely small) the roots of this are n, and $\pm\sqrt{\{n(n+1)z\}}$. The last two agree with the result obtained for a hollow only, as might be expected. Hence, when the density of the core is small the ring is stable when its aperture is very large.

When $\rho=1$ and $z=1$ (the ordinary case treated), the roots are $-n$ and $n\pm\sqrt{n}$. This is, therefore, also stable when the aperture is large.

The condition of the reality of the roots of equation (70) is

$$4n^3(n+1)^3z^3+n^4(n+1)^2z^2+18n^4(n+1)\left\{(n+1)z-\frac{2}{\rho}\right\}z+4n^5\left\{(n+1)z-\frac{2}{\rho}\right\}$$
$$>27n^4\left\{(n+1)z-\frac{2}{\rho}\right\}^2$$

or

$$(n+1)^3z^3-2n(n+1)^2z^2+n(n+1)\left(n+\frac{18}{\rho}\right)z-\frac{2n^2}{\rho}-\frac{27n}{\rho^2}>0$$

For all values of n. This may be written

$$n^3z(z-1)^2+n^2\left\{3z^3-4z^2+\left(1+\frac{18}{\rho}\right)z-\frac{2}{\rho}\right\}+n\left\{3z^3-2z^2+\frac{18}{\rho}z-\frac{27}{\rho^2}\right\}+z^3>0 \quad (71)$$

Call this expression Y_n.

Then the condition for stability of steadiness is $Y_1>0$, or

$$z(z-1)^2+\left\{z(3z-1)(z-1)+\frac{18}{\rho}z-\frac{2}{\rho}\right\}+\left\{3z^3-2z^2+\frac{18}{\rho}z-\frac{27}{\rho^2}\right\}+z^3>0$$

and

$$Y_n-Y_1=(n-1)\left[(n^2+n+1)z(z-1)^2+(n+1)\left\{z(3z-1)(z-1)+\frac{18}{\rho}z-\frac{2}{\rho}\right\}\right.$$
$$\left.+3z^3-2z^2+\frac{18}{\rho}z-\frac{27}{\rho^2}\right]$$
$$=(n-1)\left[(n^2+n)z(z-1)^2+n\left\{z(3z-1)(z-1)+\frac{18}{\rho}z-\frac{2}{\rho}\right\}-z^3+Y_1\right]$$

Here $n=$ or >2. Hence, if $\rho\geq 1$

$$Y_n-Y_1\geq(n-1)[11z^3+16z+4+Y_1]$$

and Y_1 is positive. Hence $Y_n>Y_1$. It is therefore only needful to determine the condition of stability when $n=1$, which is given above. It is

$$8z^3-8z^2+2\left(1+\frac{18}{\rho}\right)z-\frac{2}{\rho}-\frac{27}{\rho^2}>0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

When $\rho=10^{-1}$ this is

$$(z-5.2\ldots)(z^2+4.2\ldots z+6.5\ldots)>0$$

for $\rho=10^{-2}$

$$(z-28)(z^2+27z+1206)>0$$

and in general when ρ is very small $=10^{-p}$

$$z=\tfrac{3}{4}\times 10^{\frac{2p}{3}}$$

or

$$w/v=10^{\frac{p}{3}}\sqrt{\tfrac{3}{4}}=1{\cdot}22\times 10^{\frac{p}{3}}$$

Coming back to the general equation of condition, it is clear that, since $w^6\rho^3$ occurs on the left with a positive coefficient, whereas the highest on the right is $w^4\rho^3$, with a given value of x, it is possible to satisfy the condition of stability in the state of the ring defined by x, by taking $w^2\rho$ sufficiently large. If then $w^2\rho$ be very large, the ring would be stable up to a considerable value of x, but as the aperture increased, a point might be reached at last where it would become unstable. These considerations apply to any value of ρ. When x is very small the equation for λ/v becomes (writing $y=\lambda/v$, $w^2\rho/v^2=z^2$)

$$y^3-\frac{2\rho}{1+\rho}y^2-\frac{1}{1+\rho}\{n(n+1(z^2+(n^2-3n+4)\}y+\frac{2n}{1+\rho}\{(n+1)z^2+n-3\}=0$$

One root of this is 2; the others are given by

$$y^2+\frac{2}{1+\rho}y-\frac{n}{1+\rho}\{(n+1)z^2+n-3\}=0$$

which gives the roots already found in Case II., as was to be expected. The solid ring has two periods, whereas just as a hollow begins to be formed it suddenly possesses two additional ones, given by $\lambda=0$, and $\lambda=2$. It will be interesting to see how these are introduced. To do this we must have recourse to equations (62a) and determine the amplitudes of the inner and outer vibrations respectively corresponding to any given value of λ.

Taking the second of the equations (62a) and putting therein $u=0$, $x=$ small, we get

$$q\mathrm{L}\lambda(\lambda-nv)-\mathrm{M}\{(1+\rho)\lambda^2+2v\lambda-n(n+1)w^2\rho-n(n-3)v^2\}=0$$

with the value $q=0$. Hence $\mathrm{M}=0$ unless λ is one of the roots (65). In other words, as the hollow forms the outside vibrations are not immediately affected, but the internal vibrations introduce two periods peculiar to themselves, one standing vibrations ($\lambda=0$) and another ($\lambda=2v$) travelling in the direction of the cyclic motion with velocity $2v/n=2\mathrm{U}/rn$, where r is the radius of the outside surface.

19. In the general case

$$\{p\lambda^2+\mathrm{A}a\lambda-n(np+1)u^2+n\mathrm{A}au\}\{(p+\rho)\lambda^2-\mathrm{A}a\lambda-n(np-1)v^2-n\mathrm{A}av-n(n+1)w^2\rho\}$$
$$-q^2(\lambda^2-n^2u^2)(\lambda^2-n^2v^2)=0$$

When reduced this is the biquadratic equation

$$\lambda^4+b\lambda^3+c\lambda^2+d\lambda+e=0$$

where

$$b=\mathrm{A}a\rho/(1+p\rho)$$

$$c=-\{(n^2+np+n(np+1)\rho)u^2+(n^2-np)v^2+np(n+1)w^2\rho \\ -n(p+\rho)\mathrm{A}au+np\mathrm{A}av+\mathrm{A}^2a^2\}/(1+p\rho)$$

$$d=\mathrm{A}a\{n(np+1)u^2-n(np-1)v^2-n\mathrm{A}a(u+v)-n(n+1)w^2\rho\}/(1+p\rho)$$

$$e=n^2[(n^2-1)u^2v^2+(n+1)(np+1)u^2w^2\rho+\mathrm{A}au\{(np+1)uv-(np-1)v^2 \\ -(n+1)w^2\rho\}-\mathrm{A}^2a^2uv]/(1+p\rho)$$

Now

$$\mu'=-4\pi\mathrm{A}a^3(k_2^2-k_1^2)=-4\pi\mathrm{A}a^3k_2^2(1-x)$$

$$v=\frac{\mu_1+\mu'}{8\pi a^3k_2^3}=\frac{\mu_1}{8\pi a^3k_2^3}-\tfrac{1}{2}\mathrm{A}a(1-x)$$

$$u=\frac{\mu_1}{8\pi a^3k_1^3}=\frac{\mu_1}{8\pi a^3k_2^3x}$$

therefore

$$v=xu-\tfrac{1}{2}\mathrm{A}a(1-x)$$

or

$$\mathrm{A}a=\frac{2}{1-x}(xu-v)$$

therefore

$$\frac{\mathrm{A}a}{v}=-\frac{2}{1-x}\cdot\frac{\mu'}{\mu_1+\mu'}$$

$$\frac{u}{v}=\frac{\mu_1}{\mu_1+\mu'}\frac{k_2^3}{k_1^3}=\frac{\mu_1}{x(\mu_1+\mu'}$$

$$\frac{w}{v}=\frac{\mu_2}{\mu_1+\mu'}$$

If then $\lambda/v=y$, the equation for y is

$$y^4+by^3+cy^2+dy+e=0$$

where now (writing $\mu'/(\mu_1+\mu')$, &c. $=\mu'$, &c.)

$$\left.\begin{aligned}
b&=-\frac{2\mu'\rho}{(1-x)(1+p\rho)}\\
c&=-\left\{(n^2+np+n(np+1)\rho)\frac{\mu_1^2}{x^2}+n^2-np+np(n+1)\rho\mu_2^2\right.\\
&\qquad\left.+2n\frac{p+\rho}{x(1-x)}\mu_1\mu'-\frac{2np}{1-x}\mu'+\frac{4\mu'^2}{(1-x)^2}\right\}\frac{1}{1+p\rho}\\
d&=\frac{2\mu' n}{(1-x)(1+p\rho)}\left\{(np-1)-(np+1)\frac{\mu_1^2}{x^2}+(n+1)\mu_2^2\rho-\frac{2\mu'}{1-x}\left(1+\frac{\mu_1}{x}\right)\right\}\\
e&=\frac{n^2}{1+p\rho}\left[(n^2-1)\frac{\mu_1^2}{x^2}+(n+1)(np+1)\frac{\mu_1^2\mu_2^2}{x^2}\rho-\frac{2\mu'\mu_1}{x(1-x)}\left\{(np+1)\frac{\mu_1}{x}-np+1\right.\right.\\
&\qquad\left.\left.-(n+1)\mu_2^2\rho\right\}-\frac{4\mu_1\mu'^2}{x(1-x)^2}\right]
\end{aligned}\right\}\quad(73)$$

The general investigation of the question of stability would be very laborious, but it may be useful to put down the values of the coefficients when the aperture becomes very large, or $x=1$. They are

$$\left.\begin{aligned}
b&=-n\mu'\\
c&=-n(n+1)\mu_2^2-\left(n^2+\frac{n}{\rho}\right)\mu_1\\
d&=\frac{n^2\mu'}{\rho}\{(n+1)\mu_2^2\rho-\mu_1(1+\mu_1)-2\}\\
e&=n^2\mu_1\left\{(n+1)\mu_1\mu_2^2-\frac{2\mu'}{\rho}(1+\mu_1)\right\}
\end{aligned}\right\}\quad\ldots\ldots\quad(74)$$

In which

$$\mu_1+\mu'=1$$

20. *Pulsations.*—Let the pulsations be given by

$$k=k_1(1+\xi)\qquad k=k_2(1+\eta)$$

Then since the volume of the core remains unaltered

$$k_1^2\xi=k_2^2\eta.$$

Since the stream function is now many-valued, it will be advisable to use the potential function for the part of the motion due to this: denoting it by ϕ, it is of the form

$$\phi=\sqrt{C-c}\{AP_0+BQ_0\}$$

where A and B are constants determined by the equation

$$\frac{\delta\phi}{\delta u}\frac{du}{dn} = \text{vel. inwards at the two surfaces}$$

$$= \frac{dn}{dt} = \frac{dn}{du}\frac{du}{dk}\dot{k} = -\frac{dn}{du}(\dot{\xi} \text{ and } \dot{\eta})$$

Hence

$$\frac{\delta\phi}{\delta u} = -\frac{a^3}{(C-c)^2}\dot{\xi} = -4a^3k^2\dot{\xi}$$

Now

$$\frac{\delta\phi}{\delta u} = \frac{S}{2\sqrt{(C-c)}}(AP_0+BQ_0)+\sqrt{(C-c)}\left\{A\frac{dP_0}{du}+B\frac{dQ_0}{du}\right\}$$

$$= \frac{A}{2\sqrt{(C-c)}}\left\{SP_0+2(C-c)\frac{dP_0}{du}\right\}+\frac{B}{2\sqrt{(C-c)}}\left\{SQ_0+2(C-c)\frac{dQ_0}{du}\right\}$$

$$= A\sqrt{2} \quad \text{to the lowest order.}$$

Hence

$$A\sqrt{2} = -4a^3k_1^3\dot{\xi} = -4a^3k_2^2\dot{\eta}$$

Therefore

$$\phi = -2\sqrt{2a^3k_1^2\sqrt{(C-c)}}.\dot{\xi}P_0 \quad \ldots\ldots\ldots \quad (75)$$

Again the circulation is unaltered. A term must therefore be added to ψ; let it be denoted by χ. Then χ is of the form

$$\chi = \frac{1}{\sqrt{(C-c)}}\Sigma\{A_0R_0+B_0T_0+\ldots\}$$

The circulation of this is

$$-\frac{\pi\sqrt{2}}{a}\Sigma A_0$$

But the circulation of ψ round the new boundary is

$$\left(U+\frac{dU}{dk}dk\right)2\pi r.$$

Along the inner boundary this is

$$= 4\pi ak_1\left\{U_1+U_1\xi+\frac{dU}{dk}k_1\xi\right\}$$

$$= 4\pi ak_1U_1-8\pi Aa^3k_1^3\xi \quad \text{by (61)}$$

Hence since the circulation is unaltered

$$-8\pi Aa^3k_1^2\xi-\frac{\pi\sqrt{2}}{a}A_0 = 0$$

$$A_0 = -4\sqrt{2}Aa^4k_1^2\xi = -4\sqrt{2}Aa^4k_2^2\eta \quad \ldots\ldots\ldots \quad (76)$$

For the pressure condition we require the stream function. Now $2\pi\psi$ is the flux across any diaphragm with circular boundary through the point in question. Take it to be the annulus $v=\pi$, from $u=u_1$ to $u=u$, and then the portion of the tore $u=u$ from $v=\pi$ to $v=v$. We need only to find the part of ψ due to ϕ. Now ϕ produces no motion across the annulus. Hence

$$2\pi\psi'=\int_\pi^v -\frac{\delta\phi}{\delta u}\frac{du'}{dn}\cdot 2\pi\rho\frac{dn'}{dv}\,dv$$

or

$$\begin{aligned}\psi' &= -a\int_\pi^v \frac{\delta\phi}{\delta u}dv \\ &= 4a^3k_1^2\dot{\xi}(\pi-v) \\ &= 4a^3k_2^2\dot{\eta}(\pi-v) \quad \ldots\ldots\ldots\ldots \quad (77)\end{aligned}$$

which is many-valued, as it ought to be.

Again, we need the flow due to χ. This is

$$\begin{aligned}\phi' &= \int_{u_1}^u \left[\frac{1}{\rho}\frac{\delta\chi}{\delta v}\frac{dv}{dn'}\frac{dn}{u}\right]_{v=\pi} du + \int_\pi^v \left[\frac{1}{\rho}\frac{\delta\chi}{\delta u}\frac{du}{dn'}\frac{du'}{dv}\right]_{u=u} dv \\ &= \int_{u_1}^u \left[\frac{1}{\rho}\frac{\delta\chi}{\delta v}\right]_{v=\pi} du + \int_\pi^v \left[\frac{1}{\rho}\frac{\delta\chi}{\delta u}\right]_{u=u} dv \\ &= \int_\pi^v \left[\frac{1}{\rho}\frac{\delta\chi}{\delta u}\right]_{u=u} dv \text{ to lowest order.} \\ \phi' &= \frac{A_0}{a}\int_\pi^v \left\{-\frac{S}{2(C-e)^{\frac{1}{2}}}R_0 - \frac{1}{4(C-e)^{\frac{1}{2}}}SP_0\right\} dv \\ &= -\frac{A_0}{2a}\left\{-\sqrt{(2)}(\tfrac{1}{2}L-1)+\frac{L}{\sqrt{2}}\right\}(\pi-v) \\ &= -\frac{A_0}{a\sqrt{2}}(\pi-v) \\ &= 4Aa^3k_1^2\dot{\xi}(\pi-v) = 4Aa^3k_2^2\dot{\eta}(\pi-v) \quad \ldots\ldots\ldots \quad (78)\end{aligned}$$

The pressure is given by

$$\begin{aligned}\frac{p}{d} &= F(t)-\dot{\phi}-\dot{\phi}' = \tfrac{1}{2}\left(U+\frac{dU}{dk}\delta k+\frac{1}{\rho}\frac{\delta\chi}{du}\frac{du}{dn}-\frac{\delta\phi}{\delta v}\frac{dv}{dn}\right)^2 \\ &\quad -\tfrac{1}{2}\left(\frac{1}{\rho}\frac{\delta\psi}{\delta v}\frac{dv}{dn}+\frac{\delta\phi}{du}\frac{du}{dn}\right)^2+A\left(\psi+\frac{\delta\psi}{\delta k}\delta k+\psi'+\chi\right) \\ &= F(t)-\tfrac{1}{2}U^2+A\psi-\dot{\phi}-\dot{\phi}'-U\frac{dU}{dk}\delta k-\frac{U}{2a^2k}\frac{\delta\chi}{\delta u}+A\left(\frac{\delta\psi}{\delta k}\delta k+\psi'+\chi\right)\end{aligned}$$

keeping only the most important terms. Substituting the values already obtained

$$\begin{aligned}\frac{p}{d}&=F(t)-\tfrac{1}{2}U^2+A\psi\\&\quad+2a^2k_1^2\frac{P_0}{\sqrt{k}}\ddot{\xi}-4Aa^3k_1^3\xi(\pi-v)\\&\quad+U(U+2Aa^3k)\xi-2UAa^2k_1\dot{\xi}\\&\quad+A\{-2a^2kU\xi+4a^3k_1^2\xi(\pi-v)+4Aa^4k_1^2(L-2)\xi\}\\&=F(t)-\tfrac{1}{2}U^2+A\psi+4a^2k_1^2L\ddot{\xi}+U(U-2Aa^2k)\xi+4A^2a^4k_1^2(L-2)\xi\end{aligned}$$

Hence along the inside

$$F(t)-\tfrac{1}{2}U_1^2+A\psi_1+4a^2k_1^2L_1\ddot{\xi}+4a^2\left\{A^2(L_1-2)a^2+\left(\frac{U_1}{2ak_1}\right)^2-\frac{U_1}{2ak_1}Aa\right\}k_1^2\xi=0$$

Whilst along the outside boundary

$$\begin{aligned}&d\left\{Ft-\tfrac{1}{2}U_2^2+A\psi_2+4a^2k_1^2L_2\ddot{\xi}+4a^2\left\{A^2(L_2-2)a^2+\left(\frac{U_2}{2ak_2}\right)^2-\frac{U_2}{2ak_2}Aa\right\}k_1^2\xi\right.\\&\quad=d'\left\{4a^2L_2k_1^2\ddot{\xi}+4a^2\left(\frac{U'_2}{2ak_2}\right)^2k_1^2\xi\right\}\end{aligned}$$

Remembering that the large terms have been already annulled, we get by subtracting the two equations and writing ξ for $k_1^2\xi$, &c., as before

$$\begin{aligned}(L_2-L_1)\ddot{\xi}+\{A^2a^2(L_2-L_1)+v^2-u^2-Aa(v-u)\}\xi\\=\frac{d'}{d}\{L_2\ddot{\xi}+w^2\xi\}\end{aligned}$$

or

$$\left\{L_1+\left(\frac{d'}{d}-1\right)L_2\right\}\ddot{\xi}+\left\{w^2\frac{d'}{d}-(v-u)(v+u-Aa)+A^2a^2(L_1-L_2)\right\}\xi=0 \quad . \quad (79)$$

whence (if $\rho=d'/d$). The time of pulsation is

$$2\pi\sqrt{\left\{\frac{L_2(\rho-1)+L_1}{\rho w^2+A^2a^2(L_1-L_2)-(v-u)(v+u-Aa)}\right\}} \quad . \quad . \quad . \quad . \quad . \quad (80)$$

The numerator of this is always positive. Hence the condition for stability is that

$$\rho w^2+A^2a^2(L_1-L_2)+(v-u)Aa>v^2-u^2$$

When the added circulation is in the same direction as that of the core—as has been supposed throughout, $v>u$.

If there are no added circulations, $u=0$, $w=v$, and

$$v=-\tfrac{1}{2}\mathrm{A}a\left(1-\frac{k_1^2}{k_2^2}\right)=-\tfrac{1}{2}\mathrm{A}a(1-x)$$

and the condition of stability is

$$\rho>1+\frac{2}{1-x}-\frac{2\log 1/x}{(1-x)^2}$$

The maximum value of the right hand member is when

$$\frac{1+x}{x(1-x)^2}+\frac{2\log x}{(1-x)^3}=0$$

or

$$1-x^2+2x\log x=0$$

whence $x=1$, which makes the condition become

$$\rho>0.$$

Now x is always <1, and since ρ is always positive, the condition is satisfied.

In general

$$w=\frac{\mathrm{U}_2'}{2\pi a k_2}=\frac{\mu_1}{8\pi a^2 k_2^2}$$

$$v=\frac{\mu_1+\mu'}{8\pi a^2 k_2^2}$$

$$u=\frac{\mu_1}{8\pi a^2 k_1^2}$$

Also

$$\mu'=-4\pi\mathrm{A}a^2(k_2^2-k_1^2)$$

Hence condition of stability is that

$$\mu_2^2\rho>\left(\mu_1+\mu'-\mu_1\frac{k_2^2}{k_1^2}\right)\left(\mu_1+\mu'+\mu_1\frac{k_2^2}{k_1^2}+2\mu'\frac{k_1^2}{k_2^2-k_1^2}\right)-4\mu'^2\left(\frac{k_2}{k_2-k_1}\right)^3\log\frac{k_2}{k_1}$$

$$>\left\{\mu_1\left(1-\frac{1}{x}\right)+\mu'\right\}\left\{\mu_1\left(1+\frac{1}{x}\right)+\mu'\right\}+2\mu'\left\{\left(\mu_1\left(1-\frac{1}{x}\right)+\mu'\right)\frac{1}{1-x}-\mu'\frac{\log 1/x}{(1-x)^3}\right\}$$

When x is small the right hand side is evidently negative and the condition is satisfied. When $x=1$ the right hand member is 0.

Consequently

$$\mu_2^2\rho>0$$

In general

$$\mu_2{}^2\rho > \mu_1{}^2\left(1-\frac{1}{x^3}\right)+2\mu_1\mu'\left(1-\frac{1}{x}\right)+\mu'^2\left(1+\frac{2}{1-x}+\frac{2\log x}{(1-x)^3}\right),$$

all these terms are negative. Hence the core is always stable for pulsations.

Substituting for u, v, w the time of pulsation is given by

$$16\pi^2a^2k_2{}^2\sqrt{\left\{\frac{\rho L_2+\frac{1}{2}\log 1/x}{\mu_2{}^2\rho-\mu_1{}^2\left(1-\frac{1}{x^3}\right)-2\mu_1\mu'\left(1-\frac{1}{x}\right)+\mu'^2\left(1+\frac{2}{1-x}+\frac{2\log x}{(1-x)^3}\right)}\right\}} \quad . \quad (81)$$

For a coreless ring this is

$$\frac{16\pi^2a^2k^2}{\mu_2}\sqrt{\log\frac{4}{k}}$$

which agrees with the value obtained in [I. § 14].

[Added April 3, 1886.—The cubic giving the times of vibration of a hollow ring of the same density as the fluid, and with no extra circulations, can be solved. The equation is

$$y^3-\frac{1-x^n}{1-x}y^2-\left\{n^2-\frac{n(1+x^n)}{1-x}+\frac{2(1-x^n)}{(1-x)^2}\right\}y+\frac{2n}{1-x}\left(n-\frac{1-x^n}{1-x}\right)=0.$$

The roots are

$$y=-n,\quad \tfrac{1}{2}\left(n+\frac{1-x^n}{1-x}\right)\pm\tfrac{1}{2}\sqrt{\left\{\left(n+\frac{1-x^n}{1-x}\right)^2-\frac{8}{1-x}\left(n-\frac{1-x^n}{1-x}\right)\right\}}$$

The corresponding times of vibration are $\frac{4\pi^2r^3}{\mu y}$, where r is the radius of the section.]

ERRATA IN PAPER ON "STEADY MOTION AND SMALL VIBRATIONS OF A HOLLOW VORTEX," PHIL. TRANS., VOL. 175.

1. Page 188, line 6 from bottom, the coefficient of k^2 in U is wrong, since the full value of ψ_0 was not substituted.
2. Page 191, line 4, *for* $1-2k^2$ *read* $1+2k^2$.
3. „ „ 6, *for* $(L-\frac{1}{2})$ *read* $L-\frac{1}{2}$.
4. „ „ 7, the coefficient of k^2 is $\frac{1}{4}(L-\frac{1}{2})^2$.
5. In § 13 the effect of the surface velocity in modifying the normal motion of the wave motion has been neglected. The time of vibration there given is therefore wrong. The correct value is given in the foregoing discussion in § 17.

XVI. *On the* CLARK *Cell as a Standard of Electro-motive Force.*

By Lord RAYLEIGH, *F.R.S.*

Received Jan. 7,—Read Jan. 21, 1886.

§ 39. The importance of a convenient standard of electro-motive force is now fully recognised. It gives the most available means of measuring currents, especially of large amount, and has been used for this purpose by several experimenters. I may refer to my investigation on the Constant of Magnetic Rotation of Light in Bisulphide of Carbon,* in which the currents were all measured by reference to a CLARK cell, whose value was originally obtained by absolute measurements and verified at intervals by the silver voltameter. CLARK cells are exceedingly convenient in use, and would doubtless be generally employed, could confidence be felt in their permanence, and in the equality of cells set up by different persons from the same recipe. To these questions I have given much attention; and the result of a large experience is very favourable to the trustworthiness of the cells, if reasonable precautions be observed in charging them. I believe that any one who takes the trouble to set up three or four cells and compares them occasionally, will be in possession of a standard of E.M.F. which he may trust to about $\frac{1}{1000}$th part.

The present memoir is to be regarded as supplementary to that on the Electrochemical Equivalent of Silver, and on the Absolute Electro-motive Force of CLARK Cells,† and the paragraphs are numbered accordingly. The total number of cells experimented upon is large. Of my own construction there have been about 60 of the ordinary kind (with solid zincs), and about 30 of the H-pattern (§ 28) with zinc amalgam. In addition to these some 40 cells made by others have been compared, with very interesting results to be given later.

Before entering into details it may be convenient to summarise the principal sources of error. The E.M.F. may be too high, (1) because the paste is acid, (2) because the paste is not saturated with zinc sulphate. The first fault tends to cure itself, and is rarely found after the cells are a month old. The second is the origin of the more serious discrepancies that have been met with in commercial cells. If the E.M.F. is too low, the cause may be, (1) that the cell has become dry, in which case the drop will probably be progressive, (2) the solution is *super*-saturated with zinc sulphate, (3) the mercury is impure.

* See *ante*, p. 343.

† Phil. Trans. Part II., 1884.

Believing that these cells are capable of affording standards of a high degree of precision, and that they ought to be in general use, I have gone into considerable detail as to the procedure which may be adopted. This may give the impression that the preparation is troublesome, but in reality the method that I propose is much simpler than those hitherto employed and thought to be necessary. To show how easy it is to set up these cells, I may refer to two large ones, contained in glass cylinders of about 4 inches diameter, and provided with wooden covers by which the electrodes are carried. Enough common mercury was poured in to cover the bottom, contact being made with it by means of a platinum wire sealed in a glass tube. The jar was then filled to a height of about 4 inches with saturated solution of commercial zinc sulphate with which some mercurous sulphate had been rubbed up in a mortar. The zinc electrode was cut from ordinary sheet metal, and was suspended horizontally near the top of the liquid by a projecting tail. After the first few weeks these large cells have never deviated from the standard by much more than $\frac{1}{1000}$, and have been found very convenient for certain purposes on account of their comparatively small resistance. They have also been used for preliminary comparisons with cells whose value was unknown, in which case there was danger of more current passing than is desirable to allow through delicate standards.

§ 40. The method followed for making the recent comparisons is the same in substance as that described in § 28. The use of a high resistance galvanometer gave a greater facility of reading, a change of $\frac{1}{10000}$ in the E.M.F. under measurement giving a motion of the spot of light which could be seen without a telescope from across the room.

The accompanying table (XIII.) gives the values of most of the older cells in continuation of that contained in the note to § 30 (p. 459). Cells (4), (8), (9) were, I think, left at Cambridge; (18) and (19) were observed at intervals during 1885, but the E.M.F. was found to fall. When about three parts per thousand too low, they were removed for examination, and found to be dry. The water had exuded, or evaporated, through cracks in the paraffin wax. The cells of the H-pattern, H_8, H_{13} were broken in a manner to be presently explained. On the other hand some new cells of the H-pattern, a, b, ... f are included. They are those referred to in the previous paper as having been fitted up by Mr. THRELFALL, and are more than a year old.

The agreement exhibited in Table XIII. is very remarkable. In many cases the cells may be depended on not to vary relatively more than 2 or 3 parts in 10,000, notwithstanding considerable changes of temperature. It is, indeed, doubtful whether even the whole of the small variations recorded are real. 1° C. influences the E.M.F. about 8 parts in 10,000, and differences of temperature of two or three-tenths of a degree may well have occurred, since the cells were variously mounted, and no particular precautions were taken beyond the avoidance of readings at times when the temperature of the room (immediately under the roof) was changing rapidly.

Table XIII.

	Jan. 2, 1885.	Jan. 26, 1885. 56°.	March 2, 1885. 58°.	April 7, 1885. 56°.	June 4, 1886. 71°.	June 29, 1885. 64°.	Aug. 14, 18, 1885. 63°.	Sept. 3, 4, 1885. 64·5°, 62·5°.	Oct. 23, 23, 24, 1885. 57°, 57°, 56°.	Nov. 23, 24, 1885. 54°, 52°.	Dec. 26, 1885. 52°.	April 27,* 1886. 58°.
Clark 1	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000	1·0000
,, 5	1·0000	1·0002	1·0002	1·0002	1·0004	·9998	1·0000	1·0001	1·0001	·9999	·9999	·9999
,, 10	1·0002	1·0000	1·0000	1·0000	1·0001	1·0000	1·0001	..	1·0002	1·0002	1·0005	1·0003
,, 11	1·0002	1·0000	1·0002	1·0000	1·0003	1·0002	1·0001	..	1·0001	1·0001	1·0002	1·0000
,, 12	1·0004	1·0002	1·0002	1·0001	1·0002	1·0002	1·0001	..	1·0004	1·0003	1·0006	1·0001
,, 13	1·0000	·9998	·9998	1·0000	·9998	1·0000	1·0000	..	1·0000	1·0001	1·0003	1·0001
,, 14	1·0000	1·0000	1·0000	·9999	·9998	1·0000	·9999	1·0000	1·0000	1·0001	1·0003	1·0000
,, 15	1·0006	1·0005	1·0005	1·0002	1·0002	1·0003	·9998	1·0005	1·0006	·9999	1·0006	1·0007
,, 16	1·0004	1·0001	1·0001	1·0002	·9998	1·0000	1·0000	1·0000	1·0001	1·0003	1·0005	1·0000
H_5	1·0000	·9998	·9998	·9998	1·0001	1·0002	·9999	1·0001	·9999	·9999	1·0001	1·0001
H_{10}	·9998	·9998	·9998	1·0002	1·0002	1·0002	1·0001	1·0003	1·0001	1·0003	1·0005	1·0003
H_{11}	·9998	·9996	·9998	1·0002	1·0002	1·0002	1·0003	1·0001	1·0001	1·0003	1·0004	1·0004
H_{13}	·9995	1·0003	1·0001	·9996	1·0002	1·0003	broken					
a	1·0002	1·0000	·9999	1·0002	1·0004	1·0004	1·0001	1·0003	1·0003	1·0003	1·0001	1·0003
b	1·0002	1·0002	1·0000	1·0002	1·0004	1·0004	1·0001	1·0003	1·0001	1·0002	1·0004	1·0003
c	1·0000	·9998	·9998	1·0000	1·0002	1·0002	1·0000	1·0000	1·0000	1·0001	1·0004	1·0003
d	1·0000	1·0000	1·0000	1·0000	1·0002	1·0004	1 0001	..	1·0000	1·0001		
e	1·0010	1·0004	1·0008	1·0010	1·0010	1·0010	1·0004	1·0009	1·0009	withdrawn		
f	1·0002	1·0000	1·0001	1·0004	1·0006	1·0006	1·0003	1·0004	1·0004			

* Added since the reading of the paper.

It may be convenient to recall that cells 1, 5, were made in Oct., 1883; 10-13 and 14–19 in May, 1884; H_8 in March, 1884; H_{10}, H_{11} also in March, 1884.

§ 41. In cells of the ordinary type the principal source of weakness is imperfect sealing at the top, due to cracks in the paraffin wax. As pointed out by Dr. ALDER WRIGHT,* a better result is obtained if the whole cell be imbedded in a large mass of wax than when (as in my cells) the wax is applied merely inside the tube, above the cork sustaining the zinc. During the last year I have replaced paraffin by marine glue, which, so far as can be judged at present, may be relied upon to effect a complete seal. The procedure will be described presently more in detail.

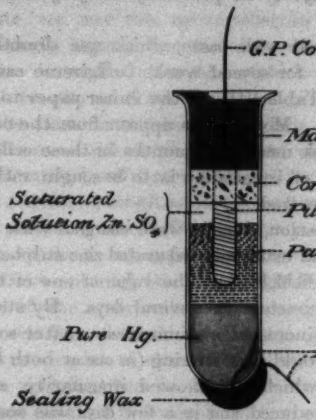
The cause of failure in the H-cells is of a different nature. Many of the earlier cells had been found to break in the amalgam leg, and the trouble was attributed to a hardening and expansion of the contents (§ 29). Such a hardening had, in fact, been observed in one or two cases. More recent experience, however, has proved that the cause must be looked for elsewhere, several cells having failed in which no trace of solid amalgam was to be found. Nevertheless the amalgam is the cause of the trouble, for out of a large number of breakages *not one* has occurred in the leg containing pure mercury. It would appear that some alloying takes place with the platinum wire in contact with the amalgam, and that this gradually extends itself with fatal results to the part of the platinum sealed into the glass, from which place the cracks are always observed to radiate. It is hoped that a cure will be found in a plan, adopted for some recent cells, of melting in a little cement (marine glue has been used) so as to protect from the amalgam the part of the platinum which lies nearest to the glass; but it is too soon to speak with certainty.

§ 42. The H-form lends itself to hermetical sealing, and at one time I anticipated advantage from this course. There is, however, such a large amount of spare liquid that there is no likelihood of trouble from desiccation, even if the corks allow a little evaporation. Indeed, by withdrawing the corks a fresh supply of liquid could be introduced at any time. It happened on one occasion that an H-cell to which a large excess of salt had been added, was so far crusted up next the metallic surfaces that it began to show signs of failing E.M.F., much as if it were going dry. The mass was so compact that no impression could be made upon it with a glass rod; but it was bored through with a steel reamer, when the E.M.F. at once recovered its normal value. In such cases the accessibility is advantageous, especially for purposes of experiment. It is well, however, to avoid such a large excess of salt as was present in this case. By alternate melting and crystallisation as the temperature rises and falls, there is a tendency to aggregation, of which the cell above referred to affords an extreme example.

In the construction of cells with solid zinc electrodes, I have fallen back upon a simplified pattern—nothing more in fact than a small tube with a platinum wire sealed through its closed end. See figure.

* Phil. Mag., July, 1883, p. 32.

The only objection to this form is that the cells cannot, without precaution, be supported from underneath. Most of mine are held at the centre by a spring (cut from sheet metal) against a piece of board mounted on its edge. In this case the copper electrodes are secured in sealing-wax to the wooden stand. For single cells, when portability is desired and convenience of immersion in water or ice, it is a good plan to enclose the whole in a rather long and narrow test-tube. A little cotton wool supports the cell and prevents it from shaking about laterally. The gutta-percha covered leads pass through a piece of cork inserted near the top of the test-tube, and a little marine glue poured over the cork makes all tight. In order to give mechanical support to the platinum wire, which is liable to break where it passes through the glass, the external application of sealing-wax is recommended—a precaution applicable also to the H-cells.



§ 43. In charging the cells the first step is to pour in sufficient *pure* * mercury to cover the platinum effectively. The paste (of which more presently) is next introduced, with the aid of a small funnel, care being taken not to soil the sides above the proper level. The zincs, cut from rods of pure zinc, as supplied by HOPKINS and WILLIAMS, and not recast, are soldered to copper wires and cleaned in the lathe. Just before use they are dipped in dilute sulphuric acid, washed in distilled water, and dried with a clean cloth or filter paper. Each zinc is mounted in a short piece of cork fitting the tube (but not too tightly), and nicked in order to allow of the passage of air. The cork is pushed gradually down until its lower face is almost in contact

* Except when the contrary is stated, mercury distilled *in vacuo* has been used for CLARK cells. There is, I believe, a difficulty in purchasing mercury thus treated; but every physical laboratory should be provided with an apparatus for this purpose. That employed by me was distilled at Cambridge in an apparatus set up by Mr. W. N. SHAW.

with the paste. The object is to leave but little air, and at the same time to avoid squeezing up the paste between the cork and the glass. The whole is now made tight by pouring marine glue over the cork high enough to cover the zinc and soldering, and leave only the wire projecting. The tube should rise high enough to receive the glue, and thus secure a good adhesion.

The marine glue is melted over the gas flame in a small pot or basin, and stirred, until uniform, with a small stick. It should be fluid enough to pour by its own weight. If necessary, a little benzole may be added, but the cement should be pretty hard when cold.

In the operation of pouring in the marine glue the glass is heated by the glue sufficiently for adhesion; but this heat does not extend appreciably below the cork. *Neither in this, nor any other stage of the process of charging, is heat applied to the paste.*

§ 44. The earlier cells, prepared with paste, which was doubtless strongly acid, frequently gave irregular results for several weeks. Extreme cases are afforded by 15 and 16, which are shown by Table VIII. of the former paper to have been at first more than 2 per cent. too strong. Moreover, as appears from the continuation of this table in the notes (p. 459), it took nearly two months for these cells to settle down to their normal values. The cause of irregularity is to be sought rather at the mercury than at the zinc (or amalgam) electrode.

In order to examine this question, H-cells were charged with pure mercury and paste in both legs, and filled up as usual with saturated zinc sulphate solution. There should, of course, have been no E.M.F.; but the value of one of the cells was ·0041 CLARK, and remained tolerably constant for several days. By stirring with a glass rod, the E.M.F. could be either increased or diminished. After some weeks the cells had come sensibly to zero, and would bear stirring (in one or both legs) without much disturbance. To another cell, which still showed irregularity, zinc carbonate was added. The E.M.F. was much reduced, and in a few days was scarcely sensible even on stirring.

When the paste is neutralised in the first instance with zinc carbonate, the irregularities are much reduced. Two cells thus prepared had an E.M.F. less than ·0001 CLARK, and were scarcely affected by stirring. On Jan. 27, 1885, a piece of zinc wire was poked through the paste, so as momentarily to touch the mercury in one leg of one of these cells. A large E.M.F. was thus developed, which remained operative for half-an-hour or more; but on Jan. 28 the E.M.F. was only ·0003 CLARK, and on Jan. 31 ·0002 CLARK. It is clear that the mercurous sulphate has the property of freeing the mercury from the smallest contamination with zinc.

§ 45. In consequence of these observations more recent cells have been prepared with neutralised paste. This course has the advantage that the cells attain their normal values in a few days, sometimes within one day, of charging. So far as I can judge, however, there is no difference in the ultimate value whether the paste be acid

or neutral. In the former case the cell probably neutralises itself by dissolution of zinc, a certain amount of gas being liberated.

It is convenient to keep a stock bottle of saturated solution of zinc sulphate. This may be prepared in a flask by mixing distilled water with about twice its weight of crystals. A little carbonate of zinc is added to neutralise free acid, and solution is effected with the aid of *gentle* heat. If time can be afforded, it is a good plan to let the solution stand, as a good deal of iron is usually deposited, even when "pure" zinc sulphate is used. The solution may then be filtered in a warm place into the stock bottle. When it is intended to charge H-cells, or prepare paste, the bottle should be exposed to a gentle warmth for a few hours, and the solution should be drawn with a pipette from near the crystals at the *bottom of the bottle*. Otherwise there is no security that the liquid used will be saturated.

To prepare paste we may rub up together in a mortar 150 grms. mercurous sulphate (as purchased), 5 grms. zinc carbonate, and as much of the saturated zinc sulphate solution as is required to make a thick paste.* Carbonic anhydride is liberated, and must be allowed a sufficient time to separate. I have found it convenient to leave the paste in the mortar for two or three days, rubbing it up at intervals with additions of zinc sulphate solution until the gas has escaped. By the addition of a small crystal, and by evaporation, we have security that the paste is saturated, and will remain so, notwithstanding such moderate elevation of temperature as the cells are expected to bear. The paste may then be transferred to a tightly-corked bottle, and, so far as my experience extends, will remain available for many months at least. Before pouring, the bottle of paste should be well shaken up.

The performance of the newer cells has been satisfactory (one irregularity will be mentioned later), and the substitution of marine glue for paraffin wax promises a longer life. A large number of observations have been recorded, but it does not appear necessary to give them here in detail.

§ 46. I have been anxious to compare with my cells some prepared by others, and have to thank many physicists for the opportunity of doing so. Dr. ALDER WRIGHT was good enough to send me several of his cells, with which comparisons of especial value have been made. I shall have occasion to remark more at length upon the results obtained with these cells, and for the moment will only say that the difference between them and mine is under $\frac{1}{1000}$th part. Cells prepared by Mr. THRELFALL (not those previously mentioned), by Dr. FLEMING, and by Prof. G. FORBES also agreed well.

Dr. FLEMING'S cells, of which six remained in my hands, were at first irregular, and even now show somewhat larger variations than the best cells are liable to. The cause is, I believe, to be found in insufficient purity of mercury, as is suggested by

* It is usually found that on neutralisation the mercurous sulphate turns yellow, so that the paste presents ultimately somewhat the appearance of mustard. I do not know whether the change of colour is normal, or is to be attributed to impurity.

the appearance of the metallic surface. When the mercury is quite pure, the surface is as bright as in a thermometer bulb. I have instituted special experiments in order to ascertain the effects of impure mercury. On two occasions three cells have been charged alike in all respects, except that in the first vacuum-distilled mercury was employed, in the second ordinary mercury, and in the third mercury purposely contaminated with tin-foil (probably containing also lead). The results were not very distinct; but they indicate that impurity in the mercury is apt to depress the E.M.F. (*e.g.* by ½ per cent.), and especially to make it irregular. I am disposed to attach great importance to purity of mercury, and believe it to be more essential than purity of zinc,* although I should not recommend the use of common zinc when the purer metal can be obtained so easily. Other cells have been prepared with mercury to which a little silver and copper (filings from a silver coin) were added. After the first week or two the E.M.F. of these cells was normal (to within $\frac{1}{1000}$th part). It is probable that metals more oxidisable than mercury are removed from it by the paste, as certainly happens in the case of zinc.

§ 47. It has been abundantly proved by v. Helmholtz, Alder Wright and other workers, that the E.M.F. of Clark's, Daniell's, and similar cells, rises as the zinc sulphate solution is diluted. In some such cells to be discussed presently (39, 40), the E.M.F. is about 1½ per cent. higher than for a normal Clark. Dr. Hopkinson, some of whose cells have been compared with mine, writes that he is pretty clear that the worst irregularities of Clark's cells are due to the zinc sulphate not being saturated. A cell 2 per cent. in excess could be made right by simply introducing crystals of $ZnSO_4$. It is evident that sufficient care has not been taken in this respect in the preparation of cells sold to the public.

§ 48. In this matter of saturation there is another danger to be encountered, to which my attention was first drawn in connexion with some cells prepared with great care by Mr. Mortimer Evans, and left in my charge by Sir W. Thomson. Of these, two were in practical agreement with mine; but the other eight, though in close agreement with one another, were too low (according to my standard) by rather more than four parts per thousand. And this state of things persisted without the smallest change for two or three months, during which tests were applied at intervals.

Being anxious to examine a phenomenon to which my experience had afforded no parallel, I opened carefully one of the abnormal cells (T_2) to the extent of withdrawing the zinc. My idea was that possibly the zinc solution was *supersaturated*, in which case the E.M.F. might be expected to be too low. Attached to the zinc, however, were found what appeared to be crystals of sulphate; but in order to be on the safe side, a few particles from the stock bottle were added to the cell, and the zinc was replaced. After the lapse of a few hours the E.M.F. was tested, *and was found to be normal*, as it has remained ever since.

* Of ten cells, prepared at the same time as a set to be used in series, five were made with pure zinc and five with common sheet zinc. No difference in the performance can be detected.

There was now little doubt but that the solution had been supersaturated, in the sense with which we are concerned. The presence of crystals is no evidence to the contrary, unless it can be proved that the crystals were those of the *normal, hepta-hydrated* salt.

At this stage I wrote to Mr. M. EVANS to inquire whether there was anything in the history of the cells that would account for the separation into two classes, and I was informed that all the cells had been prepared originally in the same manner. The mercury was twice distilled *in vacuo*, and in other respects the greatest care had been taken. When, however, the cells came to be tested, it was found that owing to contraction all but *two* were wanting in proper contact between the mercury and platinum, and that this contact could only be restored by remelting over a water-bath the whole of the paraffin wax in which the cells were imbedded. It was by this operation, no doubt, that the solution became supersaturated. The agreement with mine of the two cells which were not heated (one of them (T_3) is referred to later) is very satisfactory as showing that the great precautions exercised by Mr. EVANS lead to the same E.M.F. as I have obtained with far less trouble. I may add that a second abnormal cell (T_8) moved to equality with my standards on being opened.

§ 49. With a view to the better understanding of this matter I made myself acquainted with the beautiful researches of M. GERNEZ * upon supersaturation, conducted principally with solutions of sulphate and of acetate of soda; and have performed parallel experiments upon sulphate of zinc. A very strong solution of this salt prepared hot, and sealed up in a glass tube, will sometimes cool without any deposit. More often it throws down an abnormal (lower) hydrate. If in this condition the tube be heated pretty rapidly in boiling water, some of the salt dehydrates further to a powder (presumably mono-hydrated); if it be allowed to cool again the inferior crystalline hydrate reforms. However long the solution stands cold over this hydrate, it is still supersaturated as regards the normal hydrate, the minutest addition of which causes the supernatant liquid to become almost solid, with needles penetrating it in all directions. The experiment has been repeated many times with less strong solutions standing in open test-tubes, charged and preserved with the simple precautions indicated by M. GERNEZ.

It is evident that "supersaturation" is a term without definite meaning until further explained. GERNEZ has shown that a solution may be supersaturated with respect to one or both of two different hydrates, *i.e.*, will crystallise similarly on contact with the smallest fragment, and not supersaturated at all with respect to the anhydrous salt, the addition of which causes no effect. DE COPPET † has proved that the so-called supersaturated solution may be disposed to take up a further portion of anhydrous salt; as may, indeed, be inferred from previously known facts, since

* "Annales de l'école normale," t. iii., 1866, p. 163; t. v., 1876, p. 1.

† C.R. LXXIII., p. 1324, 1871. See also NICOL, Phil. Mag., June, 1885.

in general the anhydrous salt must tend either to leave or to enter the solution, and the former alternative is excluded by the observed behaviour on first contact.

§ 50. In view of the above facts we can hardly doubt that a CLARK cell, heated nearly to the temperature of boiling water and then cooled, would be likely to become supersaturated; but I thought it would be satisfactory actually to try the experiment. A normal cell of my own preparation and containing an excess of undissolved salt, was maintained for several hours at an elevated temperature, and tested after cooling. A temperature of 38° C. did not permanently alter the cell; neither did a temperature of 49°, nor one of 60°. But after an exposure to about 80° a permanent change set in. Immediately after cooling the value in terms of the standards was ·9914, but after one day's standing it settled to ·9943, close to which value it has since remained. It appears from the above that the cell probably requires to be heated sufficiently to decompose the normal hydrate, and not merely to bring all the immersed salt into solution. In the latter case there may well be solid particles within reach, which re-determine normal crystallisation on cooling.

A second experiment was tried with an old cell (*e* of Table XIII.) which contained a large excess of undissolved salt. This was of the H-pattern, which lends itself more conveniently to observation and experiment. On November 13 the cell was heated for several hours nearly to 100° C. After cooling a solid mass of crystals was to be seen over the metals in both legs, and it might have been supposed that the operation had been unsuccessful. On November 16, however, the E.M.F. was found to have changed (from about 1·0005) to ·9949, at which value it remained until November 25. On that day, at 5^h p.m. the corks were drawn. At 6^h no effect had been produced, and a fragment of the normal hydrate was dropped into each leg. At $6^h\ 45^m$ new crystals had formed, and the E.M.F. had risen to ·9996. A few hours later the E.M.F. was 1·0000, at which value it has since remained within 2 or 3 parts in 10,000. It was remarked that the crystalline deposit on contact with the normal hydrate was much less in amount than had been met with in experimental tubes with simple zinc sulphate. The explanation is probably that during the heating no complete diffusion of the salt was effected, so that after cooling supersaturation was limited to the lower layers. Both metals being at the bottom in this form of cell, the E.M.F. is independent of the condition of the upper parts.

It is worthy of note that all the "supersaturated" cells which I have tested are about 5 parts per 1000 too low. That they should give a definite E.M.F. is to be expected whenever the lower hydrate is formed. For the solution in contact with the lower hydrate is just as definite in composition as when it is in contact with the normal hydrate. It is, however, possible to have "supersaturated" solutions without formation of the lower hydrate, and then the E.M.F. would be indefinite. The deficiency may certainly be less than the 5 parts per 1000, and may probably be more in certain cases.

§ 51. In view of the possibility of error from under and over-saturation, the reader

may be inclined to ask whether it would not be better to prescribe a *dilute* solution for standard cells, as is conveniently done for standard DANIELL's. One advantage attending this construction is (as will presently appear) a lower temperature coefficient. Again, we should be inclined to expect a more definite dependence upon temperature. In order to bring a saturated cell to its normal condition after warming (for example), it is necessary not merely that the whole of the contents should acquire the new temperature, but also that sufficient time should be allowed for *diffusion*. If the solution in contact with the zinc be weaker than corresponds to saturation at the altered temperature, the full loss of E.M.F. will not be experienced. In this respect the H-cells, in which the excess of salt rests upon the metals, would seem to have an advantage. But I cannot say that in practice I have met with the defect due to imperfect diffusion. Cells which have stood at 10° or 12° seem to acquire their new values after an hour or two's immersion in ice. The argument weighs, however, in favour of *small* cells, through which diffusion of temperature and matter can take place quickly. Such experience as I have had of cells prepared with dilute solutions, would not lead me to prefer them, even were there no difficulty in, or necessity for, a standardising. In the case of clear solutions, such as are used for DANIELL's, the specific gravity is a convenient test of strength; but I do not see how a standard unsaturated paste could be accurately prepared without a good deal of trouble. Another objection to dilute solutions is the progressive alteration of E.M.F., due to evaporation, which must take place whenever the sealing is at all imperfect.*

In truth, there is no real difficulty in avoiding both under and over-saturation, if the experimenter will bear in mind the known properties of the materials with which he is dealing. The grosser errors, arising from the first cause, can only occur as the result of carelessness. As to the latter, it may be that supersaturation has sometimes entered as a consequence of excessive precautions against the admission of air. It cannot occur in the presence of the minutest fragment of the normal hydrate. Opinions may perhaps differ upon this point, but I am myself disposed to condemn the use of heat in charging the cells. If hot paste be brought into contact with hot mercury, and then closed hermetically, there must be some risk of supersaturation.

§ 52. The next question which I propose to consider is that of the temperature-coefficients of CLARK cells. My observations on cell No. 1 at Cambridge, § 36, gave for the proportional fall of E.M.F. per degree Centigrade in the neighbourhood of 15° the number ·00082, so that at $t°$ C. we might take

$$E = 1{\cdot}435\{1 - {\cdot}00082(t - 15)\}.$$

This number is in agreement with that found by HELMHOLTZ for saturated cells, but it differs seriously from the number (·00041) given by ALDER WRIGHT† also for

* ALDER WRIGHT. *Loc. cit.*, p. 33.

† Phil. Mag., July, 1883, p. 36.

saturated cells. These discrepancies have naturally led to the conclusion that the temperature-variations of CLARK cells are uncertain, and Dr. FLEMING has insisted upon the advantage in this respect possessed by the DANIELL.* If indeed it were a matter of chance whether the temperature-coefficient of a CLARK were ·0008, or ·0004, the utility of these cells as standards for delicate work would be seriously impaired. A glance, however, at Table XIII.† will show that such uncertainty need not exist. The results of March 2 and June 4 correspond to a difference of 19° F., or about 11° C., so that if one cell had the coefficient ·0008, and another the coefficient ·0004, the change of temperature would separate them to the extent of 44 parts in 10,000, whereas the greatest change observed (perhaps not due to this cause at all) is but 5 parts. Many observations on recent cells, made in ordinary course, point in the same direction. In one or two cases there has been an apparent rise at temperatures above 65° F., indicating a drop in the temperature-coefficient relatively to that of No. 1. I have attributed this to an insufficient excess of undissolved salt in cells prepared when the weather was cold, the result of course being a failure of saturation at high temperatures, attended (as will presently appear) by a fall of temperature-coefficient.

Being desirous of clearing up, as far as possible, any questions connected with the practical use of these cells as standards, I determined to supplement the former observations with special experiments at somewhat extreme temperatures, which should include as great a variety of constructions as possible. Most of the cells were so mounted that they could not well be tried at temperatures differing from that of the surrounding air, and I had to content myself with varying the temperature of the room by opening windows and burning gas. Care was taken that no great variation occurred within two or three hours of the comparisons. Under these circumstances tests were made of (1), (10) (Table XIII.); T_1 (one of the abnormal cells of Mr. M. EVANS'), T_3 (a normal cell of the same batch); W_{31}, W_{32}, W_{33}—three cells by Dr. ALDER WRIGHT; M_{183}, a cell sold by Messrs. CLARK and MUIRHEAD,—at temperatures from 47° F. to 69° F. These cells were all supposed to be saturated. There were also two prepared purposely with diluted paste—(39), (40). Besides these, two saturated cells of my own construction mounted in test tubes, which could be immersed in water or ice, were tested from 32° F. to 67½° F.

In order to obtain an absolute result we must have command of a standard independent of temperature-variations. At Cambridge I employed for this purpose a cell kept constantly in ice. The present observations were rather protracted, and I preferred to rely upon two cells (35), (38), mounted in test tubes and imbedded with a thermometer in a mass of sand, itself situated in an underground recess. The variations of temperature were here very small and readily determined, so that there

* On the "Use of DANIELL'S Cell as a Standard of Electro-motive Force," Phil. Mag., Aug., 1855. It appears, however, to be, by a slip of the pen, that the coefficient for the DANIELL is represented as only $\frac{1}{10}$ of that of the CLARK. The numbers given lead to the ratio $\frac{1}{4}$.

† I must apologise for the Fahrenheit degrees.

was no practical uncertainty on this account. The variation of the two cells relatively to one another was less than $\frac{1}{10000}$ during the whole month of observation. (Table XV.)

TABLE XIV.—Value of Cells referred to No. 1.

Temperature. (Fahr.)	Date, 1885.	(1).	T_1.	T_2.	W_{10}.	W_{40}.	W_{60}.	M_{183}.
47	Nov. 8 . . .	1·0000	·9957	..	·9991	·9991	·9993	..
49½	,, 2 . . .	1·0000	·9955	..	·9993	·9999	..	·9997
51	Oct. 31 . . .	1·0000	·9957	1·0001	·9996	1·0000	..	·9997
56½	,, 28 . . .	1·0000	·9957	1·0003	..	..	..	..
57	,, 27 . . .	1·0000	·9957	1·0001	1·0000	1·0000	..	..
58½	Nov. 4 . . .	1·0000	·9954	..	·9992	·9992	1·0000	·9996
61	Oct. 30 . . .	1·0000	·9957	..	·9997	·9993	·9999	·9980
67	Nov. 27 . . .	1·0000	·9951	·9996	1·0001	·9996	·9997	1·0000
69	July 20 . . .	1·0000	·9957	·9998	..	..	..	..

From the results in Table XIV., reduced to No. 1 as standard, the reader will see that there is no distinct difference of coefficient. It is interesting to note that T_1 (which there is every reason to consider supersaturated) keeps its distance from No. 1. When I first thought of supersaturation, I regarded this agreement of temperature-coefficient as an argument on the other side, not at that time recognising the probable occurrence of the lower hydrate. But what will still more arrest attention is the agreement of Dr. ALDER WRIGHT's cells with No. 1. There are same irregularities, possibly dependent upon imperfect penetration of temperature through the masses of paraffin in which the cells are imbedded; but there is no distinct evidence of a lower coefficient, and certainly no such difference as that between ·0008 per degree Cent. and ·0004. The same may be said of M_{183}, which I received from Dr. ALDER WRIGHT.

§ 53. The absolute variations with temperature of (1), (10), (39), (40) are shown in Table XV., in which all the electro-motive forces are expressed in terms of (38) at 51° F., a small correction being introduced to allow for the 1° or 2°, by which the actual temperature of (38), (35), may have differed from 51°. The coefficients for (1) and (10) are almost identical. The observed values for (1) agree pretty well with the formula

$$\cdot9970\{1-\cdot000425(t-57)\},$$

as will be seen from the adjoining column calculated therefrom. This is in Fahrenheit degrees. The corresponding formula in Centigrade degrees is

$$\cdot9970\{1-\cdot000765(t-13\cdot9)\}.$$

It appears, therefore, that the temperature-coefficient for these cells, including those of Dr. ALDER WRIGHT, is ·00077, and that the observed values are utterly irreconcilable with such a coefficient as ·0004.

TABLE XV.—In terms of (38) at 51° Fahr.

Temperature. (Fahr.)	Date, 1885.	(38) reduced to 51°.	(1) observed.	(1) calculated.	(10) observed.	(39) observed.	(39) calculated.	(40) observed.
°								
47	Nov. 8	1·0003	1·0013	1·0013	1·0014	1·0154	1·0155	1·0152
48½	„ 1 (aft.)	1·0003	1·0008	1·0006	..	..	..	..
49½	„ 2	1·0003	1·0004	1·0003	1·0006	1·0150	1·0151	1·0149
51	Oct. 31	1·0003	·9996	·9996	·9997	1·0148	1·0147	1·0147
51	Nov. 7	1·0004	·9997	·9996	1·0000	..	..	..
51½	„ 18	1·0004	·9993	·9993	..	..	..	..
52	„ 12 (morn.)	1·0004	·9994	·9991	..	..	..	..
52½	„ 17	1·0004	·9994	·9989	..	..	..	..
53	„ 21	1·0003	·9988	·9987	..	..	..	..
53½	„ 11 (even.)	1·0004	·9989	·9985	..	..	..	..
54	„ 20	1·0003	·9982	·9983	..	..	..	..
56½	Oct. 28	1·0003	·9971	·9972	·9973	1·0133	1·0135	1·1032
56½	Nov. 12 (aft.)	..	·9974	·9972	..	..	..	..
57	Oct. 27	1·0003	·9970	·9970	·9972	1·0134	1·0134	1·0133
57½	Nov. 13	1·0004	·9970	·9968	..	..	..	..
58½	„ 4	1·0004	·9964	·9965	·9963	1·0131	1·0131	1·0130
61	Oct. 30	1·0004	·9955	·9953	·9955	1·0125	1·0125	1·0125
67	Nov. 27	1·0004	·9928	·9928	·9927	1·0111	1·0113	1·0110

TABLE XVI.—In terms of (38) at 51° Fahr.

Temperature. (Fahr.)	Date, 1885.	(37) observed.	(37) calculated.	Temperature. (Fahr.)	Date, 1885.	(36) observed.	(36) calculated.
°				°			
32	Nov. 10	1·0051	1·0050	32	Nov. 11 (aft.)	1·0067	1·0067
32	„ 11 (aft.)	1·0050	1·0050	32	„ 11 (even.)	1·0066	1·0067
32	„ 11 (even.)	1·0051	1·0050	32	„ 12 (morn.)	1·0070	1·0067
40	„ 17	1·0028	1·0027	32	„ 12 (even.)	1·0068	1·0067
41½	„ 21	1·0021	1·0022	40	„ 17	1·0038	1·0041
47½	„ 8	1·0003	1·0003	41½	„ 21	1·0028	1·0035
48½	„ 12 (aft.)	·9995	·9998	47½	„ 18	1·0010	1·0013
49½	„ 12 (morn.)	·9996	·9996	49½	„ 13	1·0003	1·0005
49½	„ 13	·9993	·9996	50½	„ 2	1·0003	1·0001
50½	„ 2	·9997	·9992	50½	„ 10	1·0001	1·0001
52½	Oct. 31	·9986	·9985	53	Oct. 31	·9992	·9991
52½	Nov. 18	·9984	·9985	53	Nov. 18	·9992	·9991
57½	Oct. 27	·9966	·9966	57½	Oct. 27	·9970	·9971
57½	„ 28	·9967	·9966	57½	„ 28	·9971	·9971
61½	„ 30	·9946	·9949	61½	„ 30	·9950	·9952
66½	Nov. 4	·9926	·9928	66½	Nov. 4	·9931	·9929
67½	„ 20	·9916	·9925	67½	„ 20	·9923	·9925
67½	„ 7	·9922	·9924	67½	„ 7	·9925	·9924

For (39) and (40), prepared with unsaturated solution, the temperature-coefficient is exactly the half of that for the saturated cells. Reckoned in Fahrenheit degrees, it is ·00021, or in Centigrade degrees ·00038. The agreement with this value is shown by the column calculated from the formula

$$1{\cdot}0134\{1-{\cdot}000212(t-57)\}.$$

The coefficient ·00038 is so close to that given by ALDER WRIGHT, viz., ·00041, that one cannot help suspecting that the cells used by him for this purpose may have been unsaturated, or possibly supersaturated (without deposit of lower hydrate).

Over the above range of temperature a linear expression represents the E.M.F. sufficiently well. When, however, the values recorded for (36), and (37) for temperatures from 32° F. to 67½° F. were plotted, a distinct curvature was apparent indicating a lower coefficient at the lower temperatures. Table XVI. exhibits the observed values, and (for comparison with them) numbers calculated from quadratic expressions. For (37) the expression is

$$\cdot 9985\{1 - \cdot 00037(t - 52\tfrac{1}{2}) - \cdot 0000025(t - 52\tfrac{1}{2})^2\};$$

the equivalent of which in Centigrade degrees is

$$\cdot 9985\{1 - 000666(t - 11\cdot 4) - \cdot 0000081(t - 11\cdot 4)^2\}.$$

According to this, the change for one degree Cent. is the following linear function of temperature

$$\cdot 000674 + \cdot 0000162(t - 11\cdot 4);$$

so that the temperature-coefficient ranges from ·000489 at 0° C. to ·000813 at 20° C. At 15° C. it would be ·00073.

For the other cell (36) the observed values of E.M.F. are pretty well represented by

$$\cdot 9991\{1 - \cdot 00042(t - 53) - \cdot 0000028(t - 53)^2\},$$

from which are deduced the numbers in the column headed "calculated." In Centigrade degrees this becomes

$$\cdot 9991\{1 - \cdot 000756(t - 11\cdot 6) - \cdot 0000091(t - 11\cdot 6)^2\};$$

giving for the temperature-coefficient at $t°$ Cent.

$$\cdot 000765 + \cdot 000018(t - 11\cdot 6).$$

At 15° the value from this formula is ·00083.

It would seem that these two cells have temperature-coefficients which differ sensibly. But this way of presenting the matter is apt to give an exaggerated impression. The difference in the coefficients indicates a separation of electromotive forces at the rate of $\frac{1}{10000}$ only per degree Cent., so that the whole relative change for ordinary indoor variations would not exceed $\frac{1}{1000}$ of the whole. It will be seen from Tables XIV., XV. that, through a more limited range of temperature, a large number of various cells are satisfactorily represented by the coefficient (intermediate between those just found) ·00077; and I believe that the adoption of this number* for cells with *saturated solutions* can lead to no appreciable error in ordinary use. For

* It should be mentioned that (*loc. cit.*) Dr. FLEMING found for a CLARK cell the coefficient ·00082 at 15° C.

very special purposes it will, no doubt, be desirable to protect standard cells from the larger temperature-variations, which is very easily done by keeping them in a cellar, from which well-insulated wires will convey the E.M.F. to any desired point. As to the absolute magnitude of the variations of E.M.F. with temperature, it may be worth while to recall that they are about the *double* of those experienced by German silver resistances.

§ 54. A good many special experiments have been tried with cells variously constructed, in order to elucidate as far as possible the behaviour of CLARK's. Two of these, labelled H (1, $\frac{1}{3}$), H (1, $\frac{1}{4}$), were referred to in a short communication to the British Association at Montreal,* and have been observed at intervals since, with results which are, I think, of interest. In these cells, the legs representing the zincs were charged with a strong† but fluid amalgam of zinc and mercury; so that if the other legs had been charged with pure mercury and paste of mercurous sulphate, an ordinary CLARK would have resulted. This, however, was not done. No paste at all was used, and in place of pure mercury a dilute amalgam was substituted—in H (1, $\frac{1}{3}$) one obtained by diluting the strong amalgam with its own volume of pure mercury, and in H (1, $\frac{1}{4}$) one containing one volume of strong amalgam to three volumes of pure mercury. The cells were filled up, as usual, with zinc sulphate solution.

The electromotive forces, expressed as fractions of No. 1, are recorded in the adjoining table (XVII.). It is clear that the E.M.F. rises rapidly with temperature—something like 30 per cent. for 15° F. The large amount of this variation, and the fact of its taking place in the opposite direction to that followed by CLARK's, are very remarkable.

TABLE XVII.

Date, 1884.	Temperature. (Fahr.)	H (1, $\frac{1}{3}$).	H (1, $\frac{1}{4}$).	Date, 1885.	Temperature. (Fahr.)	H (1, $\frac{1}{3}$).	H (1, $\frac{1}{4}$).
	°				°		
July 21	..	·0041	..	Jan. 2	..	·0083	·0083
„ 22	..	·0041	..	„ 14	..	·0081	·0081
„ 23	..	·0041	..	Aug. 3	63	·0044	·0095
„ 24	..	·0041	..	Sept. 5	63	·0042	·0095
„ 29	..	·0040	·0091	Nov. 6	52	·0034	·0083
„ 30	..	·0040	·0091	„ 12	56	·0039	·0086
Aug. 6	67	·0044	·0096	„ 27	67	·0044	·0093
Sept. 4	62	·0041	·0093	Dec. 2	51	·0034	·0084
Oct. 8	63	·0040	·0093				
„ 28	59	·0037	·0090				
Nov. 17	..	·0036	·0087				
Dec. 5	..	·0037	·0089				

It would appear that, apart from changes of temperature, the cells have retained their E.M.F. tolerably constant for more than a year and a half—a result which could

* B. A. Rep., 1884, p. 651.

† As to what is to be regarded as strong, more facts will presently be given.

scarcely have been expected, in view of the very small quantities of zinc which they contain.

If we diminish still further the proportion of zinc in the weaker amalgam, the E.M.F. increases; until, if we omit the zinc altogether, the E.M.F. may reach ·8 or ·9 CLARK. In this case, however, the E.M.F. is very unsteady.

On Feb. 18, 1885, a cell (F) of the H-pattern was charged with pure mercury in one leg, and in the other with an amalgam containing one milligram of zinc dissolved in 30 grams of mercury. No paste being used, the cell was filled up with saturated zinc sulphate solution. Under these conditions the E.M.F. was unsteady—about ·7. On Feb. 23 some paste was added to the leg containing pure mercury, on which the E.M.F. became steady at ·939 CLARK, even when the amalgam was stirred with a glass rod. On Feb. 28, E.M.F. = ·935; on March 2, ·931; on March 6, ·920; on April 12, ·068; and on Dec. 2, ·0003.

Another cell (G) charged with pure mercury and paste in one leg had no zinc added to the pure mercury in the other leg, except such as it received by the passage for 3^m of the current from two large CLARKS, in whose circuit 1000 ohms of wire-resistance was also included. Under these circumstances, the deposited zinc should have been about $\frac{1}{10}$ mg. The resulting E.M.F. was ·923, falling off from day to day, as in the case of (F). Nine months afterwards the value was ·0003.

From these and other experiments it appeared that an astonishingly minute proportion of zinc in the amalgam was sufficient to give the cell most of the force of a CLARK. I had contemplated examining in greater detail the electromotive character of weak amalgams of known composition, when my attention was called by Prof. AYRTON to an interesting investigation by Messrs. HOCKIN and TAYLOR,* covering the same ground. In their cells zinc was opposed to the various amalgams; so that the circumstances were really the same as in the case of H $(1, \frac{1}{2})$, H $(1, \frac{1}{4})$, above discussed, the strong amalgam being equivalent to zinc. A few of their numbers may here be quoted. When the zinc was opposed to pure mercury, E.M.F. = 1·186 volt. (According to my experience this should have been uncertain.) The mercury being replaced by amalgams, the E.M.F. observed were as follows:—

	Volts.
Zinc 1 part, mercury 23,600,000 parts . . .	1·179
,, ,, ,, 11,800,000 ,, . . .	1·080
,, ,, ,, 7,530,000 ,, . . .	·655
,, ,, ,, 5,900,000 ,, . . .	·513
,, ,, ,, 1,800,000 ,, . . .	·214
,, ,, ,, 400,000 ,, . . .	·134
,, ,, ,, 200,000 ,, . . .	·124.

* "On the Electromotive Force of Mercury Alloys." 'Journal of Society of Telegraph Engineers,' vol. viii., 1879.

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"Mercury alloyed with any number of metals takes the place in the scale of E.M.F. of the most electro-positive metal it may contain, if the amount of the electro-positive metal present is not less than about $\frac{1}{1000000}$ in weight of the mercury."

§ 55. In order to obtain any stability of E.M.F. in these cells, it seems to be necessary that the mercury should either be alloyed with a sensible, though perhaps relatively very small, portion of zinc, or else protected with mercurous sulphate. With pure mercury without paste not only is the E.M.F. variable from hour to hour, but it can scarcely be said to be definite even at a particular time—that is, it may be altered by the passage of a very small quantity of electricity, such as should have no effect whatever upon a properly constituted cell. Sometimes when the galvanometer contact was made, a throw of the needle was observed, not followed by any corresponding permanent deflection, the cell in fact behaving like a charged condenser.

In the absence of paste a very small addition of zinc to mercury gives it a definite and tolerably permanent character. If, however, there be any mercurous sulphate, even though originally in another leg of an H-cell, the mercury is gradually repurified. To this cause is to be attributed the gradual fall to zero of the E.M.F. of the cells (F) and (G) above considered.

Indeed, the whole tendency of these observations is to suggest that the action of the mercurous sulphate may be secondary, rather than primary; so that if zinc could be opposed to really pure mercury, no mercurous sulphate would be needed in a CLARK cell. It may be, however, that in that case a minute quantity of mercurous sulphate would form itself spontaneously. In such cells as H $(1, \frac{1}{2})$ we are compelled to suppose that the chemical origin of the E.M.F. is the tendency of zinc and mercury to combine, or rather the tendency of two different amalgams to equalize themselves. There is here a close parallelism to the electromotive forces due to affinity of saline solutions of different strengths, manifested, for example, when an electrode of zinc is in contact with a strong solution of zinc sulphate, and the second similar electrode is in contact with a weak solution.

Before quitting this subject, I may mention observations on what was practically a CLARK cell, although prepared without zinc and without paste. On March 7, 1885, an H-cell (J) was charged with pure mercury in both legs, and filled up with zinc sulphate solution. A current from an external source of about $\frac{1}{100}$ ampère was then passed through it for $3\frac{1}{2}$ hours. The E.M.F. at subsequent dates were as follows:—

	CLARK.
March 9	·983
March 11	·980
April 9	·978
July 7	·973
December 12	·971

It will be seen that a tolerable approximation to a CLARK was obtained by this simple process. Zinc is thrown down upon the mercury in one leg, and mercury dissolved in the other by the passage of the current.

§ 56. The comparisons recorded in Table XIII. furnish abundant proof that there has been no special change in the E.M.F. of No. 1, but do not of themselves exclude the possibility of a general movement of the cells in one direction as the result of age, by which the relative values of cells of nearly the same date might remain unaffected. Comparisons with younger cells, indeed, go a long way towards negativing this supposition; but considering that all the older cells had undergone a journey (from Cambridge to Terling), I thought that a re-determination of the absolute E.M.F., by means of silver, would be a valuable confirmation of their constancy.

With the assistance of Mrs. SIDGWICK, two such silver determinations were effected in August, 1885, as described in § 38. Reduced to 15°, the E.M.F. of No. 1 was found on August 15 to be 1·4541 B.A. volts, and on August 19, 1·4533, giving as a mean 1·4537 B.A. volts. The value found from absolute measurements in the autumn of 1883, and the spring of 1884, was 1·4542 (§ 36). That given (note to § 37) as the result of two experiments in Nov., 1884, is 1·4534; but a slight error has been discovered in the reductions leading to the latter number. By an oversight the factor 1·00041, necessary to correct for the slight heating of the resistance by the passage of the current (§ 33), was omitted, the effect of which omission is an under-estimate of the difference of potentials at the terminals of the resistance traversed by a known current, and consequently of the E.M.F. of the cell under test. This correction being introduced, the numbers will stand thus :—

TABLE XVIII.

Date.	E.M.F. of No. 1 at 15° C. in B.A. volts.
Oct. 1883 to April 1884 . . .	1·4542
Nov. 1884	1·4540
Aug. 1885	1·4537

The slight fall in these numbers has little significance; and we may regard the E.M.F. of No. 1 as having remained constant to within about $\frac{1}{2000}$ for nearly two years.

I believe that the E.M.F. of cells constructed with reasonable precautions, especially as to saturation, will rarely differ at ordinary temperatures by more than $\frac{1}{1000}$ part from

$$1{\cdot}454\{1-{\cdot}00077(t-15)\} \text{ B.A. volts,}$$

or (the B.A. unit of resistance being taken equal to ·9867 ohm)

$$1{\cdot}435\{1-{\cdot}00077(t-15)\} \text{ true volts.}$$

§ 57. In Table X., § 32, are given some comparisons with No. 1 CLARK of a standard DANIELL with equi-dense solutions of RAOULT's pattern. As a mean at 16° C.,

DANIELL = ·7703 CLARK = 1·1046 true volt.

I have never succeeded in obtaining really good results with DANIELLS of any construction, variations in the conditions of the copper * rapidly altering the E.M.F. by two or three thousandths. The mean of a rather large number of comparisons in August, 1885, gives about ·7715 CLARK at 16°, and this number was confirmed by further observations in December, 1885. The sp. gr. of the solutions was 1·101; the zinc was amalgamated, and the copper was freshly-coated electrolytically. The effect of variations in the condition of the metals and of the solutions are discussed by ALDER WRIGHT and by FLEMING.

Some observations have also been made with apparatus and solutions lent me by Dr. FLEMING, and arranged nearly according to the description published by him.† The variations observed with this form of DANIELL were smaller than with RAOULT's, perhaps in consequence of the better protection of the copper solution from the atmosphere.‡ The mean value for 16° was about

·7674 CLARK = 1·1004 volt,

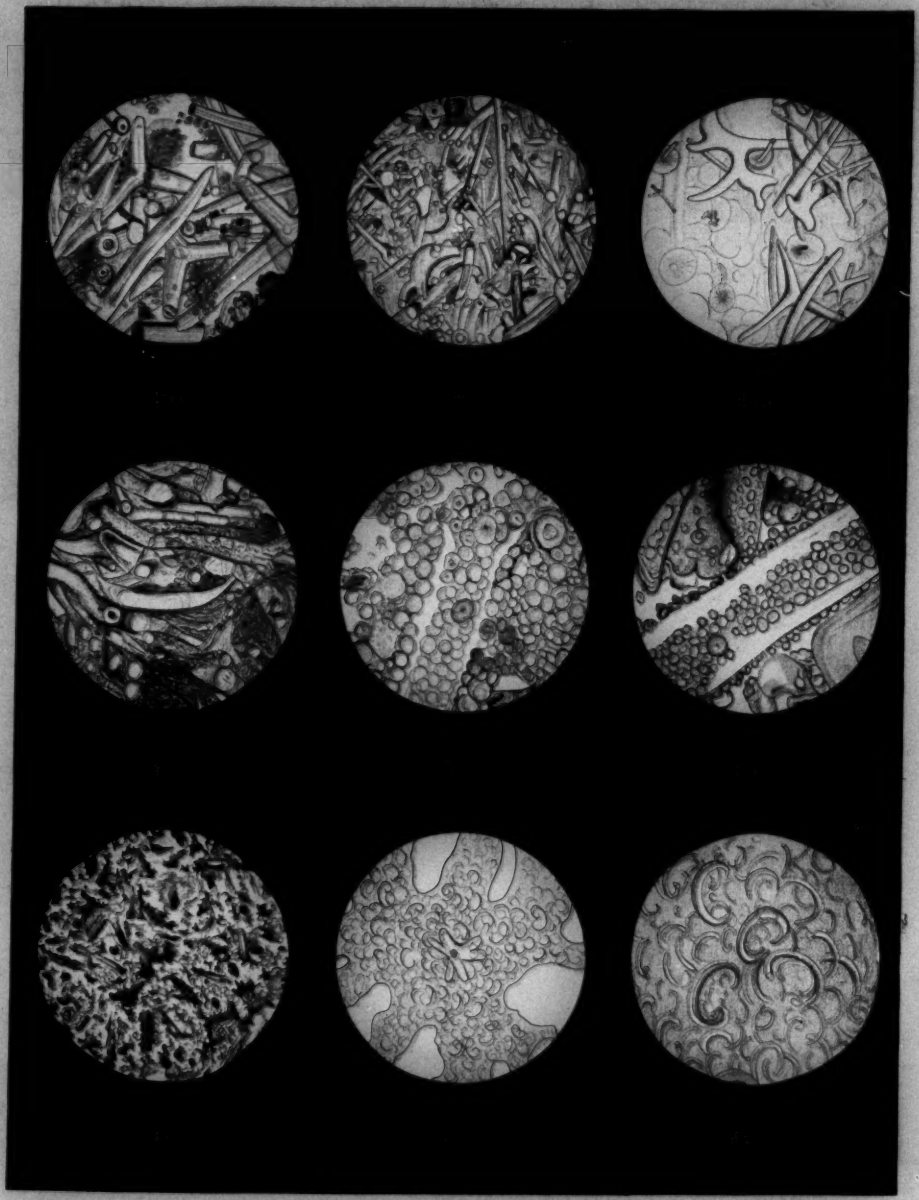
in pretty good agreement with the value found by Dr. FLEMING himself. In correcting for the effect of a small variation of temperature, I have allowed only for the variations of the CLARK, assuming that of the DANIELL to be insensible.

[P.S.—May, 1886.—I have lately had the opportunity of comparing with mine five cells set up by Dr. A. MUIRHEAD, who has had great experience in these matters. All five agreed with my No. 1 to about one part in *ten thousand*. Dr. MUIRHEAD informs me that other cells, including one prepared by himself seven or eight years ago, agree closely with these.]

* It seems not unlikely that the greater part of the uncertainty of ordinary DANIELL's might be got rid of by substitution of silver for copper. Silver *sulphate*, however, is not sufficiently soluble. But I have not made experiments in this direction, believing that the mercury of the CLARK cell is better still.

† Phil. Mag., August, 1885.

‡ In my experience the copper found its way to the zinc more readily than one would have expected. The amalgamated zinc became coated with a furry deposit, which struck a blue colour on solution in nitric acid and supersaturation with ammonia. In using RAOULT's cells I have endeavoured to hold the copper in check by placing a loose strip of zinc in the connecting Y-tube.



G. J. & E. G. HINDE del. et lith.

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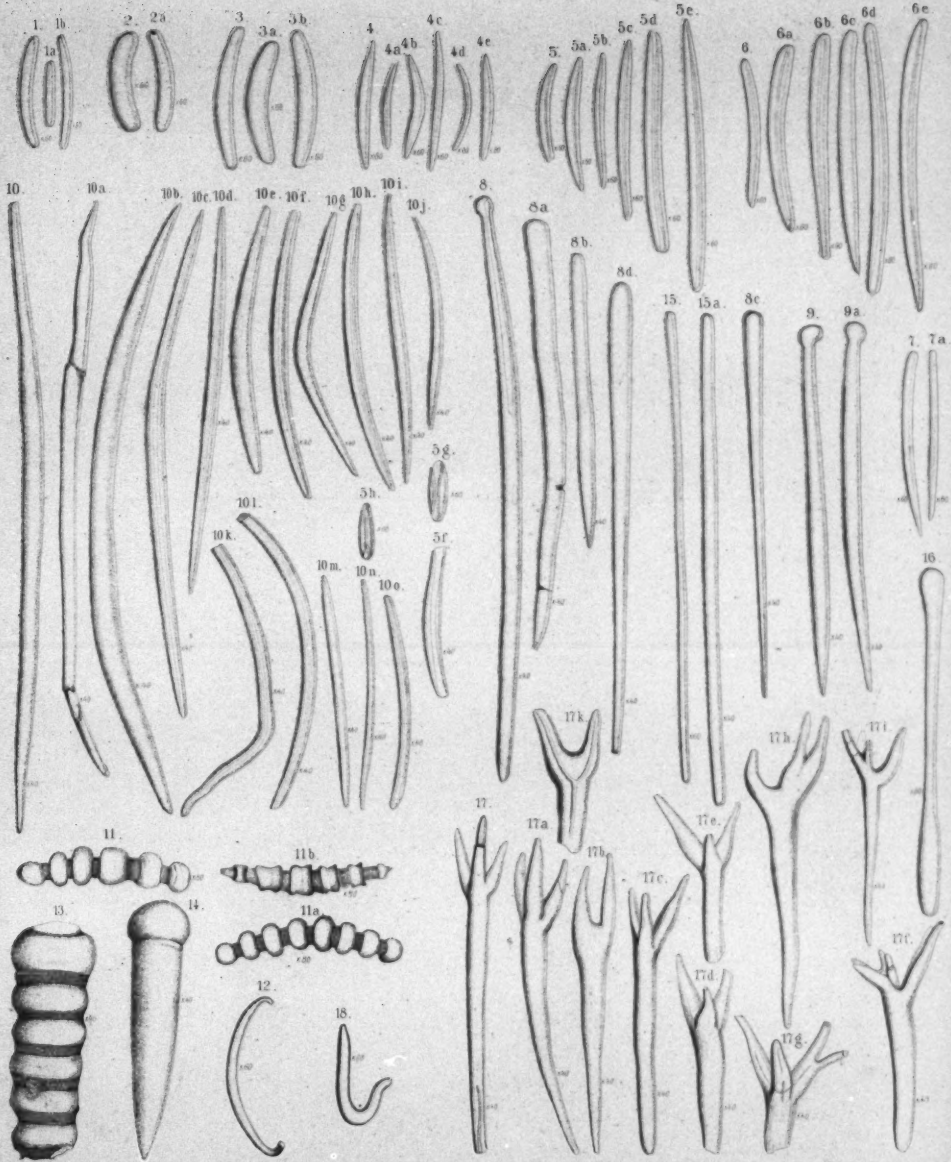
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10o.
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8c.
9.
9a.
7.
7a.
16.
11.
11b.
11a.
13.
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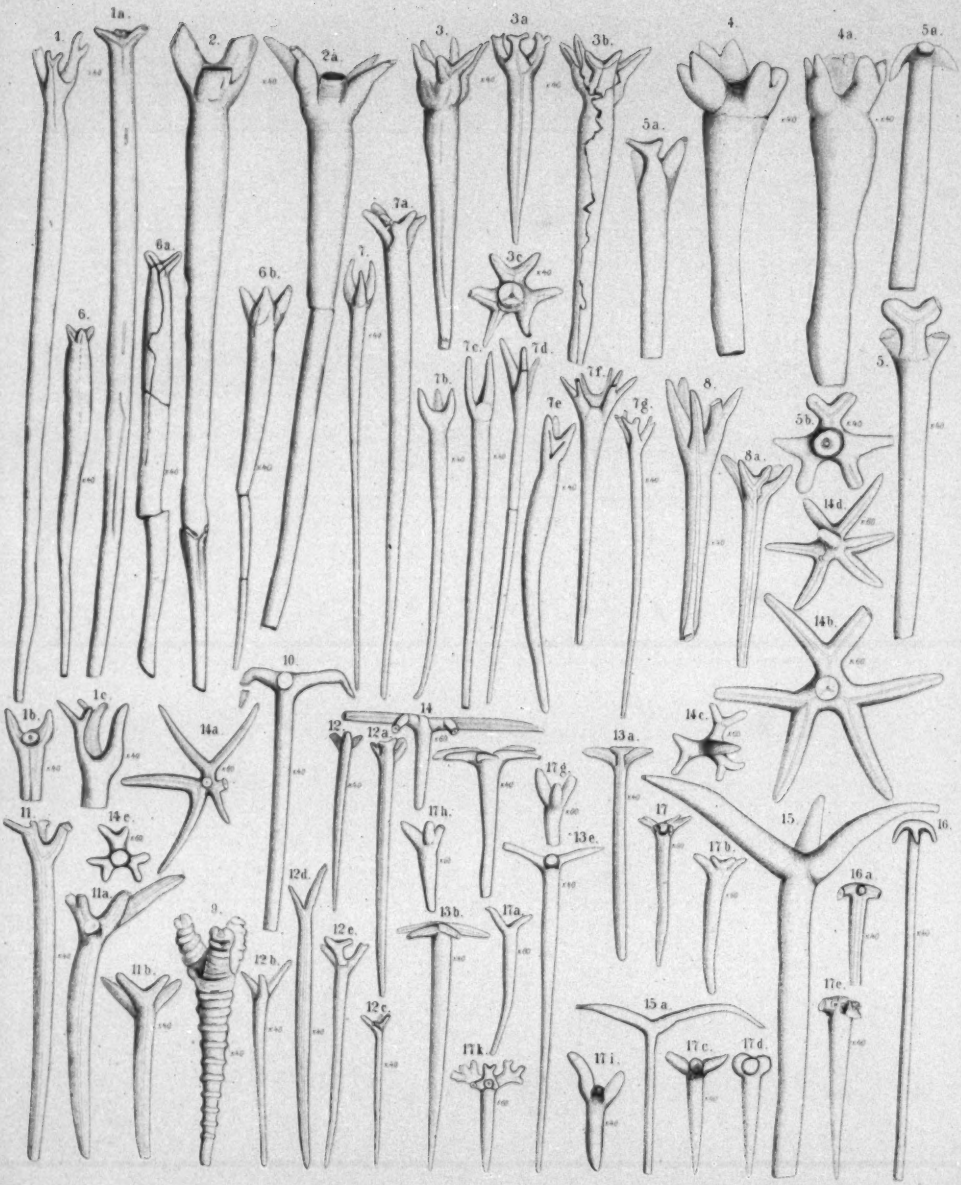
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7g.
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5b.
8a.
14d.
14b.
10.
1b.
1c
14a.
14
12.
12a.
14c.
13a.
17g.
11h.
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16.
11.
14e.
13e.
17b.
16a.
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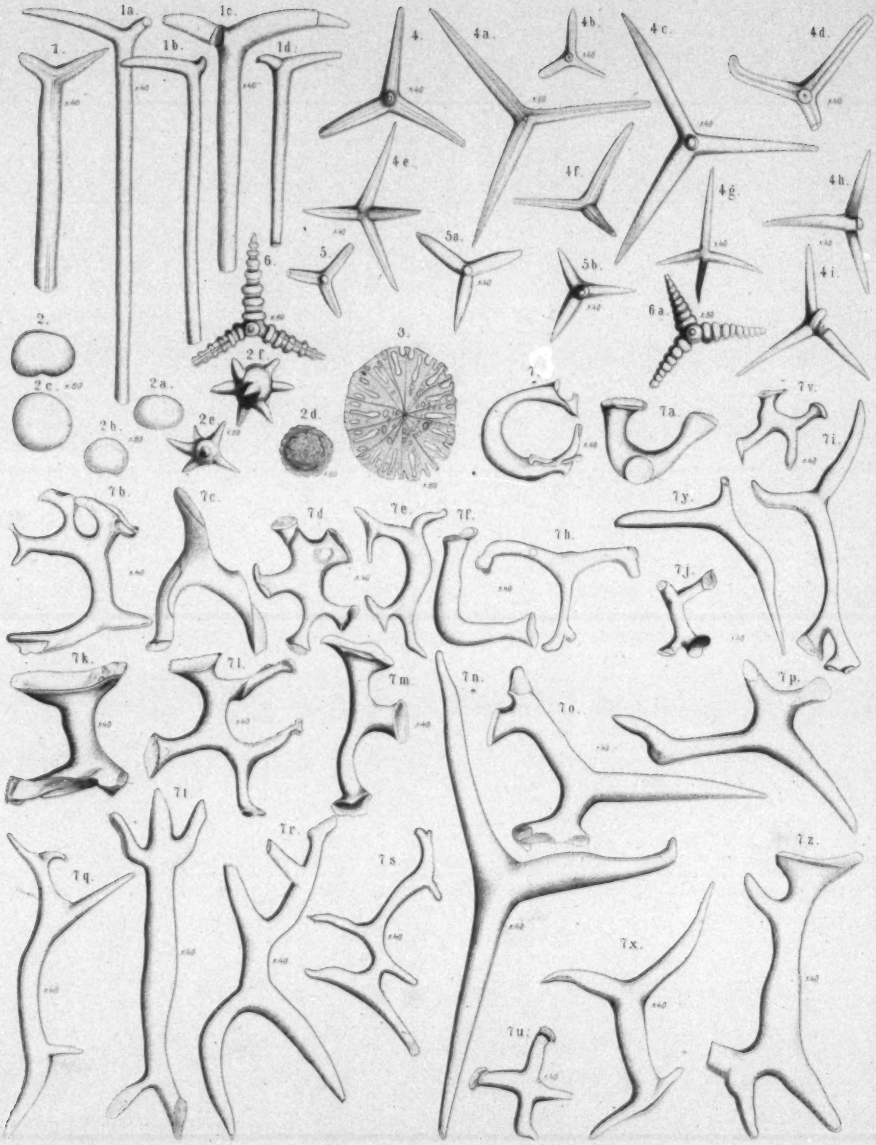
17i.
17c.
17d.



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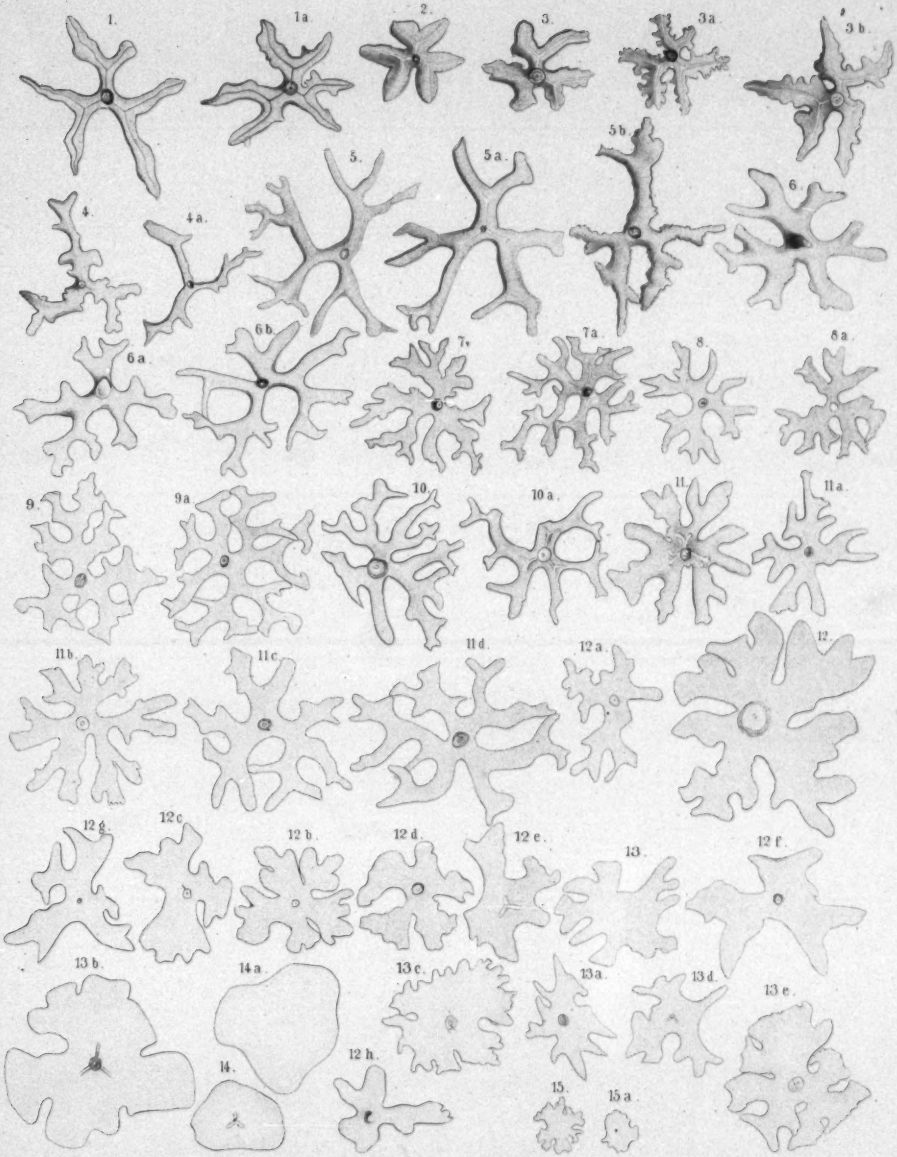

All x 60.

G. J. & J. D. HINDE del. ad nat.

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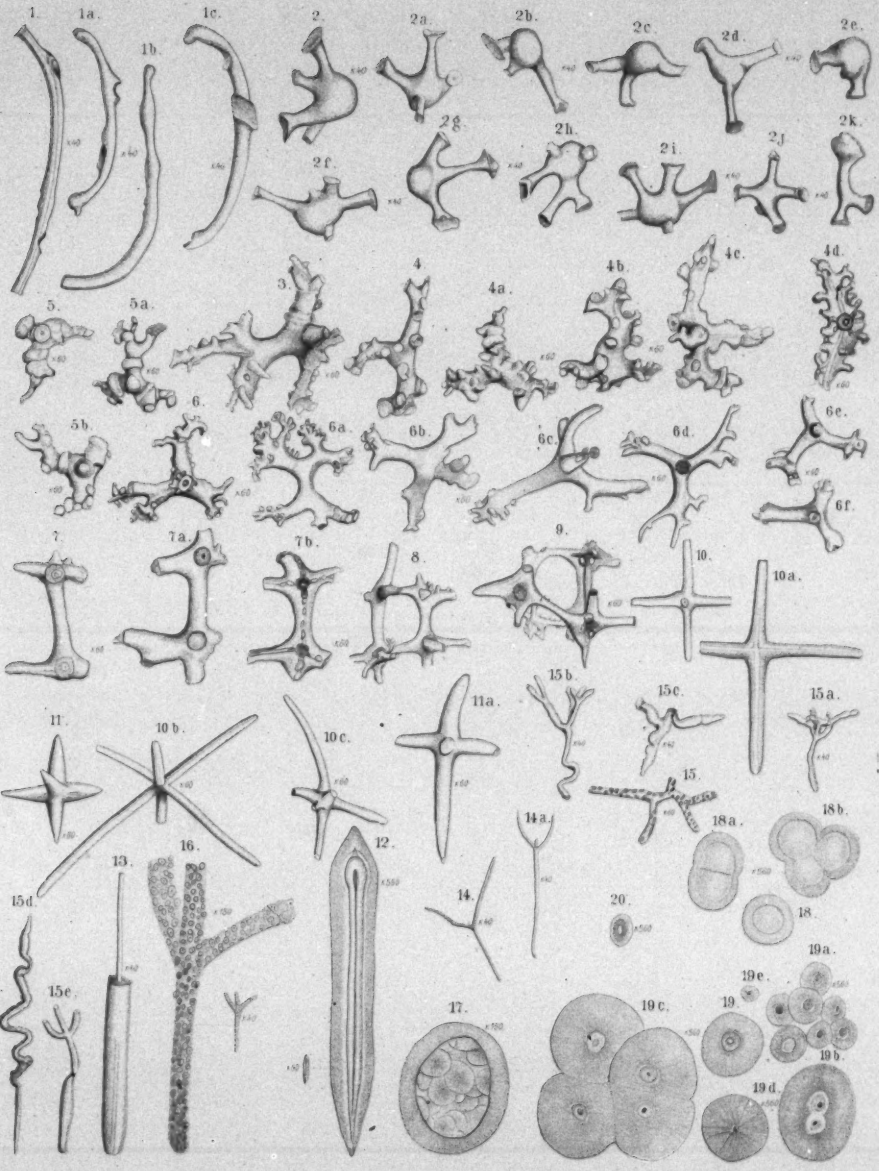
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2i.
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19e.
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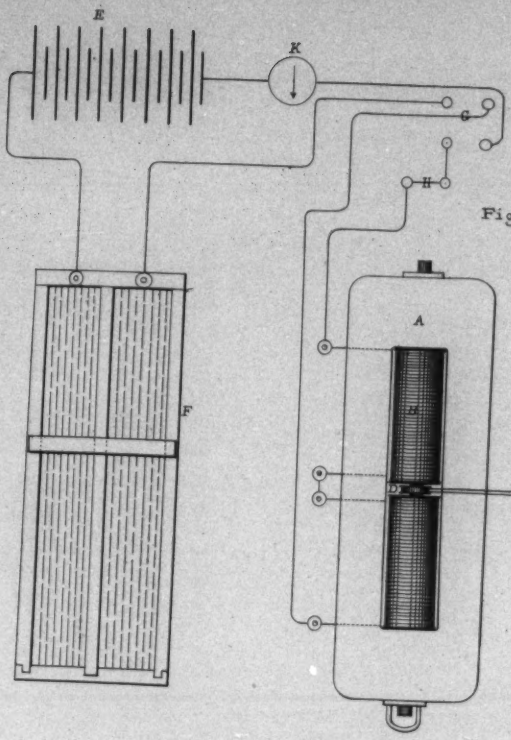


Fig. 1.

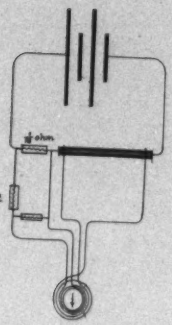
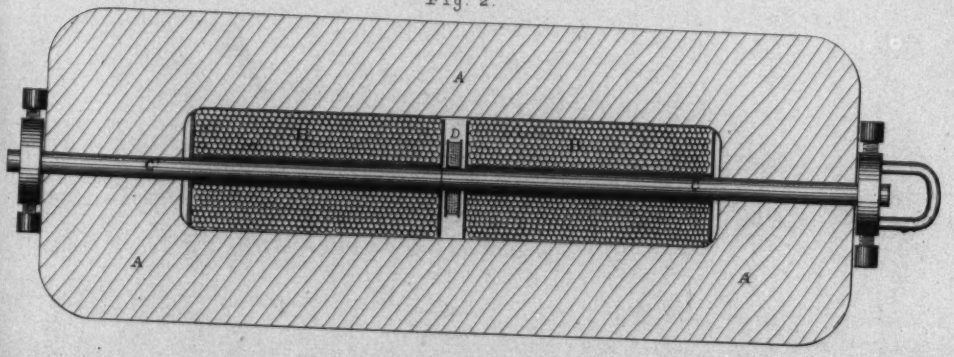
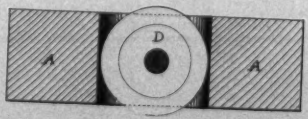


Fig. 3.

Fig. 2.





West, Newman & Co. sc.

Fig. 4.
Wrought Iron N° 1.

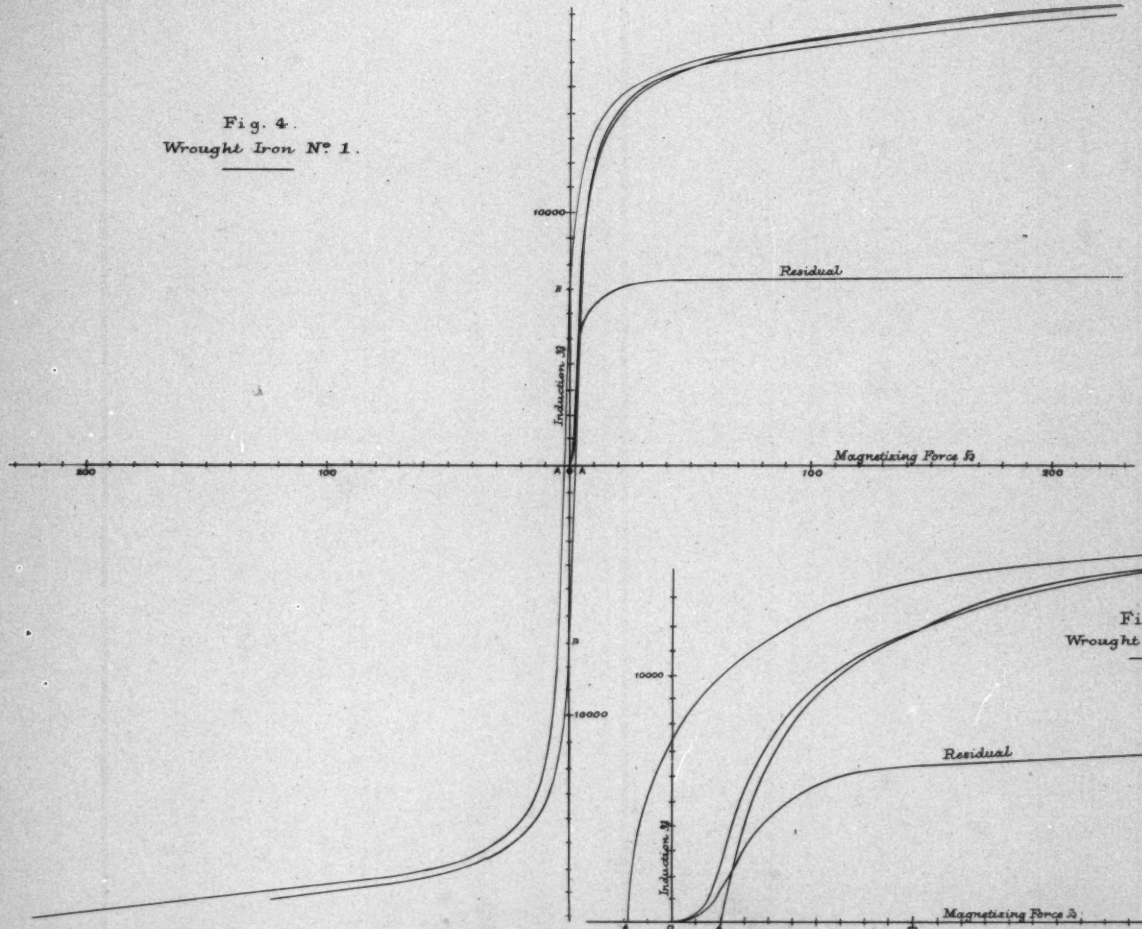
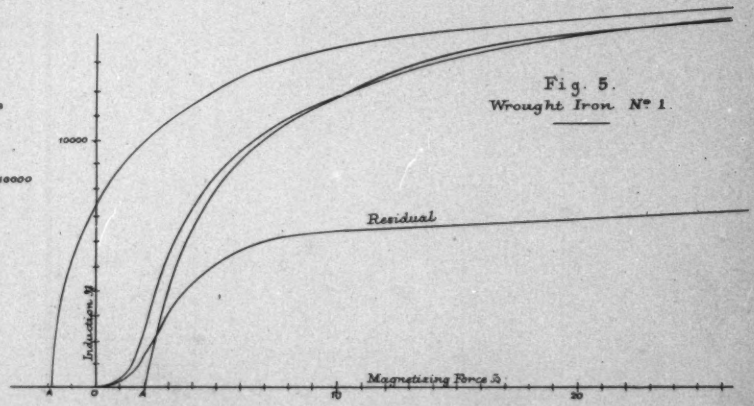


Fig. 5.
Wrought Iron N° 1.



West, Newman & Cº lith.

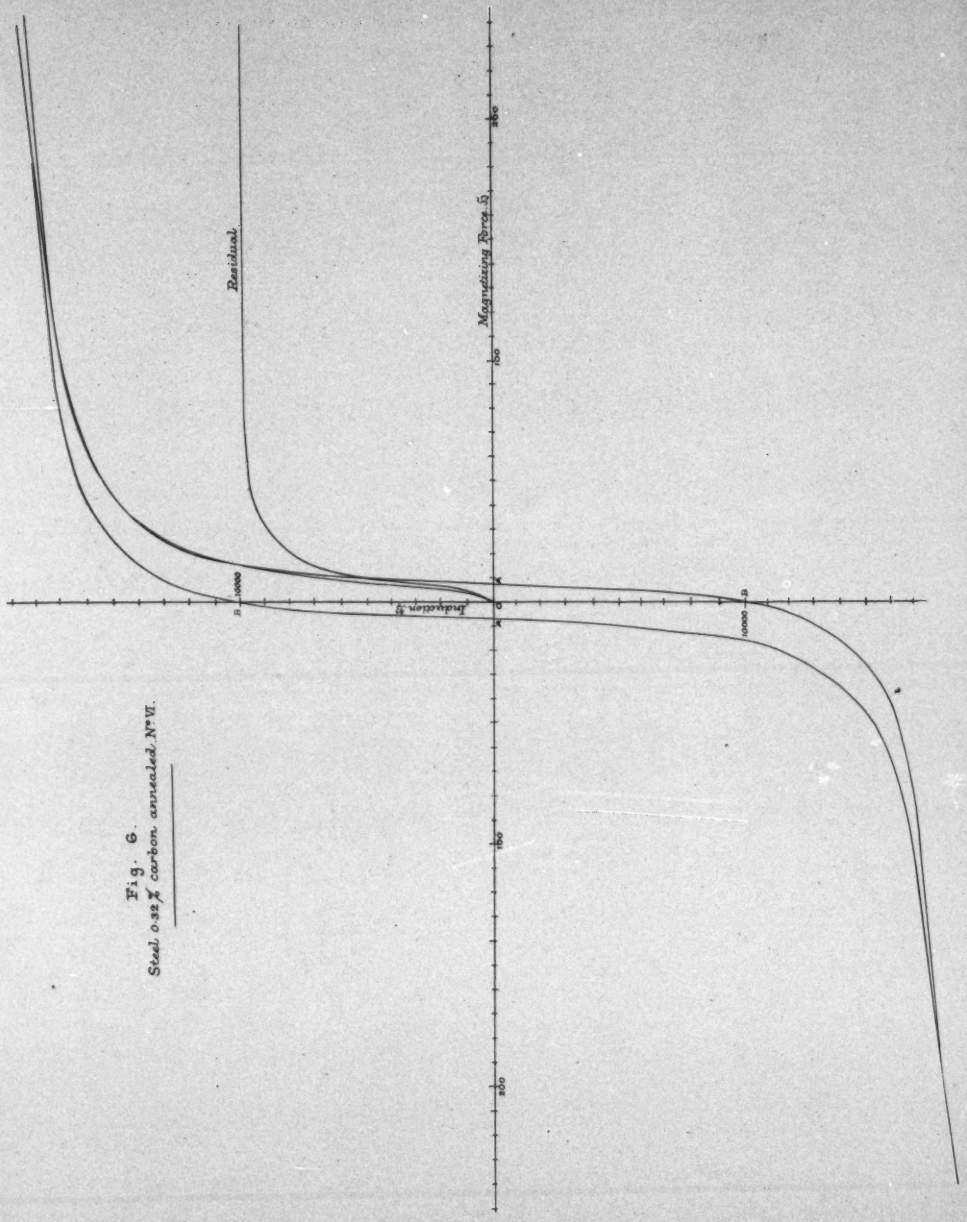


Fig. 6.
Steel 0.32 % carbon annealed. N° VI.

West, Newman & Co. lith.

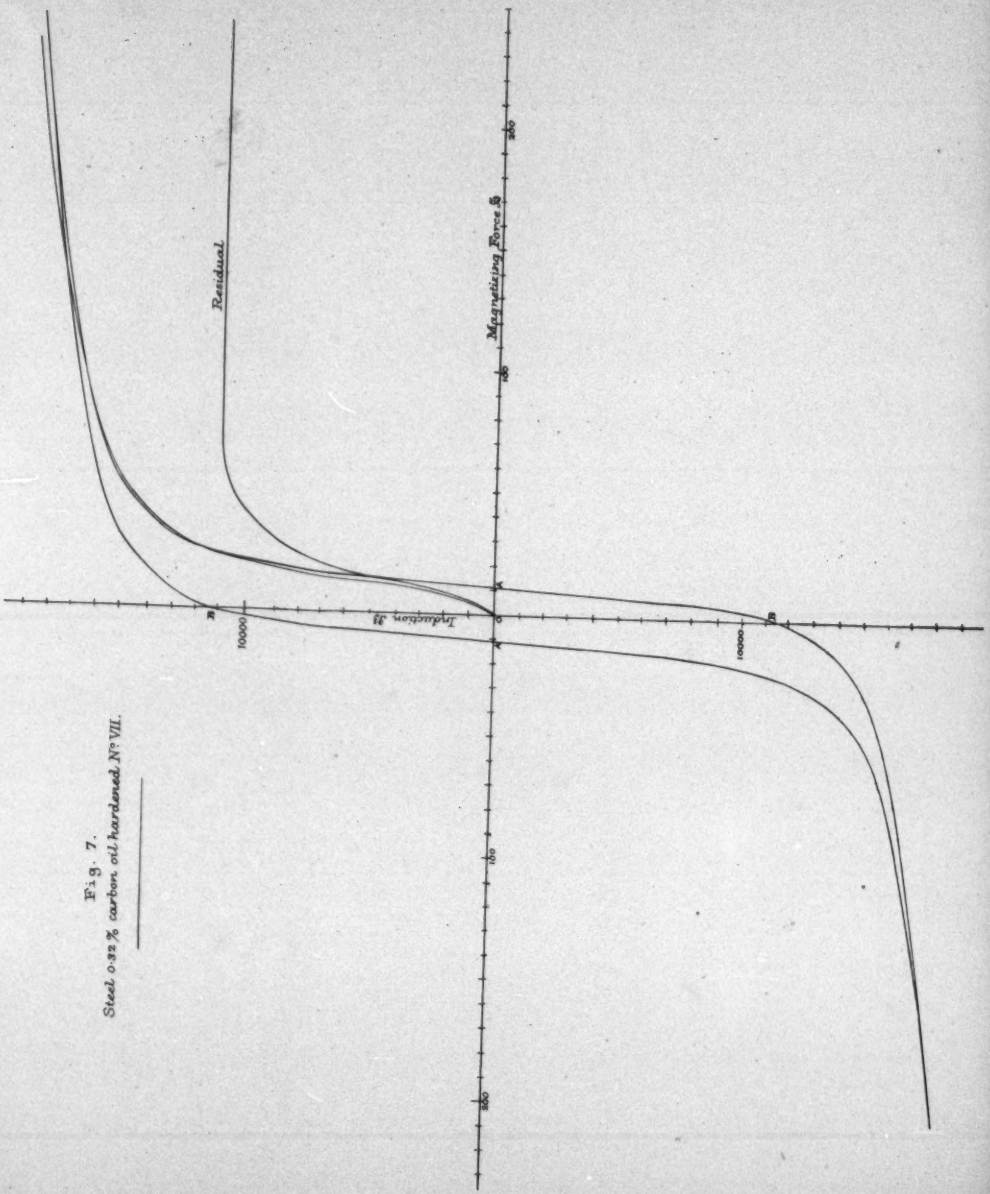


Fig. 7.
Steel. 0·32 % carbon, oil hardened. N° VII.

West Newman & C° lith.

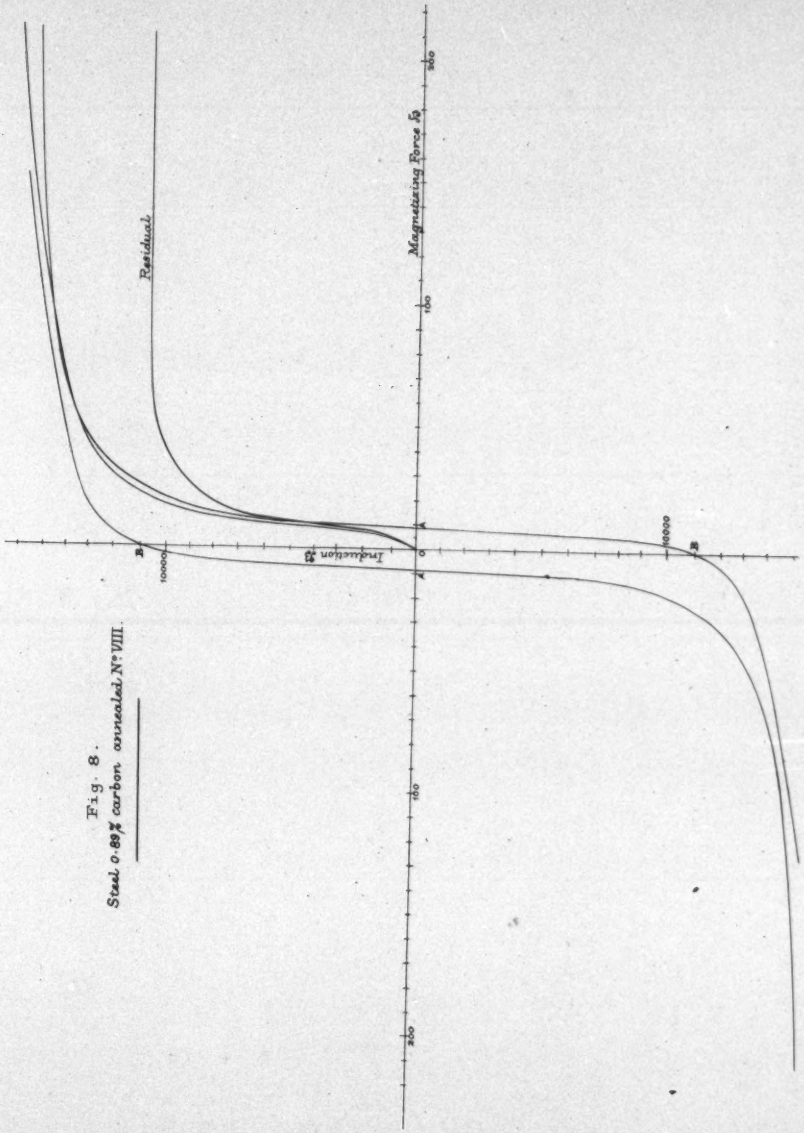


Fig. 8.
Steel 0.89% carbon annealed N° VIII.

West Newman & C° lith.

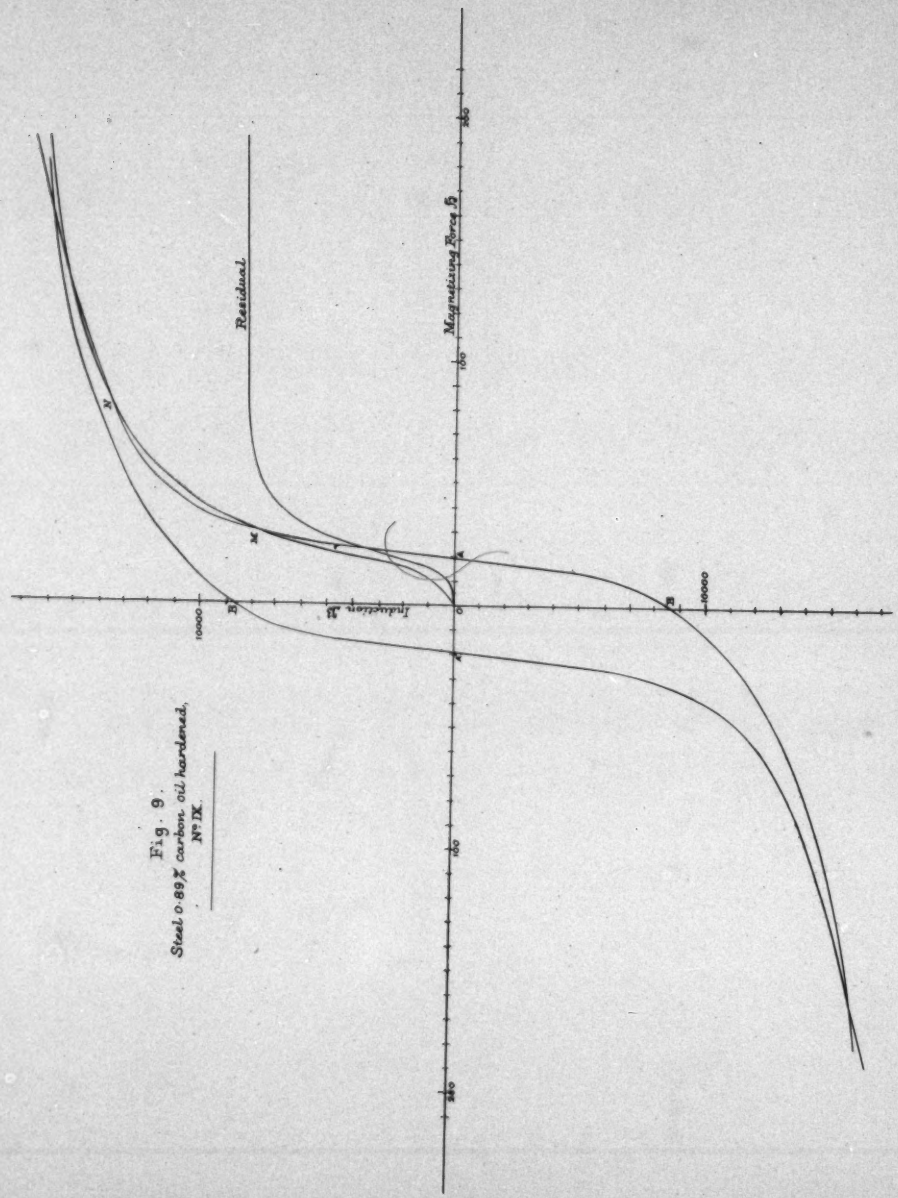


Fig. 9.
Steel 0·89% carbon. oil hardened,
N° IX.

West Newman & C° lith.

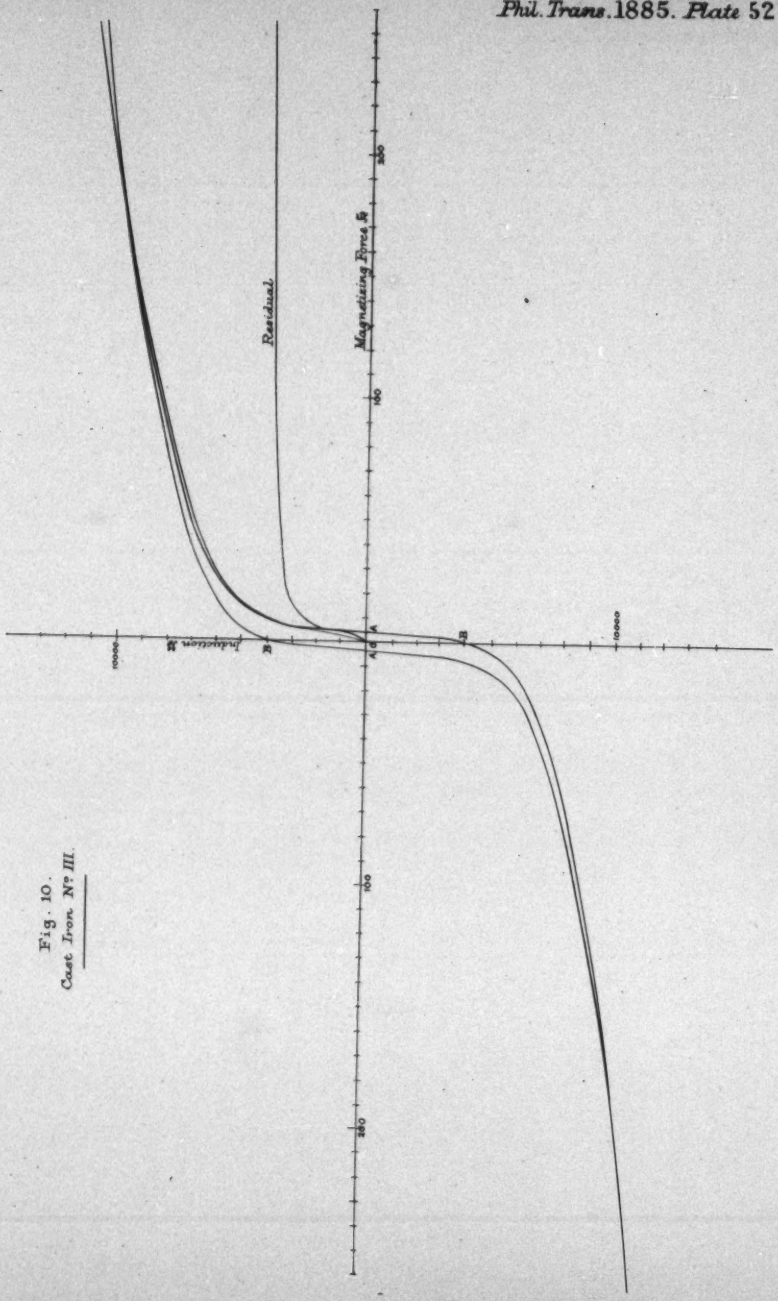
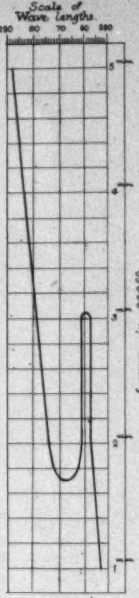


Fig. 10.
Cast Iron N° III.

West Newman & C° lith.

Aconitine.

Scale of Wave lengths.

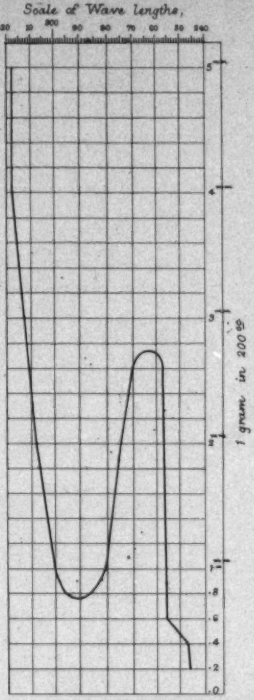


1 gram in 200 gs

Fig. 1.

English Aconitine

Scale of Wave lengths.

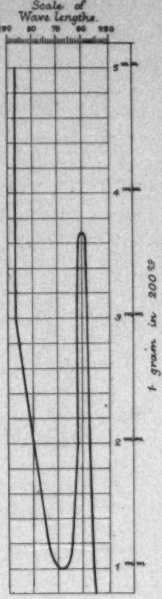


1 gram in 200 gs

Fig. 2.

Japaconitine

Scale of Wave lengths.

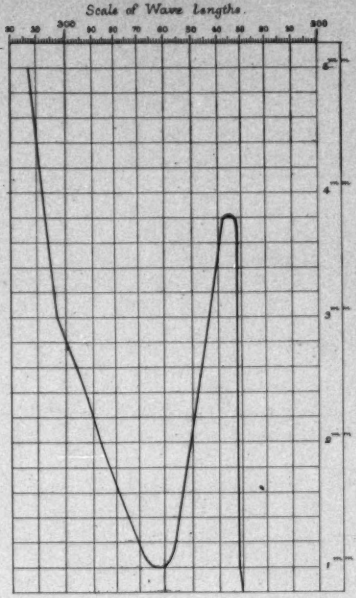


1 gram in 200 gs

Fig. 3.

Pseudaconitine.

Scale of Wave lengths.

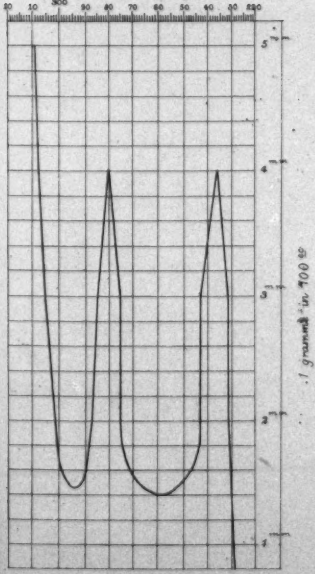


1 gram in 1000 gs

Fig. 4.

Aconitine

Scale of Wave lengths.

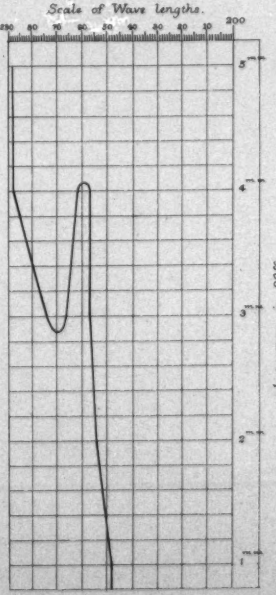


1 grain in 100 gs

Fig. 5.

Aconitine

Scale of Wave lengths.

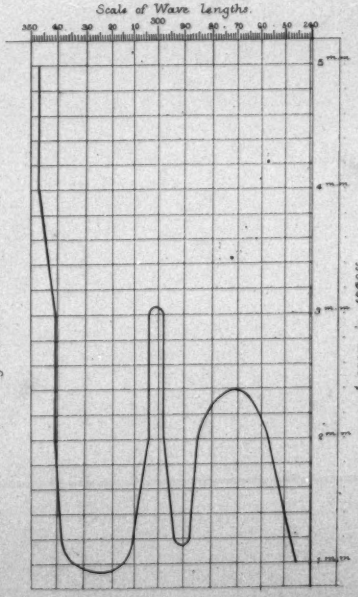


1 gramme in 20 gs

Fig. 6.

Quinine

Scale of Wave lengths.



1 gram in 1000 gs

Fig. 7.

West, Newman & Co. ad.

Quinine Sulphate

Scale of Wave lengths.

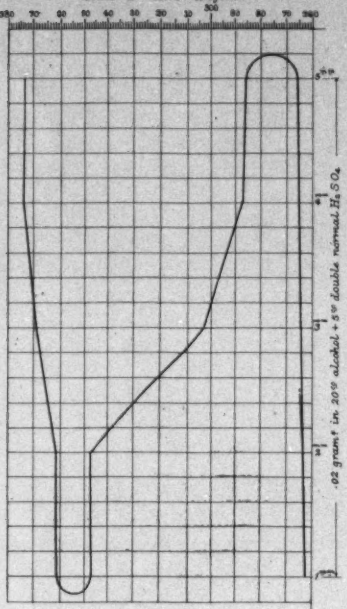


Fig. 1.

Cinchonine,

Scale of Wave lengths.

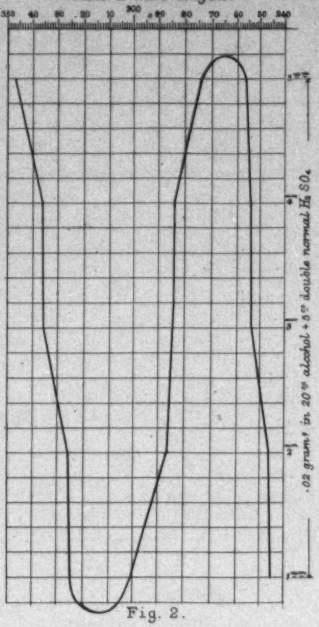


Fig. 2.

Quinidine Sulphate.

Scale of Wave lengths.

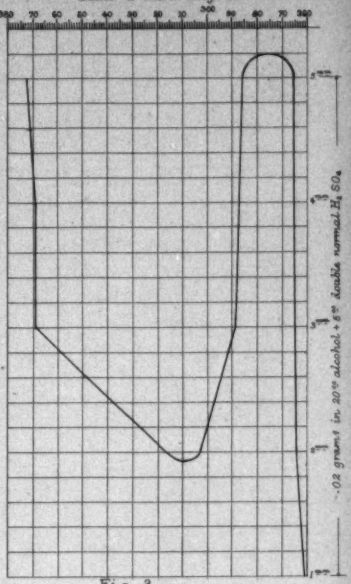


Fig. 3.

Morphine.

Scale of Wave lengths.

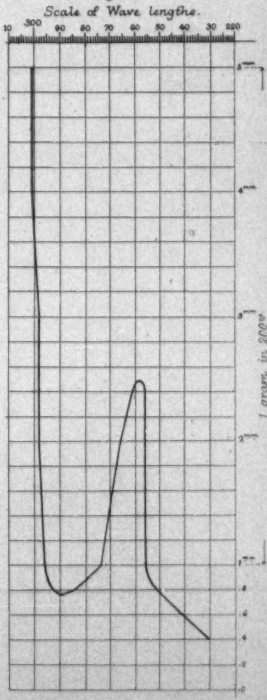


Fig. 6.

Cinchonidine Sulphate

Scale of Wave lengths.

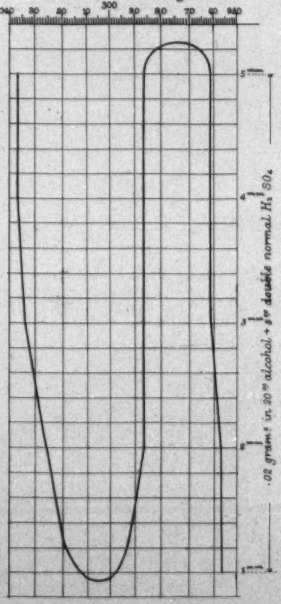


Fig. 4.

Morphine

Scale of Wave lengths.

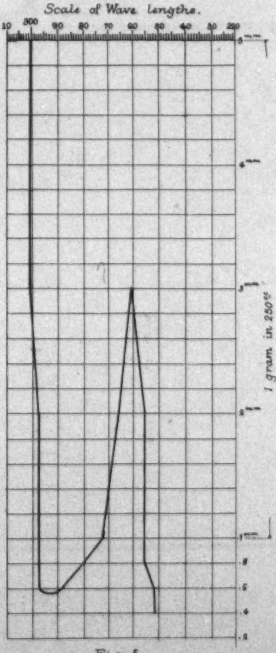


Fig. 5.

West, Newman & Co. imp.

Tetracetyl Morphine

Scale of Wave lengths.

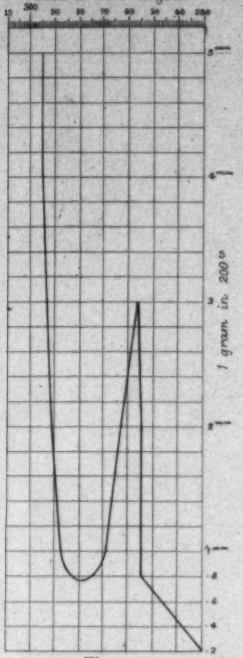


Fig. 1.

Codeine (1)

Scale of Wave lengths.

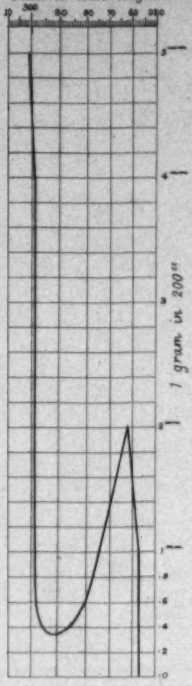


Fig. 2

Codeine (2)

Scale of Wave lengths.

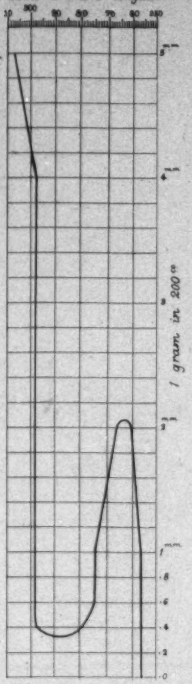


Fig. 3

Diacetyl Codeine.

Scale of Wave lengths.

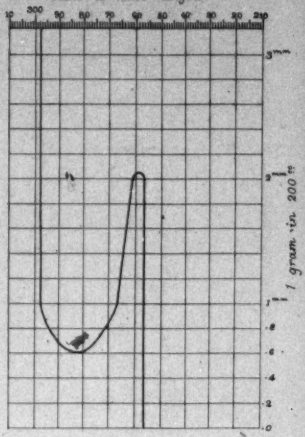


Fig. 4

Narcotine.

Scale of Wave lengths.

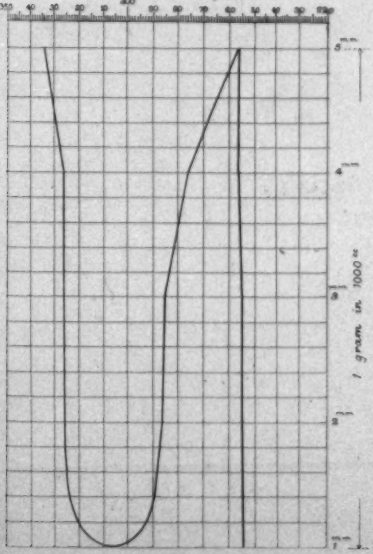


Fig. 6

Thebaine.

Scale of Wave lengths.

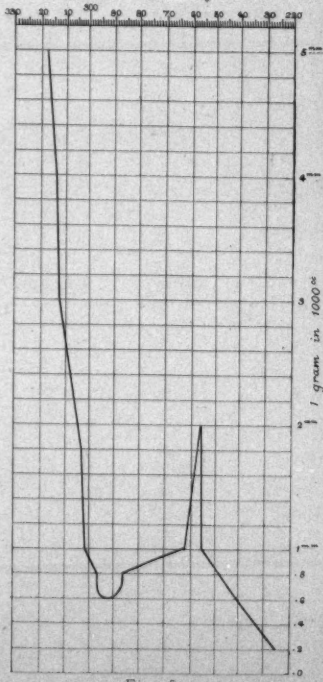
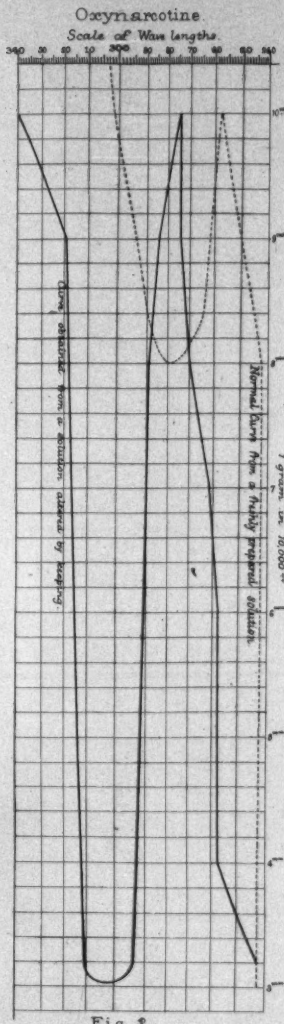
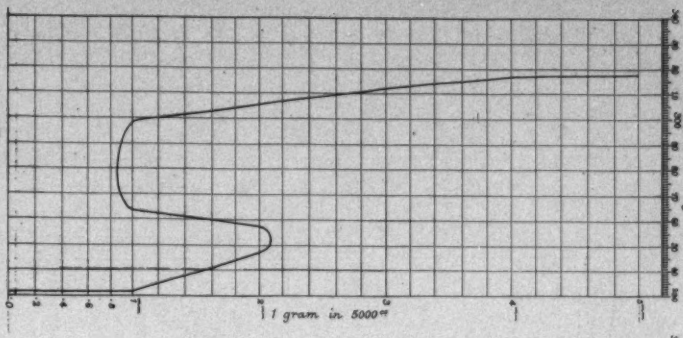
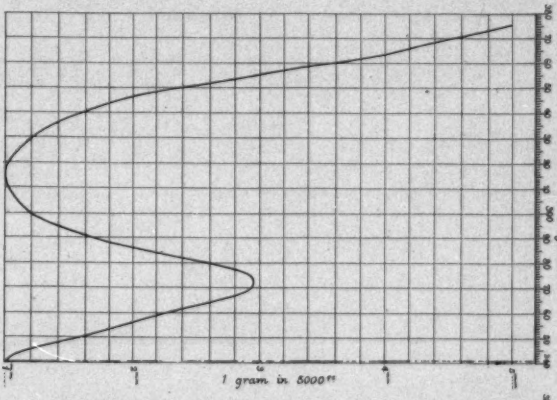


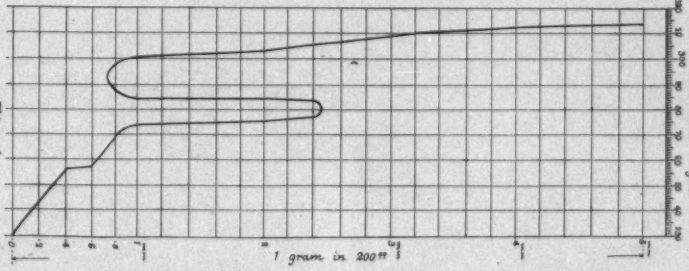
Fig. 5.

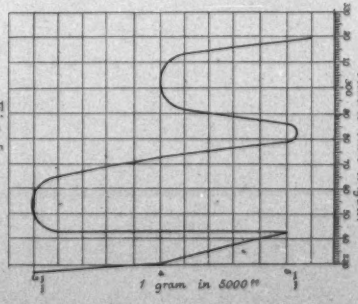
West, Newman & Co. sc.

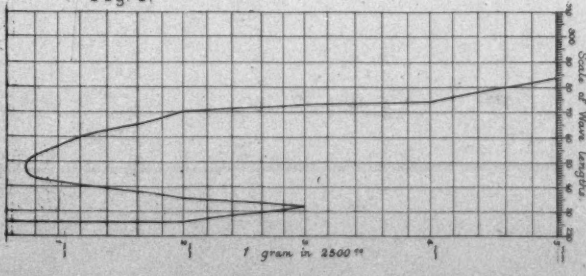

Oxynarcotine.
Scale of Wave lengths.
340 30 20 10 200 90 80 70 60 50 40
Curves obtainable from a solution, useless by keeping
Normal curve from a freshly prepared solution.
1 gram in 10000 cc
Fig. 1.
Fig. 2.





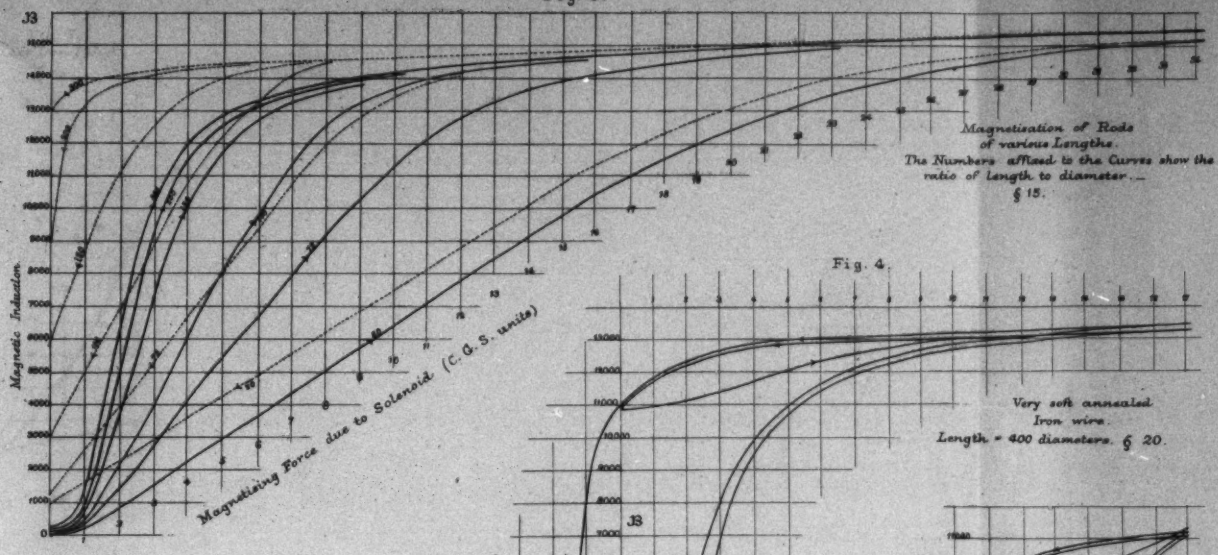






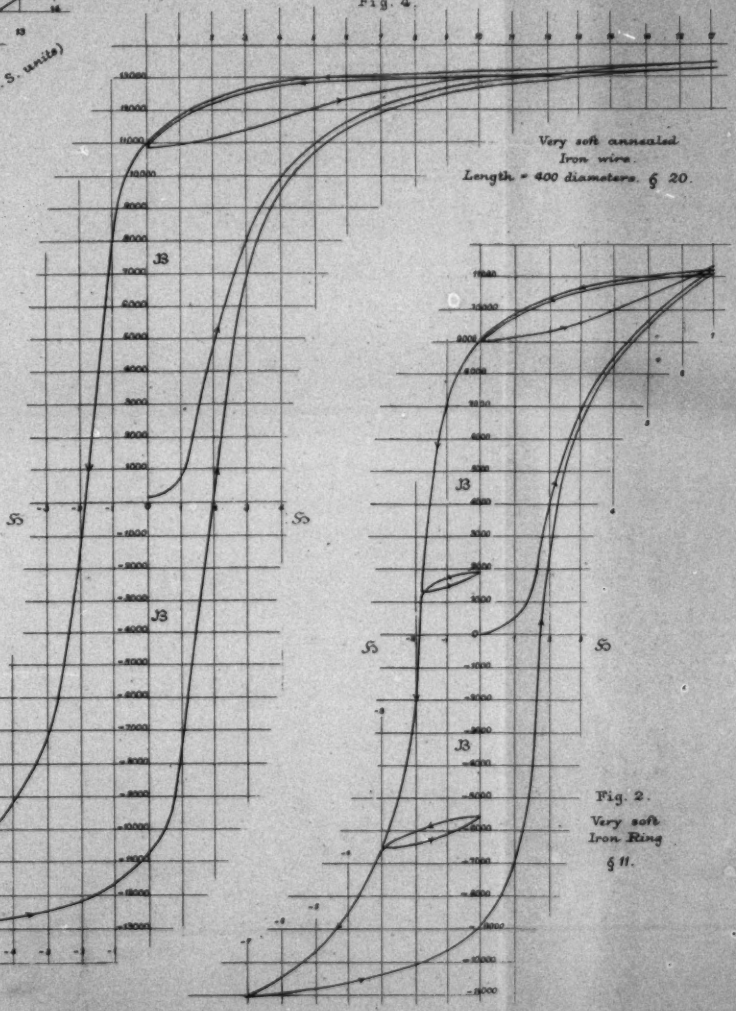
West Newman & Co. sculp.

Fig. 3.



Magnetisation of Rods of various Lengths. The Numbers affixed to the Curves show the ratio of Length to diameter.—
§ 15.

Fig. 4.



Very soft annealed Iron wire. Length = 400 diameters. § 20.

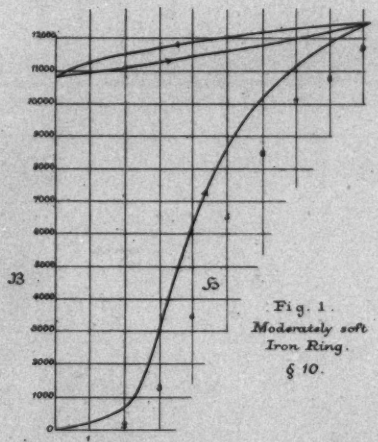


Fig. 1. Moderately soft Iron Ring. § 10.

Fig. 2. Very soft Iron Ring § 11.

West, Newman & Co. lith.

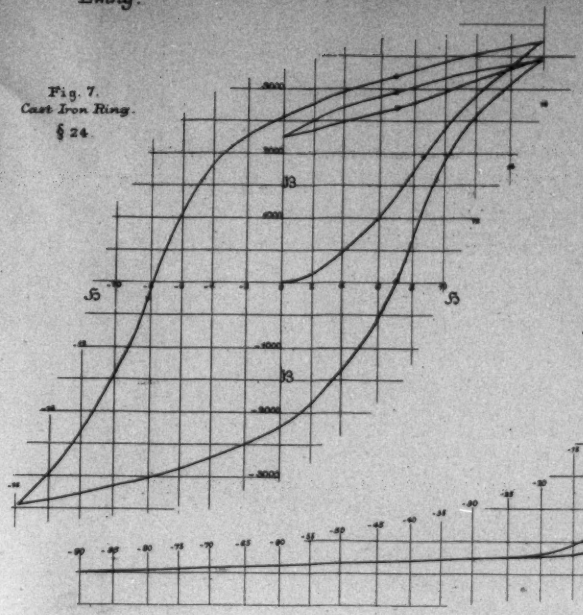


Fig. 7.
Cast Iron Ring.
§ 24.

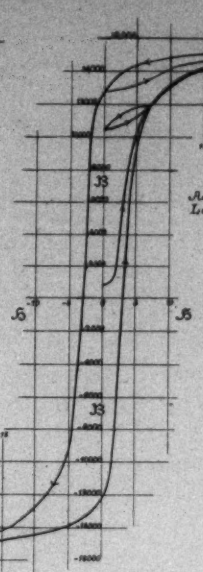


Fig. 6.
Annealed Iron wire
Length = 400 diam.s
§ 22.

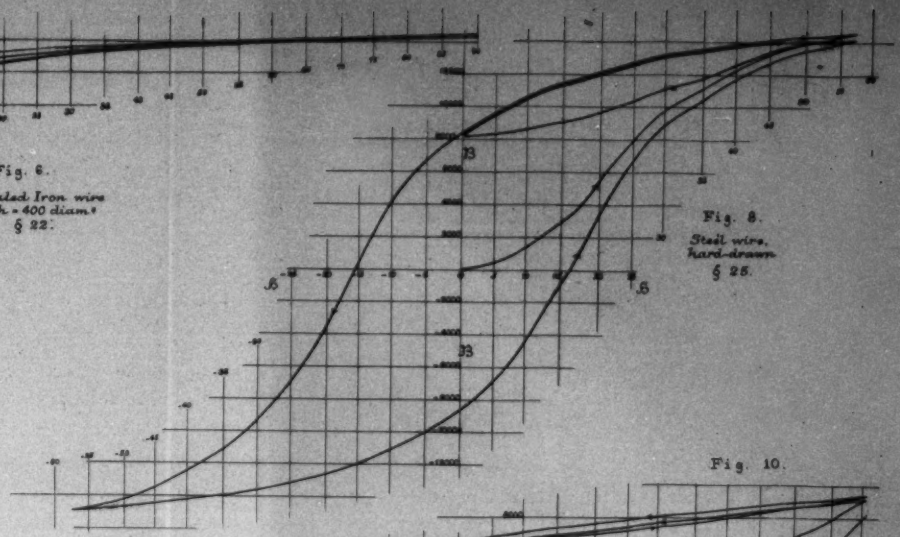
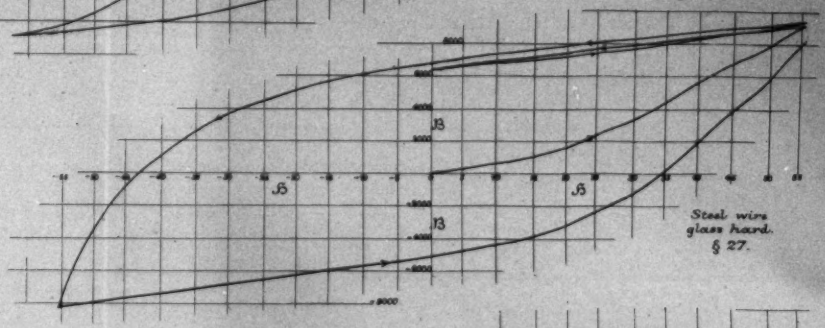


Fig. 8.
Steel wire,
hard-drawn
§ 25.

Fig. 10.



Steel wire
glass hard.
§ 27.

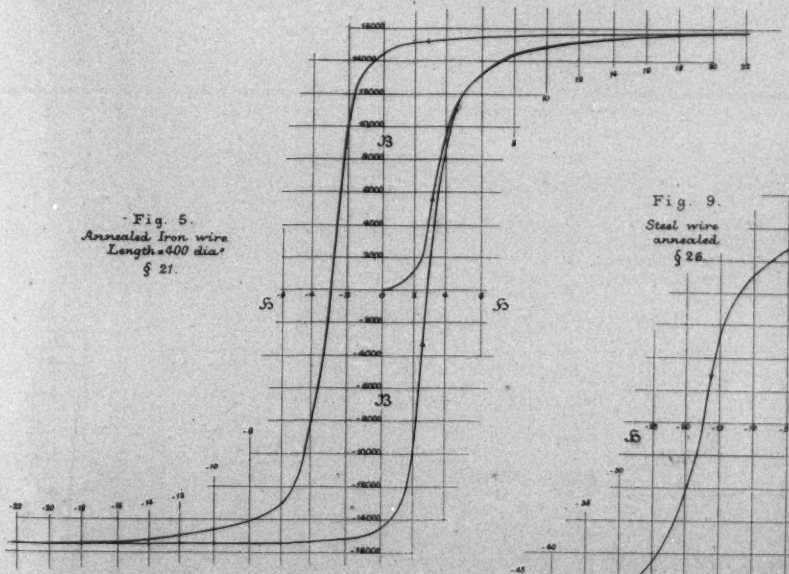


Fig. 5.
Annealed Iron wire
Length = 400 dia.s
§ 21.

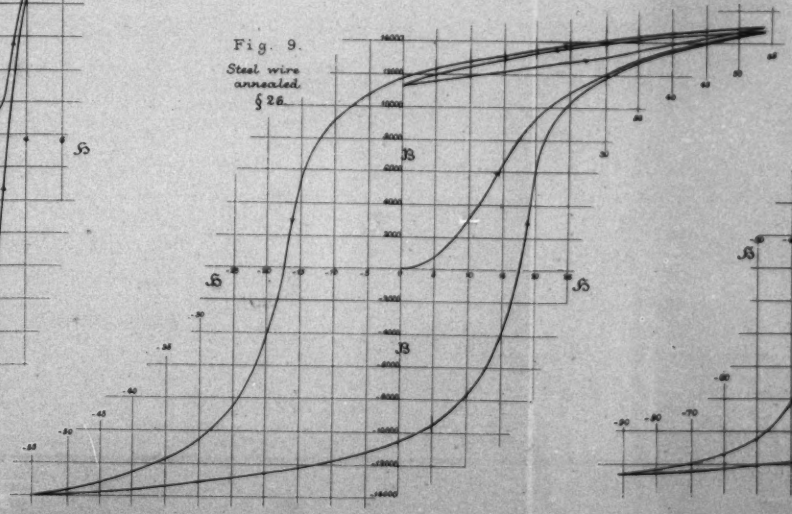


Fig. 9.
Steel wire
annealed
§ 26.

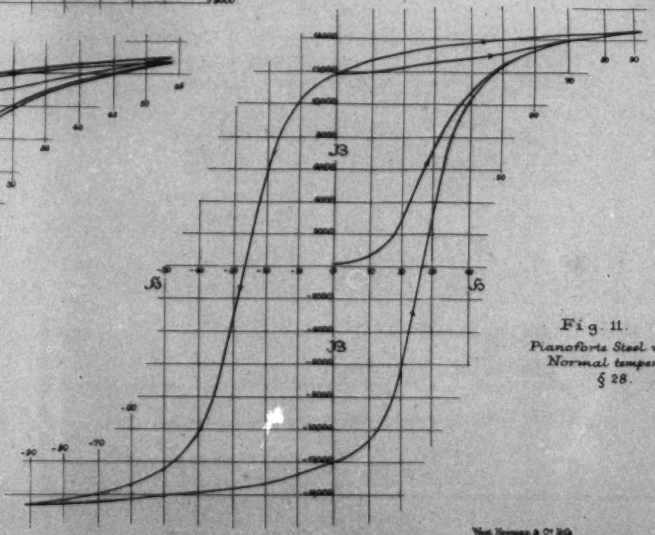


Fig. 11.
Pianoforte Steel wire
Normal temper
§ 28.

West, Newman & Co. lith.

Fig. 12.
Pianoforte Steel wire
Annealed.
§ 29.

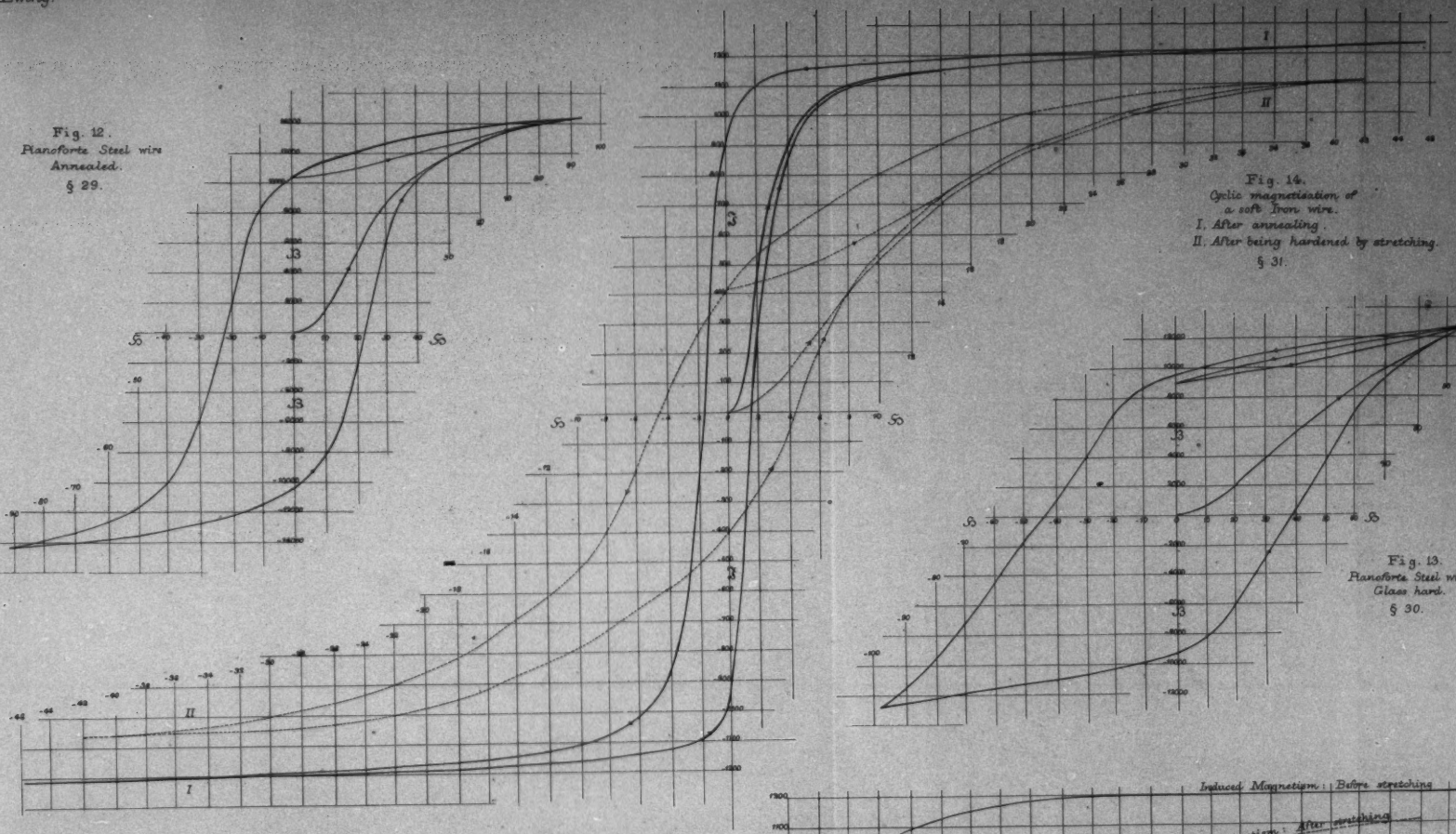


Fig. 14.
Cyclic magnetisation of
a soft Iron wire.
I, After annealing.
II, After being hardened by stretching.
§ 31.

Fig. 13.
Pianoforte Steel wire
Glass hard.
§ 30.

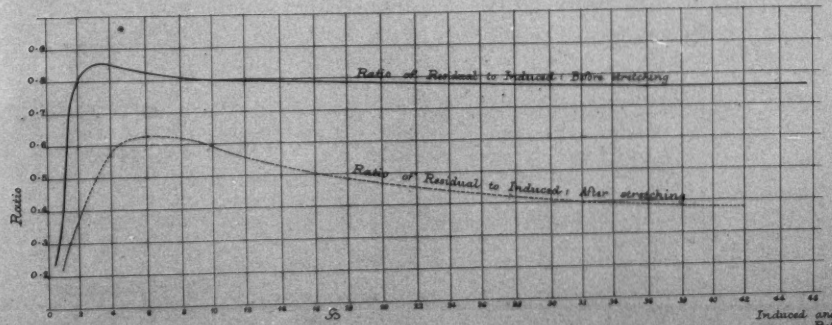
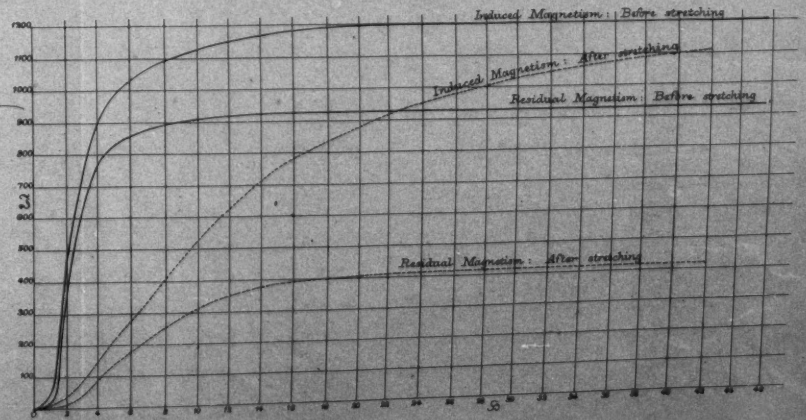


Fig. 18.


Induced Magnetism: Before stretching
Induced Magnetism: After stretching
Residual Magnetism: Before stretching
Residual Magnetism: After stretching

Induced and Residual Magnetism in Iron.
Before stretching ——— § 41.
After stretching ---------

West, Newman & Co. imp.

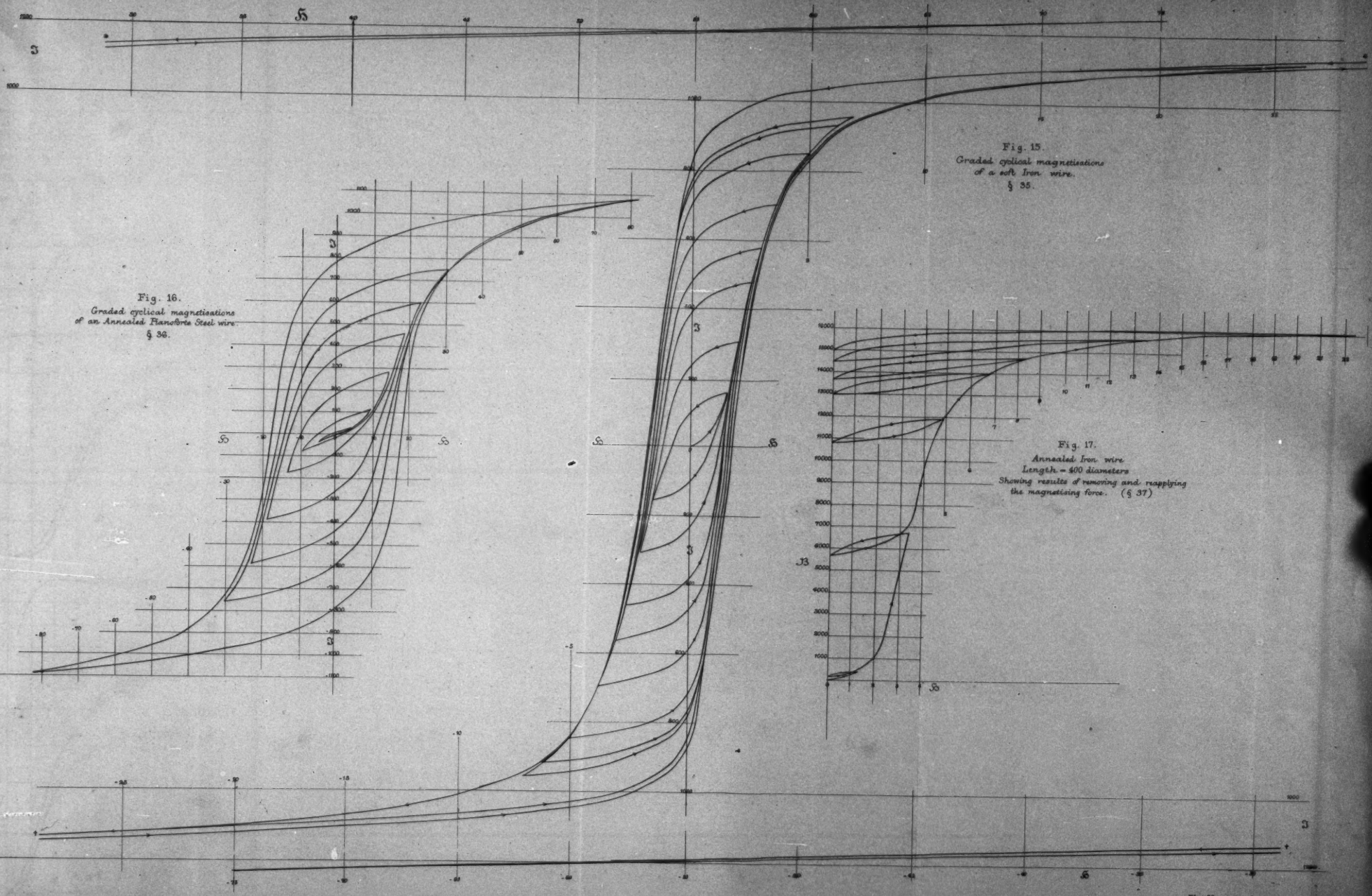


Fig. 15.
Graded cyclical magnetisations of a soft Iron wire.
§ 35.

Fig. 16.
Graded cyclical magnetisations of an Annealed Pianoforte Steel wire.
§ 36.

Fig. 17.
Annealed Iron wire.
Length = 400 diameters
Showing results of removing and reapplying the magnetising force. (§ 37)

West, Newman & Co. lith.

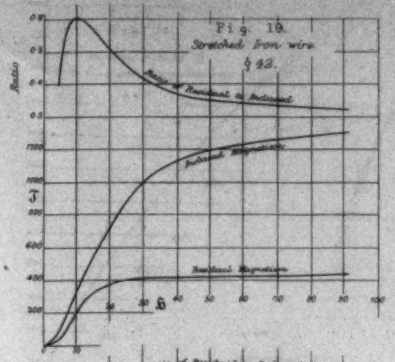


Fig. 19.
Stretched Iron wire.
§ 42.

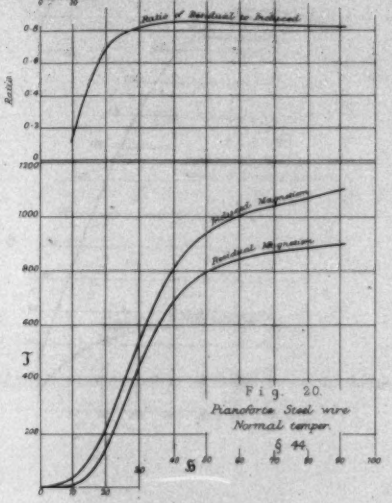


Fig. 20.
Pianoforte Steel wire
Normal temper.
§ 44.

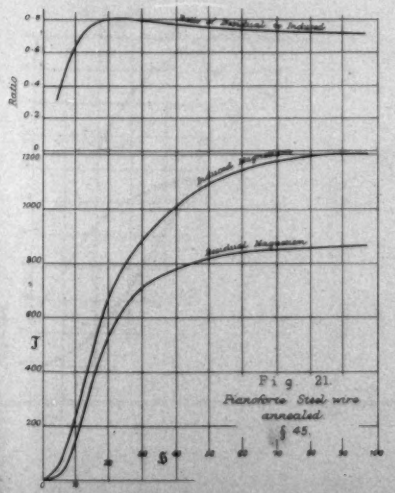


Fig. 21.
Pianoforte Steel wire
annealed.
§ 45.

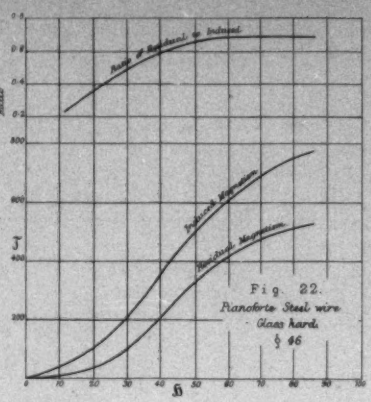


Fig. 22.
Pianoforte Steel wire
Glass hard.
§ 46.

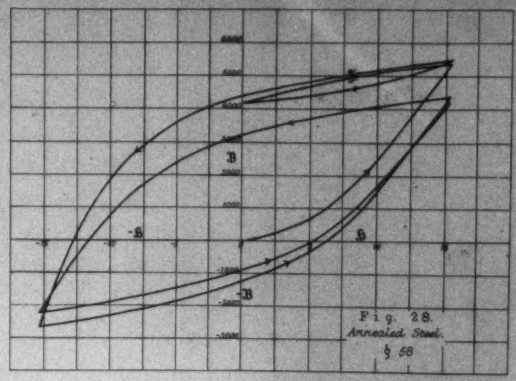


Fig. 28.
Annealed Steel.
§ 58.

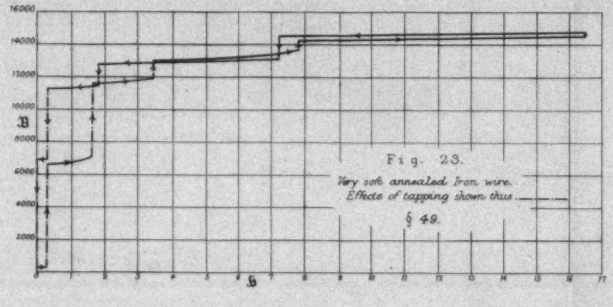


Fig. 23.
Very soft annealed Iron wire.
Effects of tapping shown thus ——
§ 49.

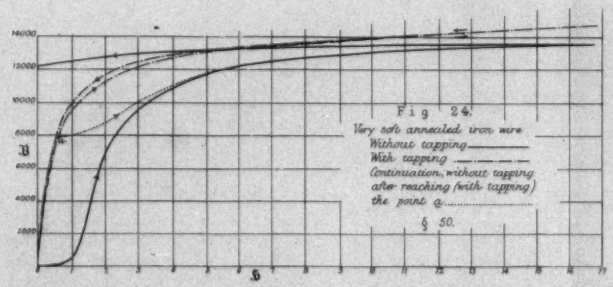


Fig. 24.
Very soft annealed iron wire
Without tapping ——
With tapping — — —
Continuation without tapping
after reaching (with tapping)
the point a ……
§ 50.

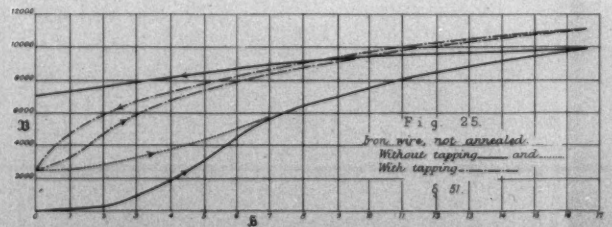


Fig. 25.
Iron wire, not annealed.
Without tapping —— *and*
With tapping — — —
§ 51.

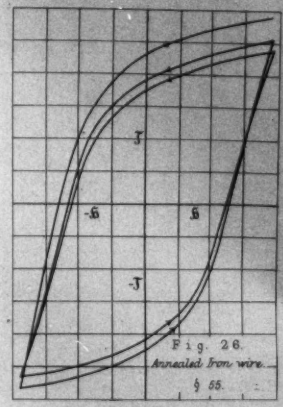


Fig. 26.
Annealed Iron wire.
§ 55.

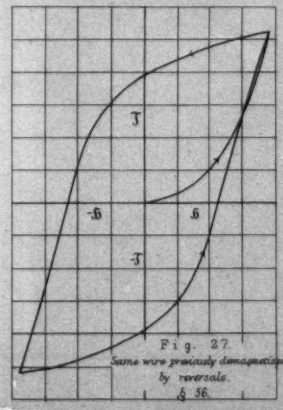


Fig. 27.
Same wire previously demagnetized
by reversals.
§ 56.

West Newman & Co. lith.

Fig. 29.
Relation of B to $\mathfrak{B}$ in two pieces of annealed iron.
§ 59.

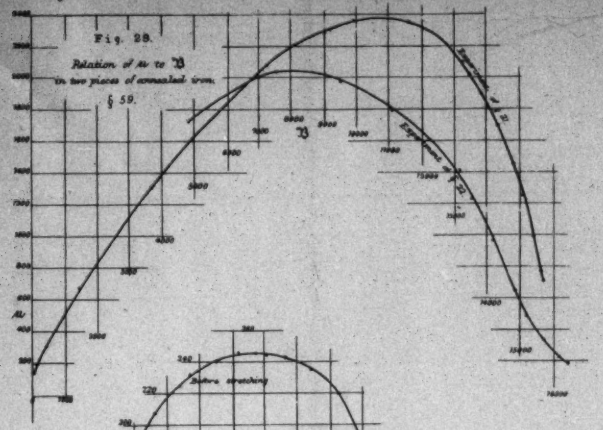


Fig. 30.
Relation of K to $\mathfrak{J}$ in Iron before and after stretching
§ 60.

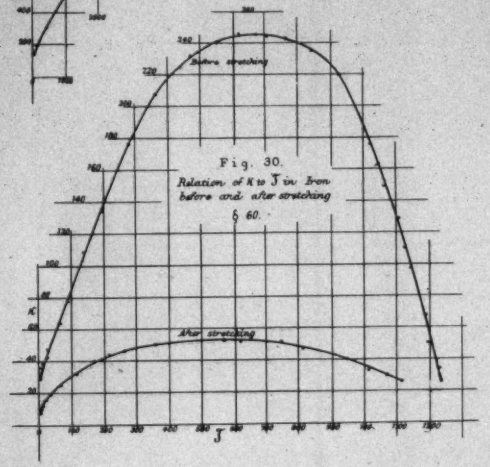
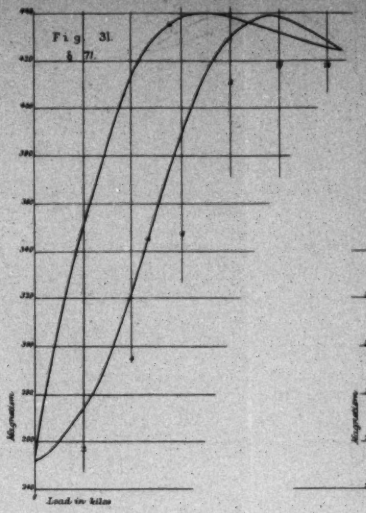


Fig. 31.
§ 71.



Effects of Stress on a Stretched Iron Wire in a Constant Magnetic Field. (0·36 c.g.s. units.)

Fig. 32.
§ 72.

Order of operations,
0-1-2-3-24-5-6-7-8-9-8-9
corresponding to
a-3-a-2-a-7-g-h-i-j-k

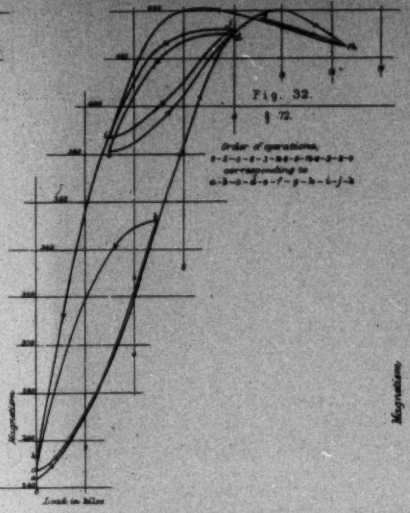


Fig. 33.
§ 73.
On and off curves with vibration.

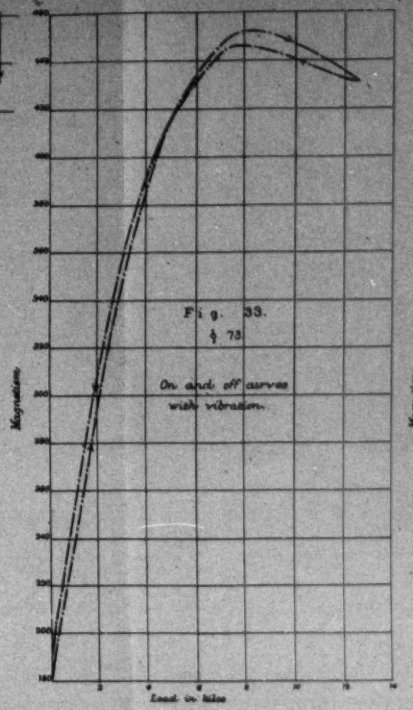


Fig. 34.
§ 74.
Effects of tapping while the load was kept constant are shown thus

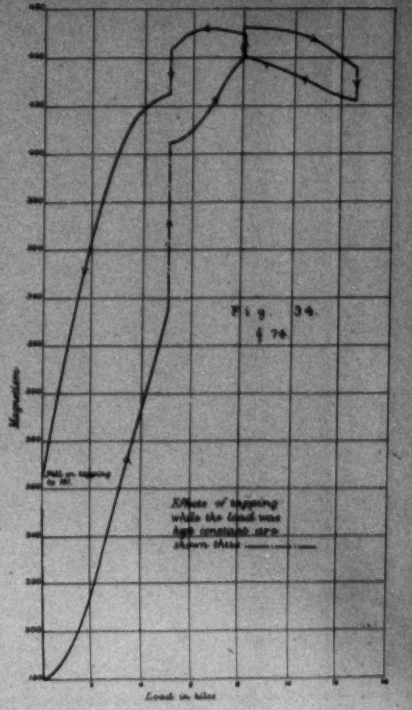
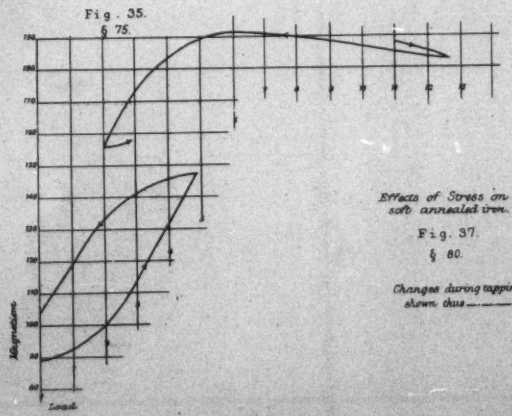


Fig. 35.
§ 75.



Effects of Stress on soft annealed iron.
Fig. 37.
§ 80.
Changes during tapping shown thus

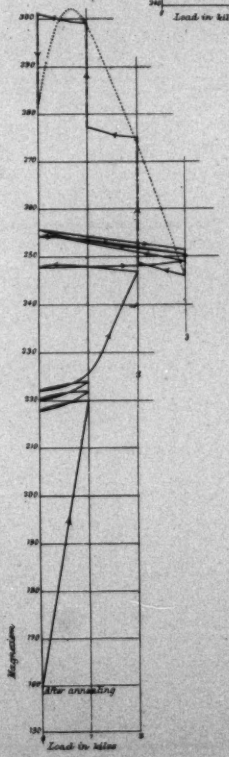
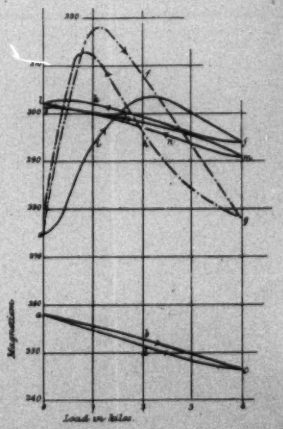
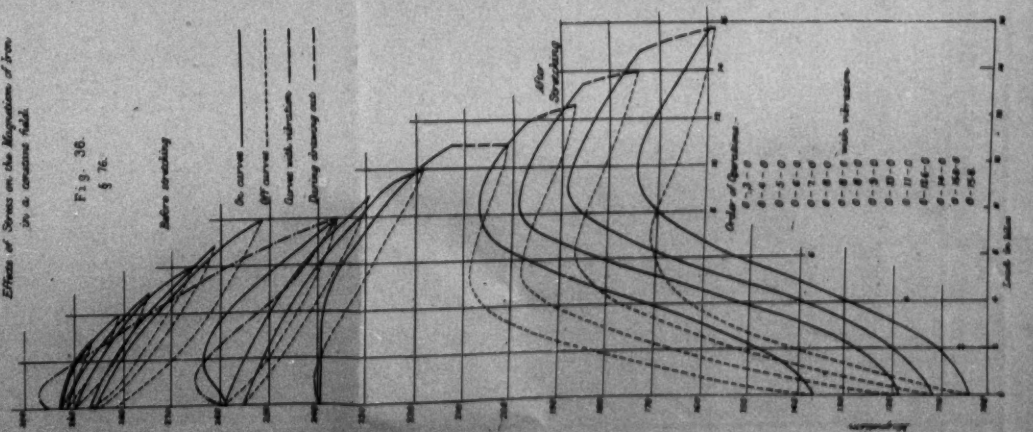


Fig. 38.
§ 82.



Effects of Stress on the Magnetism of Iron in a constant field.
Fig. 36.
§ 76.



West, Newman & Co. lith.

Phil. Trans. 1885. Plate 63.

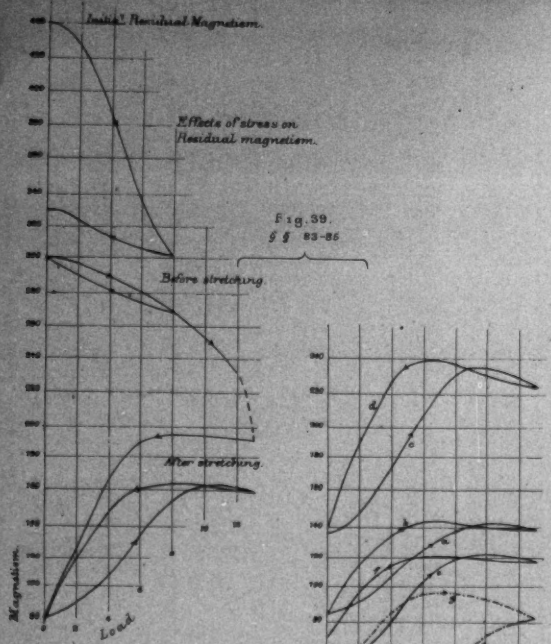
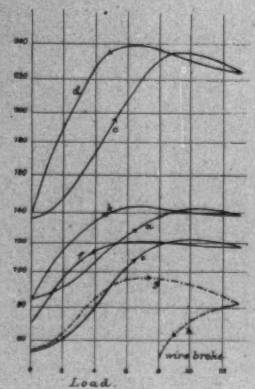
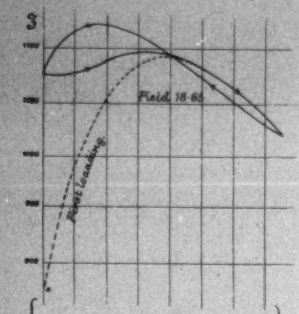


Fig. 39.
§§ 83-85





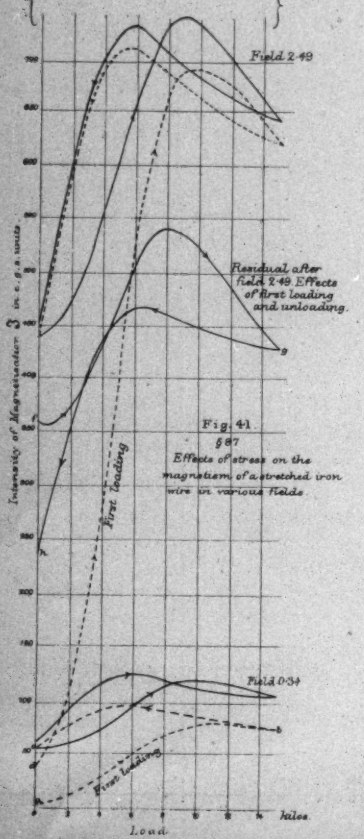
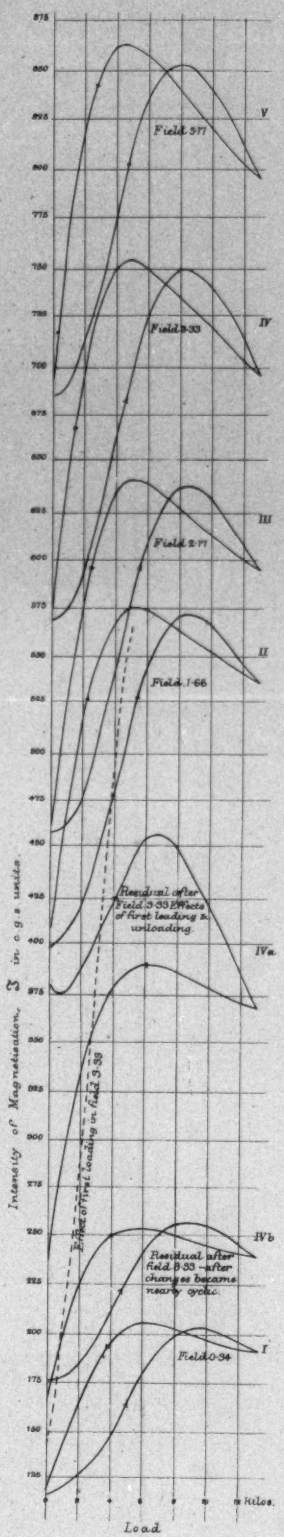


Fig. 41
§ 87
Effects of stress on the magnetism of a stretched iron wire in various fields.

Fig. 40.
§ 86
Effects of stress on the Magnetism of a stretched iron wire in various fields.



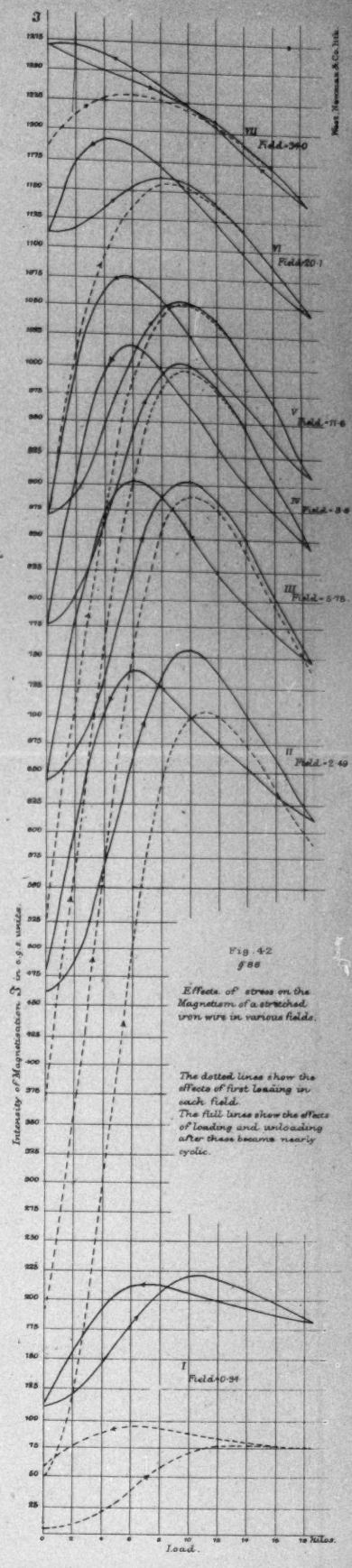


Fig. 42
§ 88
Effects of stress on the Magnetism of a stretched iron wire in various fields.

The dotted lines show the effects of first loading in each field.
The full lines show the effects of loading and unloading after these became nearly cyclic.

Ewing.

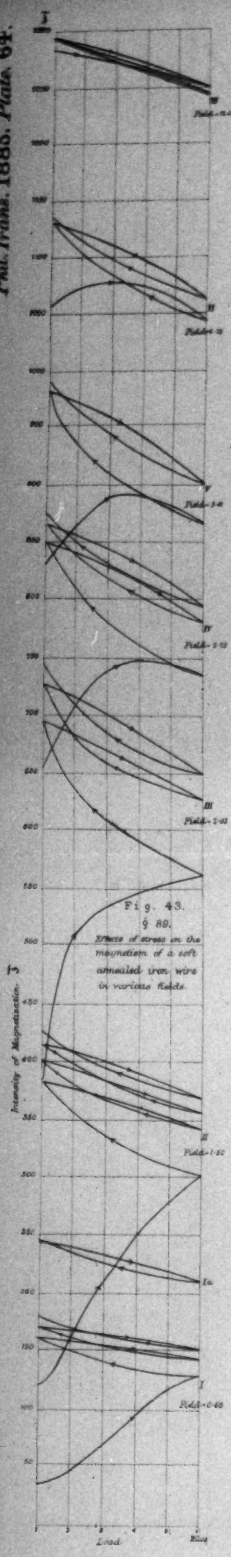


Fig. 43.
§ 89.
Effects of stress on the magnetism of a soft annealed iron wire in various fields.

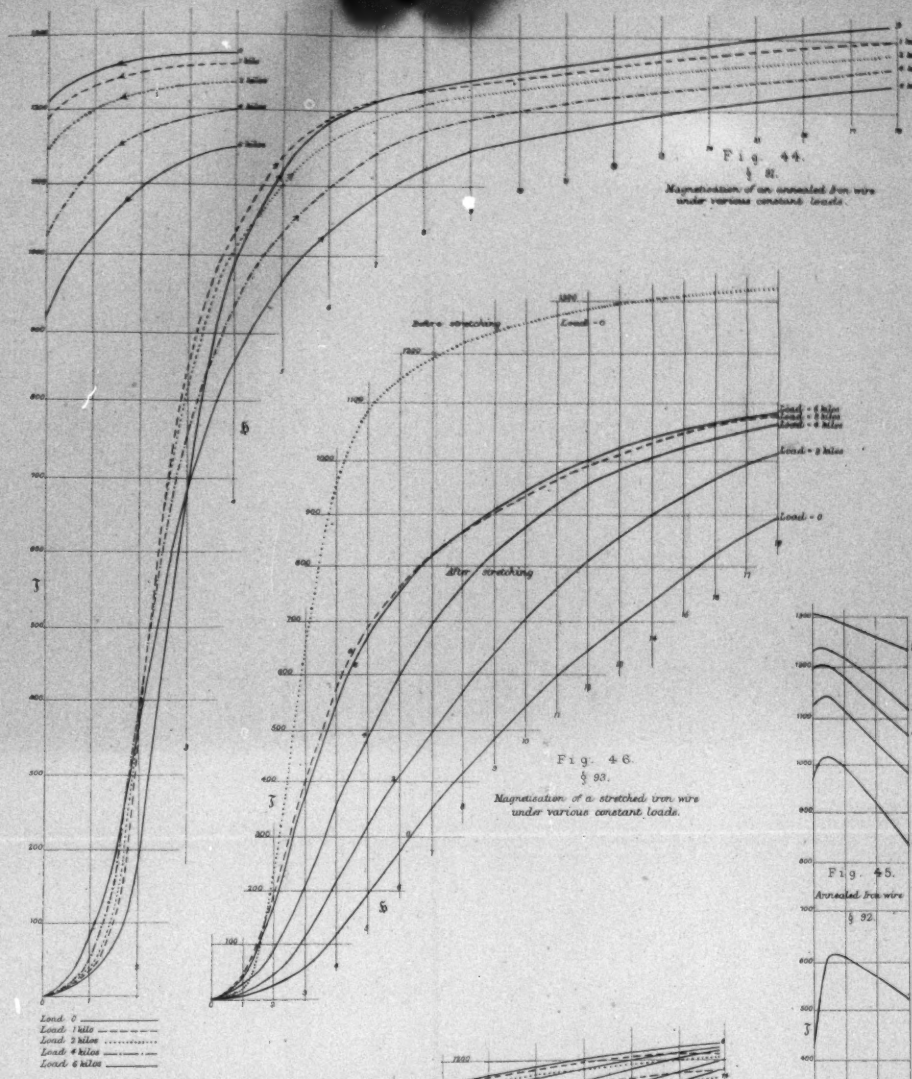


Fig. 44.
§ 81.
Magnetisation of an annealed iron wire under various constant loads.

Fig. 46.
§ 93.
Magnetisation of a stretched iron wire under various constant loads.

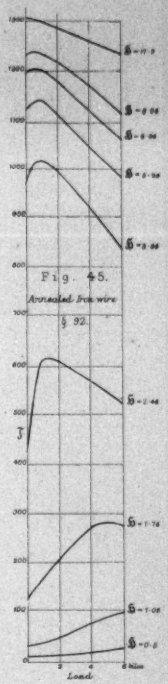


Fig. 45.
Annealed Iron wire
§ 92.

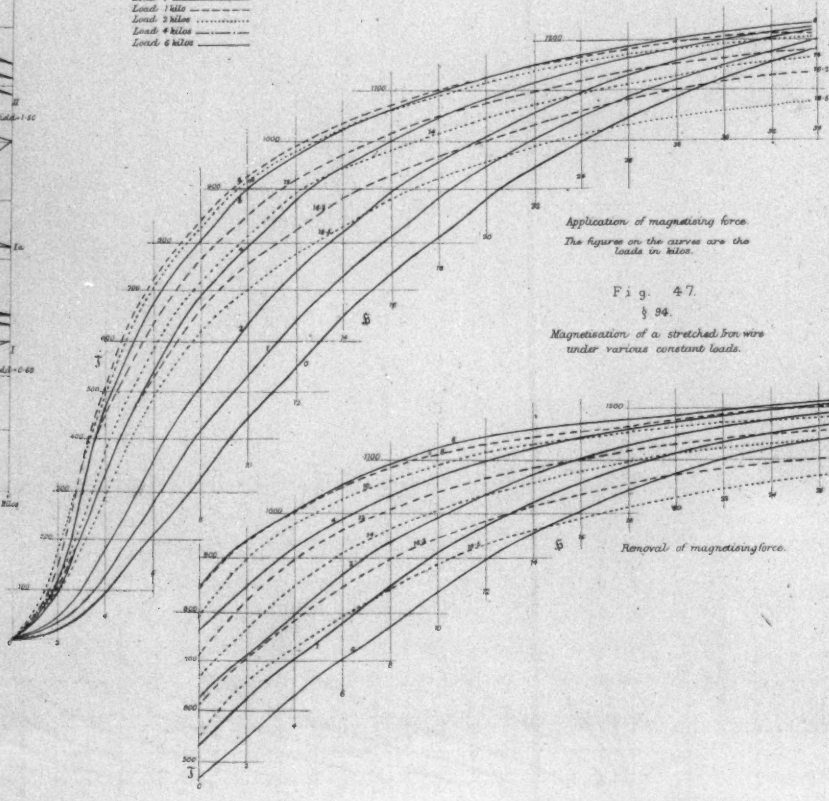
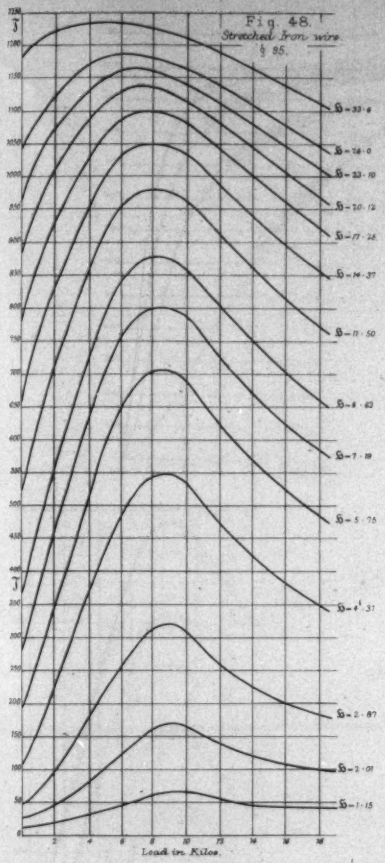


Fig. 47.
§ 94.
Magnetisation of a stretched iron wire under various constant loads.

Ewing.



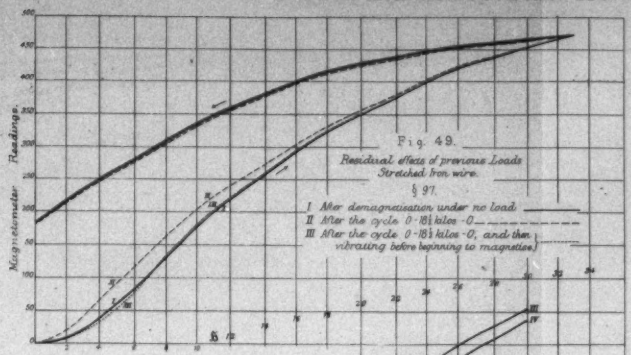


Fig. 49.
Residual effects of previous Loads
Stretched Iron wire.
§ 97.
I After demagnetisation under no load.
II After the cycle 0-18½ kilos-0
III After the cycle 0-18½ kilos-0, and then vibrating before beginning to magnetise.
Magnetometer Readings.

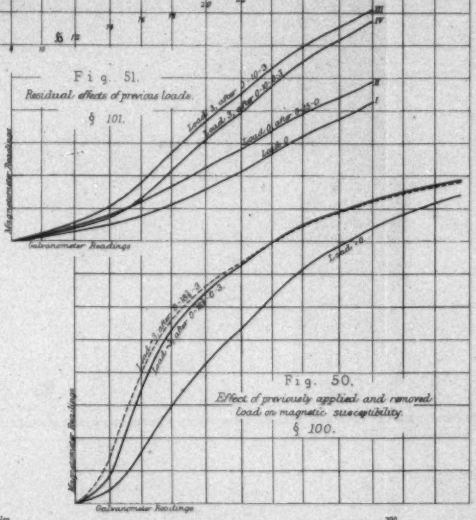


Fig. 50.
Effect of previously applied and removed load on magnetic susceptibility.
§ 100.

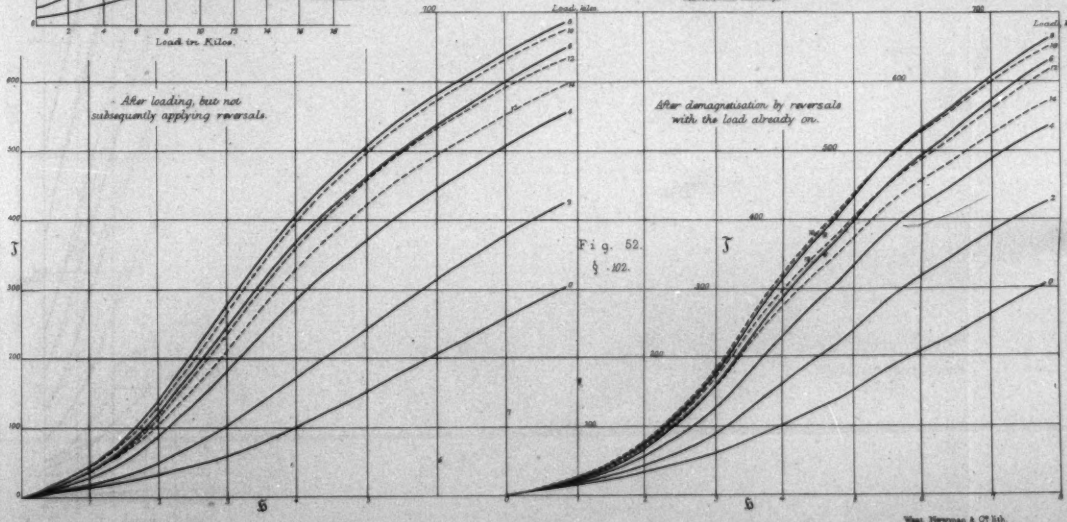

After loading, but not subsequently applying reversals.
After demagnetisation by reversals with the load already on.
Load, kilos.

Fig. 52.
§ 102.

West, Newman & Co. lith.

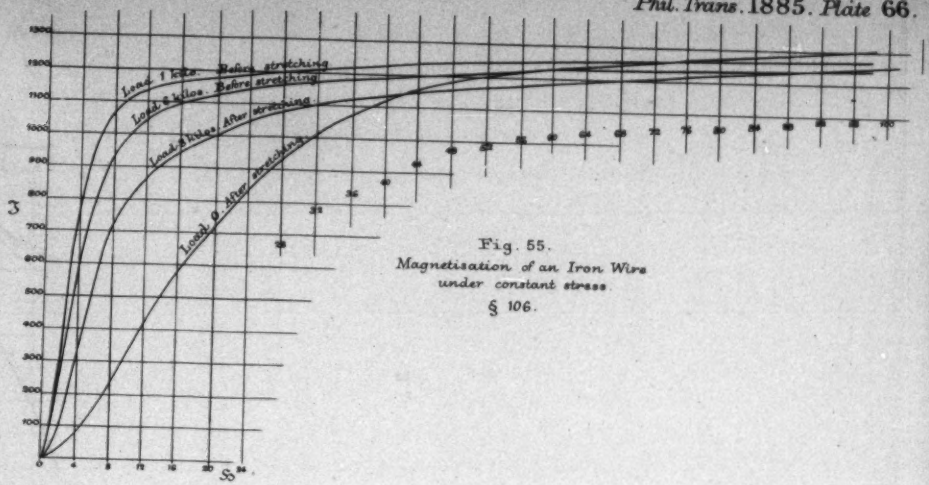


Fig. 55.
Magnetisation of an Iron Wire under constant stress.
§ 106.

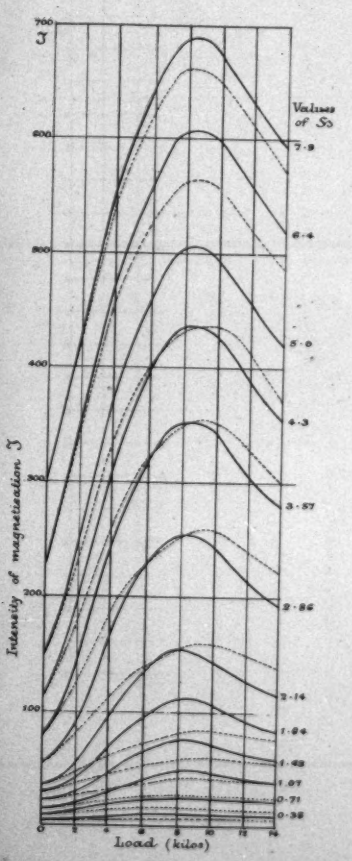


Fig. 53.
Magnetisation of a stretched Iron Wire under constant stress.
§ 103.

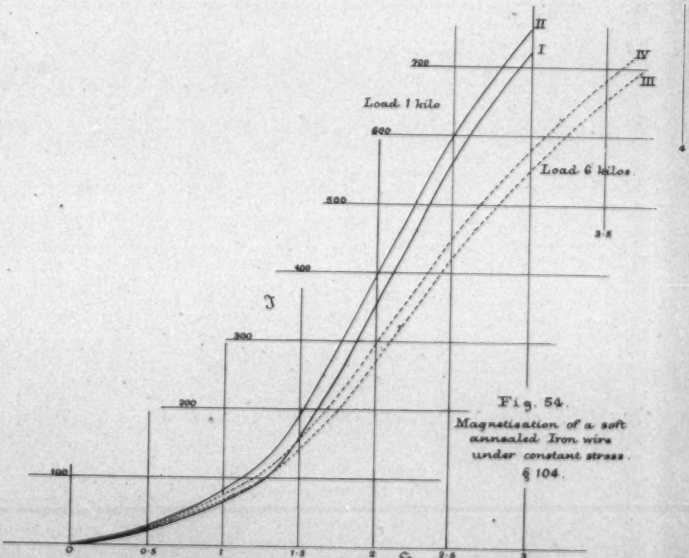


Fig. 54.
Magnetisation of a soft annealed Iron wire under constant stress.
§ 104.

West, Newman & Co. imp.

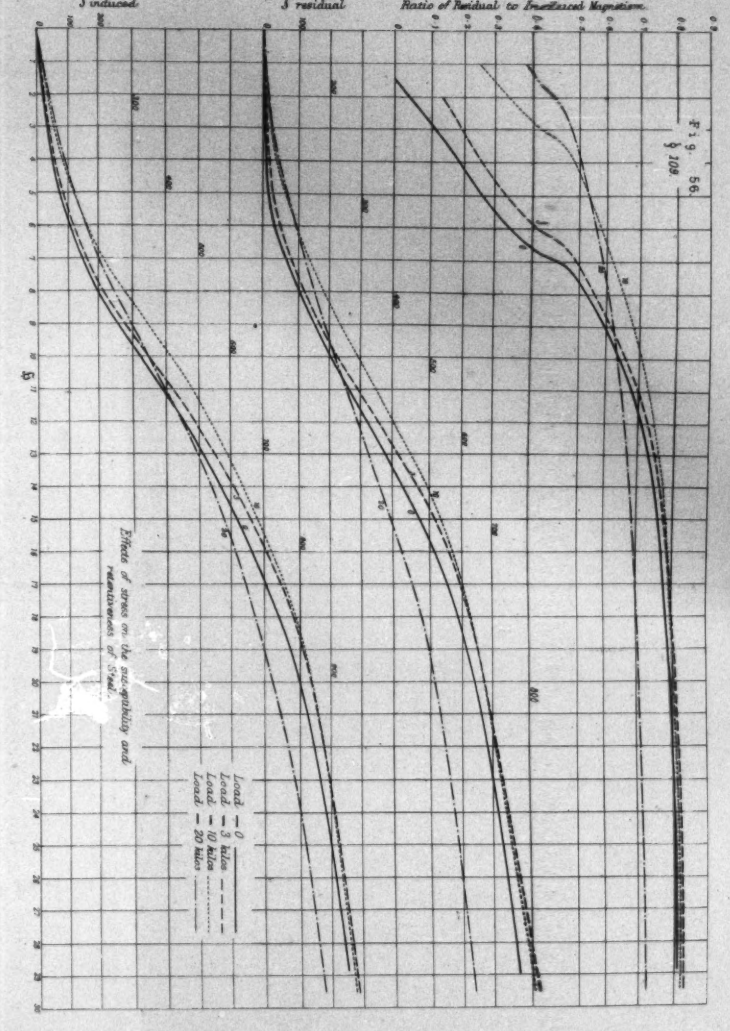

F residual
F induced
Ratio of Residual to Induced Magnetism
Effects of stress on the susceptibility and retentiveness of Steel.
Load = 0
Load = 3 kilos
Load = 10 kilos
Load = 20 kilos

Fig. 56.
§ 109

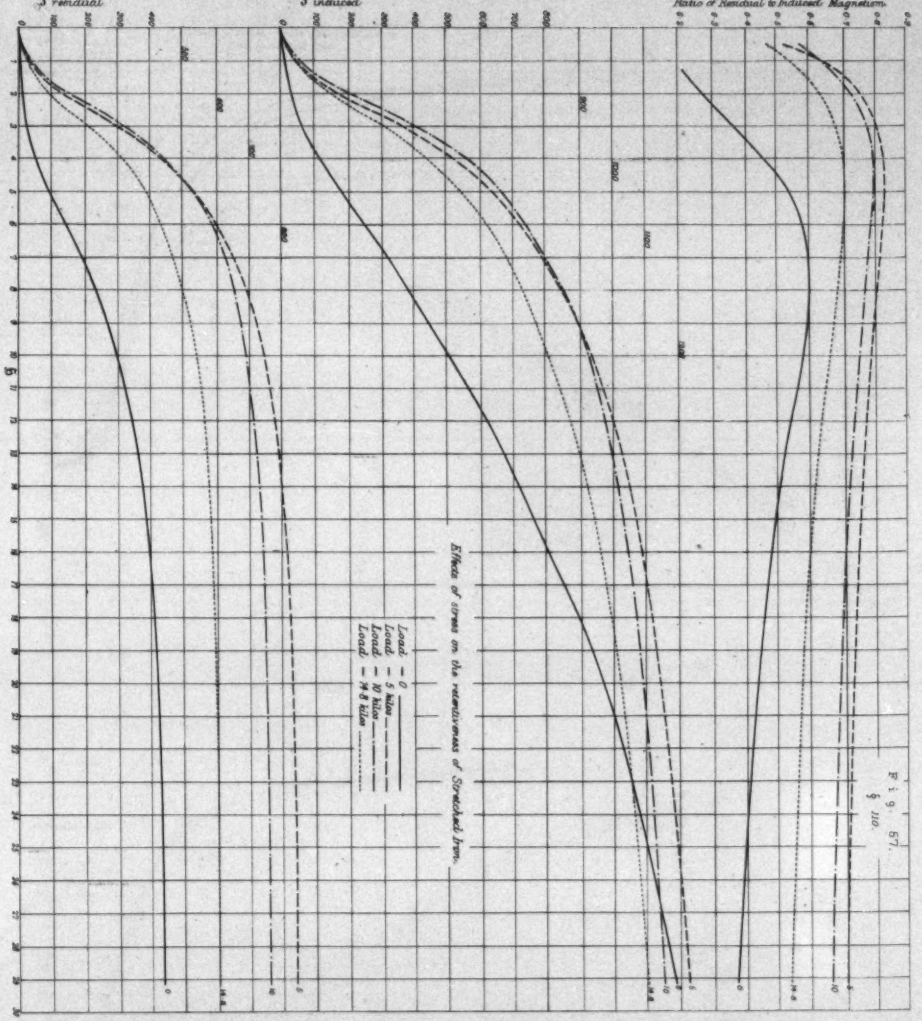

F residual
F induced
Ratio of Residual to Induced Magnetism
Effects of stress on the retentiveness of Steel.
Load = 0
Load = 5 kilos
Load = 10 kilos
Load = 14·8 kilos

Fig. 57.
§ 110.

Waterlow Newman & Co. Ltd.

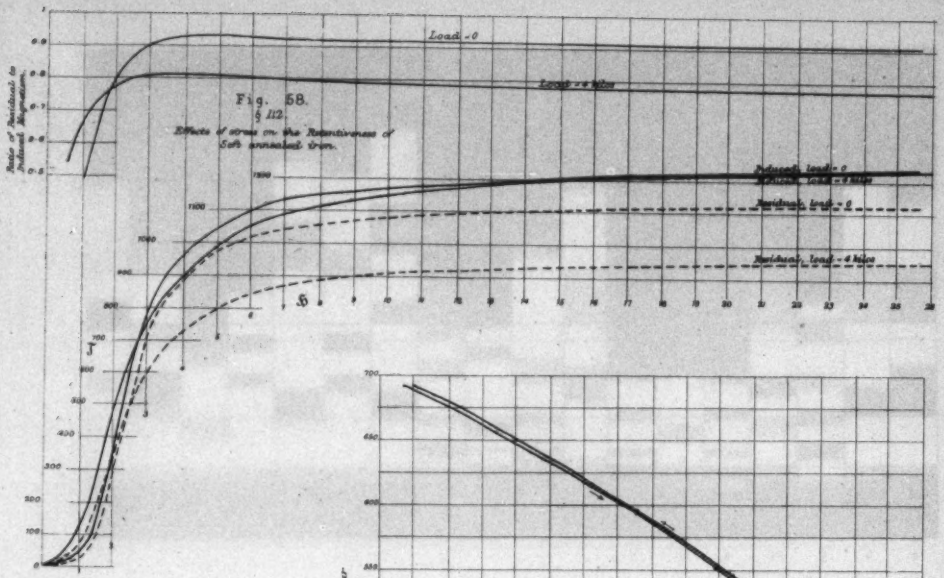


Fig. 58.

§ 112.

Effects of stress on the Retentiveness of Soft annealed Iron.

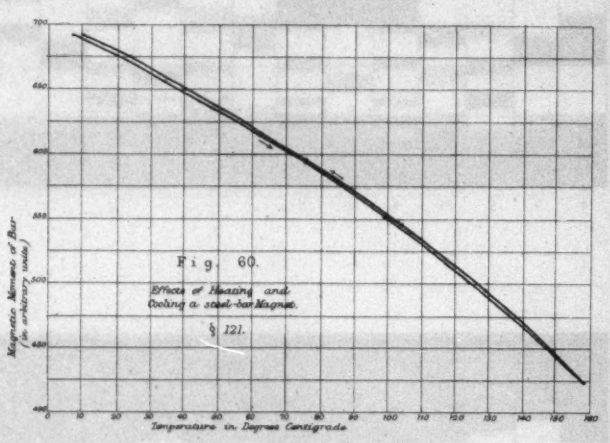


Fig. 60.

Effects of Heating and Cooling a steel-bar Magnet.

§ 121.

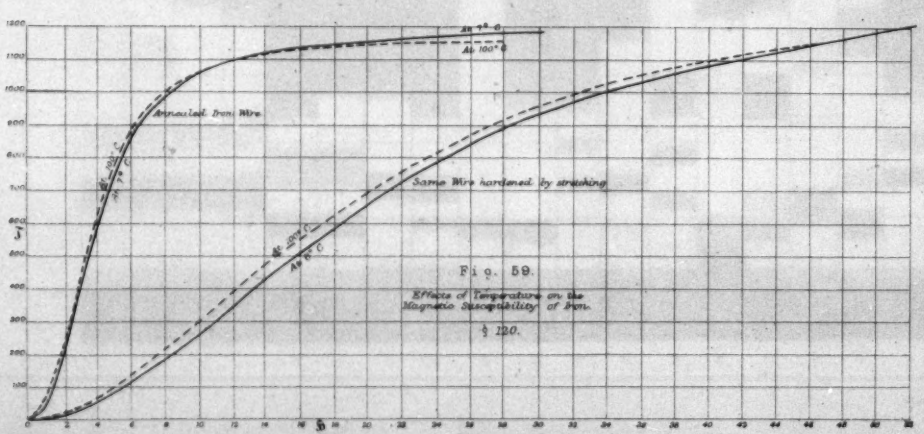
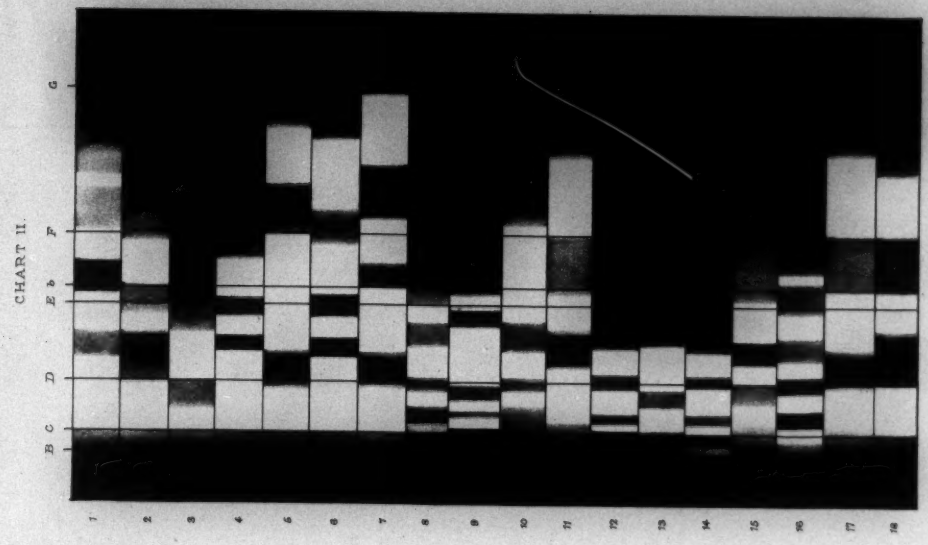


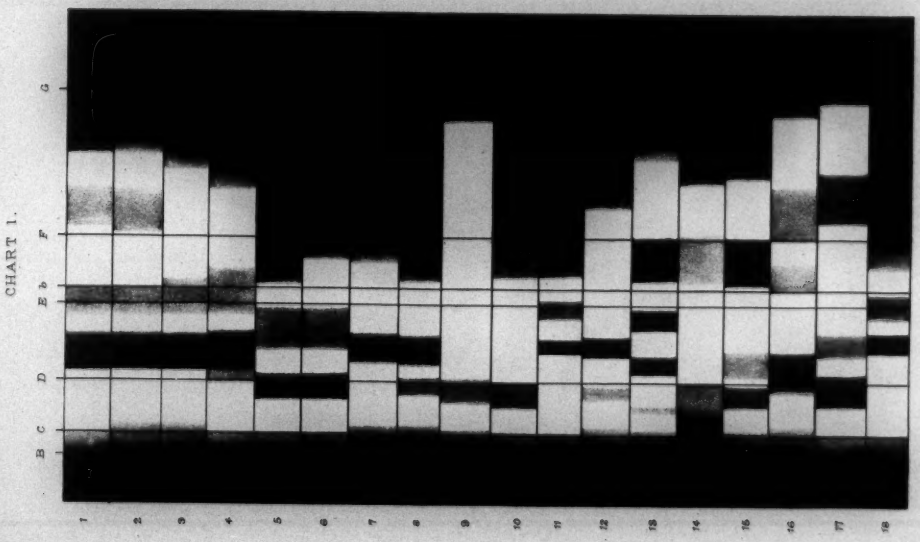
Fig. 59.

Effects of Temperature on the Magnetic Susceptibility of Iron.

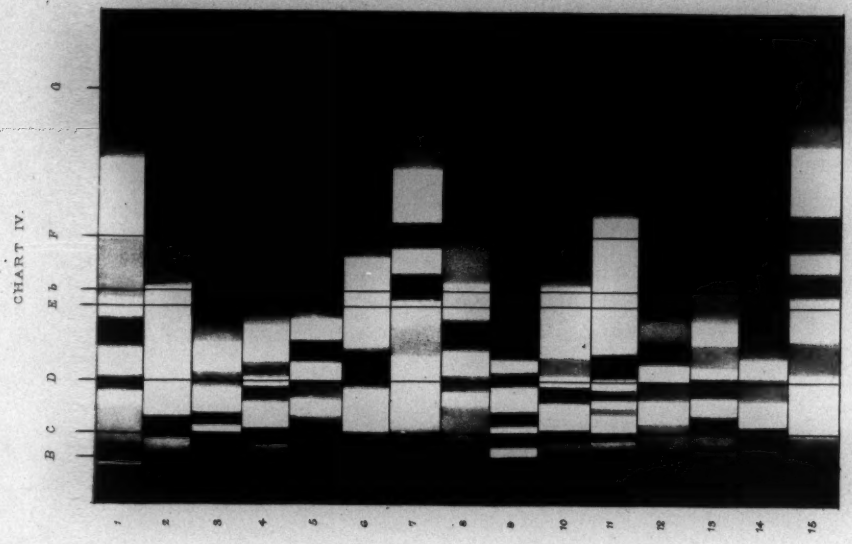
§ 120.

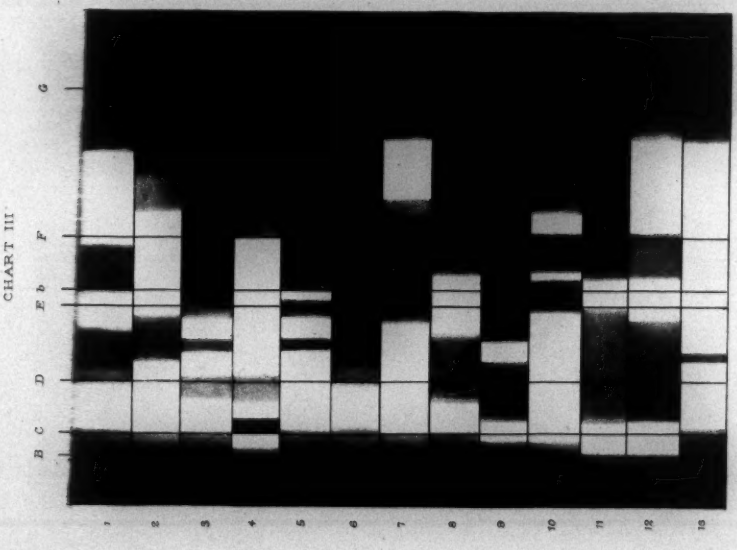
West, Newman & Co. lith.



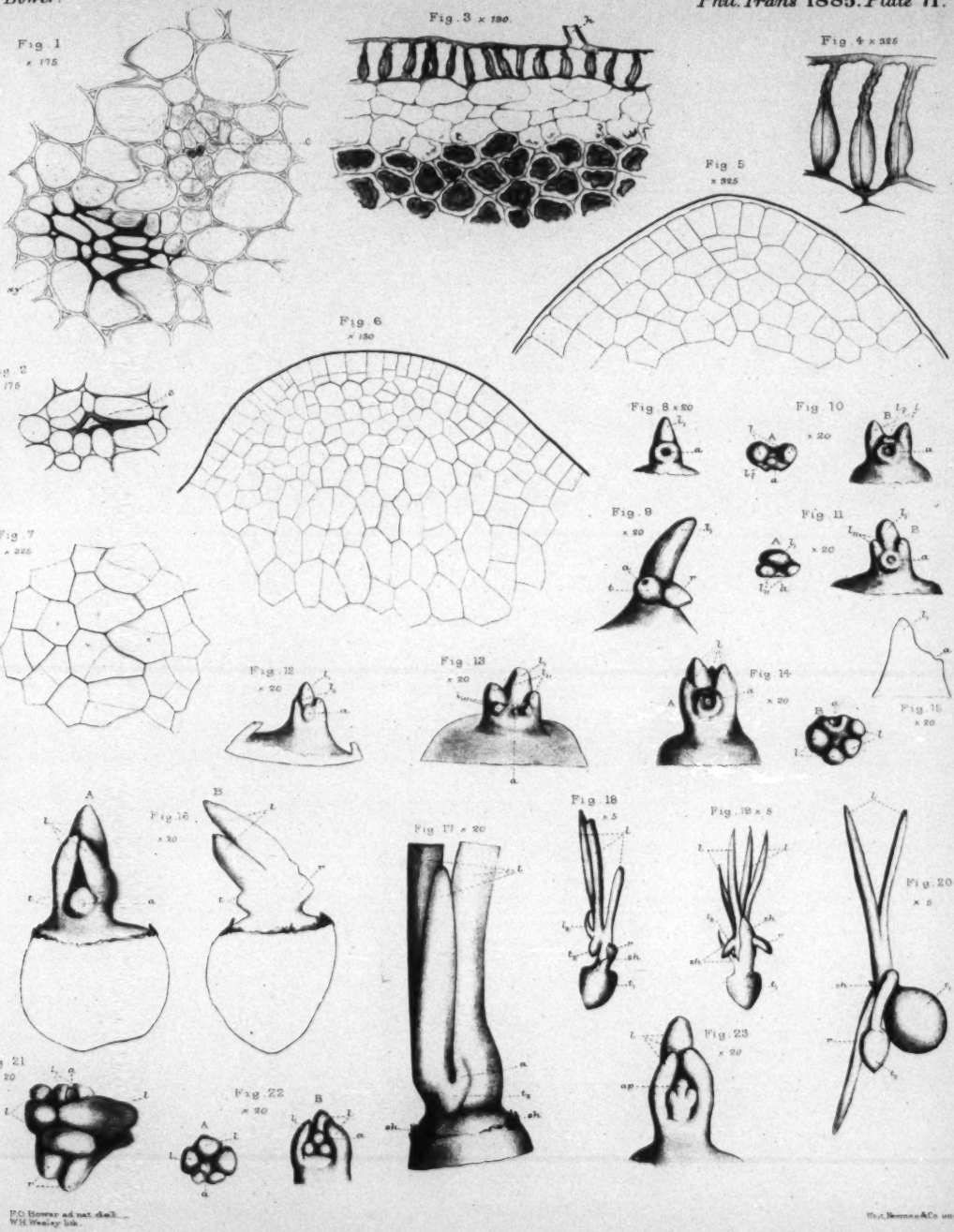


West, Newman & Co. lith.

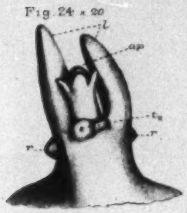


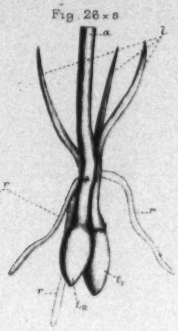


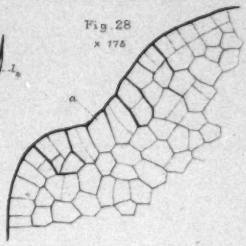
West, Newman & Co. lith.

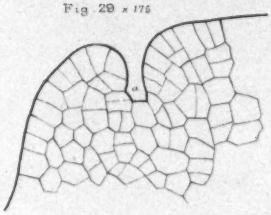


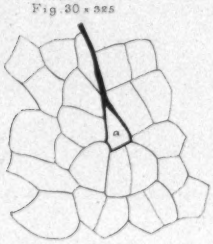
P.O. Bower ad nat. del.
W.H. Wesley lith.
Wm. Newman & Co. imp.

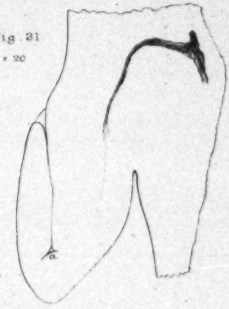


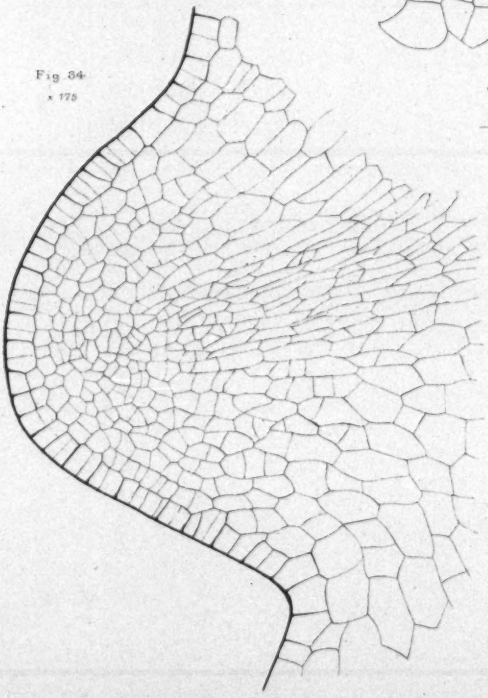


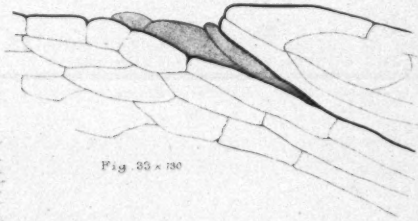


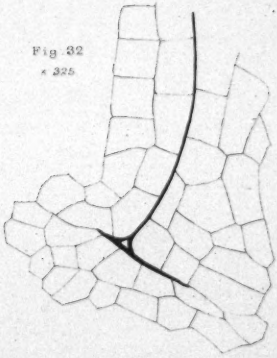












F. O. Bower ad nat. del.
W. H. Wesley lith.

West, Newman & Co. imp.

Fig. 35
× 175

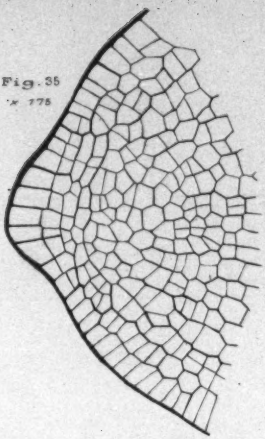


Fig. 38.
× 325

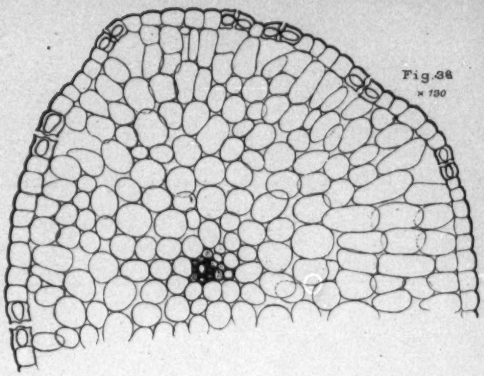


Fig. 36
× 130

Fig. 37 × 130.

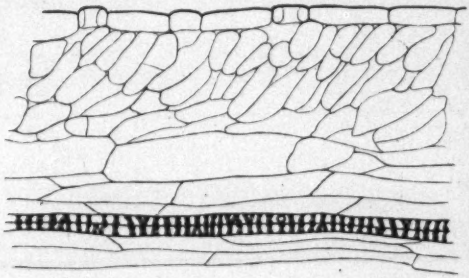
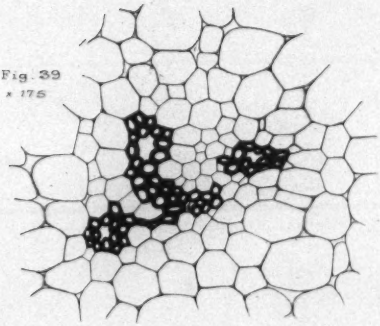


Fig. 39
× 175



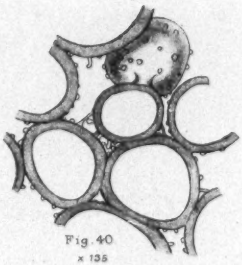
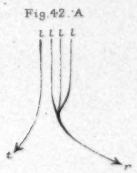


Fig. 40
× 135

Fig. 42. A



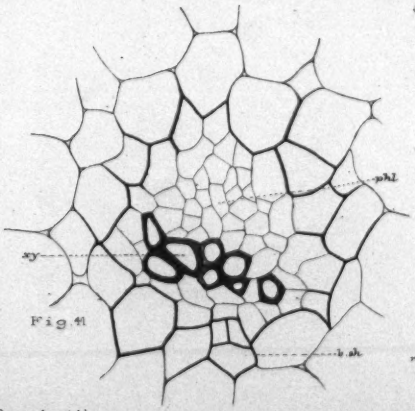
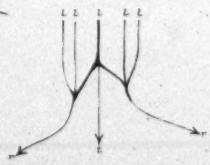


Fig. 41

Fig. 42. B



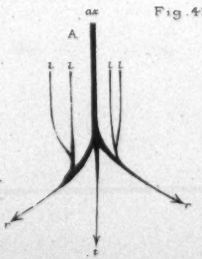
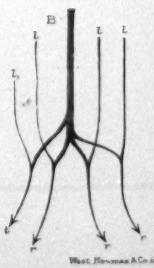


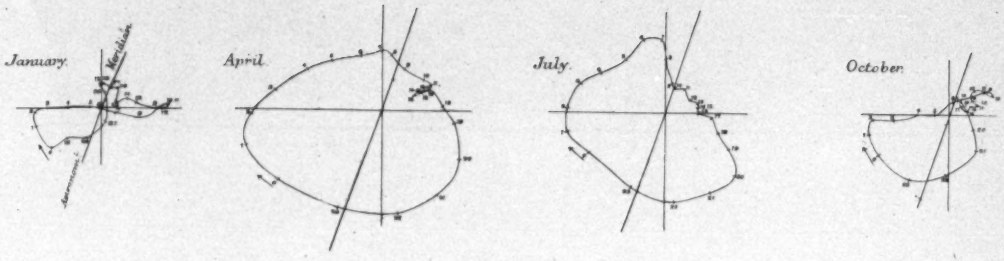
Fig. 43.



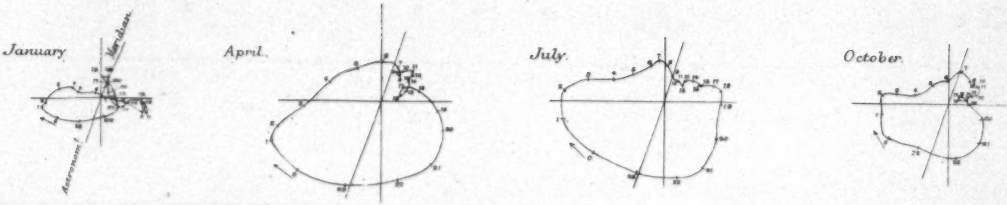
F.O.Bower, del. et lith.
W.H.Wesley, lith.

West, Newman & Co. imp.

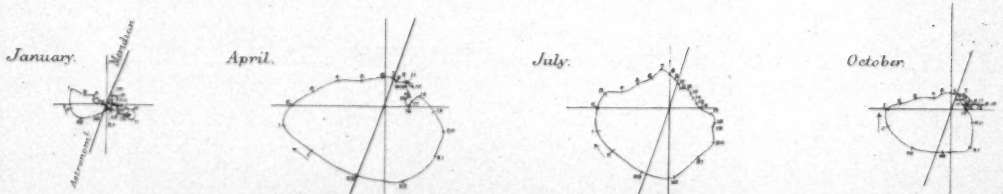
1873.



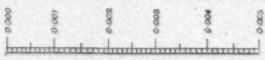
1874.



1875.



Scale in terms of Mean Horizontal Force.

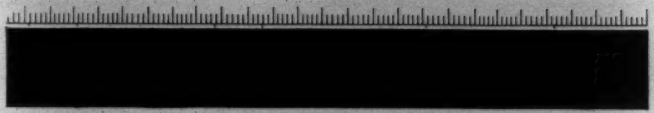


Curves representing the Diurnal Change in Magnitude and Direction of the Magnetic Forces in the Horizontal Plane acting on the North End of the Needle at the Royal Observatory, Greenwich for each of the Months January, April, July, October, in the Years 1873, 1874, 1875.

4

The Yttria Spectrum.

Vide Phil. Trans. Vol. 17. 1883. *(p. 891.)*



Sm

Fig. 3.

Sm 50
La 50

Fig. 5.

Sm 50
Pb 50

Fig. 6.

Sm 44
Yt 56

Fig. 7.

Sm 42
Yt 58

Fig. 8.

Sm 39
Yt 61

Fig. 9.

Sm 20
Ca 80

Fig. 4.

Sm 1
Ca 100.000

Fig. 18.

Sm 1
Ca 2.500.000

Fig. 21.

West, Newman & C° lith.